



**Northern Beaches Secondary College**

**Manly Selective Campus**

**2011  
HIGHER SCHOOL CERTIFICATE  
TRIAL EXAMINATION**

# Mathematics Extension 2

## **General Instructions**

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen
- Board-approved calculators and templates may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

## **Total marks – 120**

- Attempt Questions 1- 8
- All questions are of equal value

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**Marks**

**Question 1** (Answer in a separate booklet)

**15**

(a)  $\int x \sec^2(x^2) dx$  (1)

(b)  $\int \frac{dx}{x(1+x^2)}$  (3)

(c) Use the substitution  $t = \tan \frac{x}{2}$  calculate  $\int_0^{\frac{\pi}{2}} \frac{dx}{5 + 3 \sin x + 4 \cos x}$  (4)

(d)  $\int \frac{dx}{\sqrt{x+5}\sqrt{4-x}}$  for  $x < 4$  using the substitution  $u^2 = 4 - x$ . (3)

(e)  $\int \sin(\ln x) dx$  using  $u = \ln x$  (4)

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**Marks**

**Question 2** (Answer in a separate booklet)

**15**

(a) Simplify:

(i)  $(4 + 3i)^2$  (1)

(ii)  $\frac{7 - 2i}{3 + i}$  (1)

(iii)  $2 \operatorname{cis} \frac{\pi}{6} \times 3 \operatorname{cis} \frac{\pi}{3}$  (1)

(b) Find the roots of  $z^5 - i = 0$  and sketch them on an argand diagram. (3)

(c) (i) In the same diagram, sketch the locus of both  $|z - 2| = 2$  and  $|z| = |z - 4i|$ . (2)

(ii) What is the complex number represented by the point of intersection of these two loci? (1)

(d) Let  $y = i(1 - i\sqrt{3})(\sqrt{3} + i)$

(i) Express  $y$  in cis form. (2)

(ii) Hence find  $y^3 + y^{-3}$  in the form  $A + iB$ . (2)

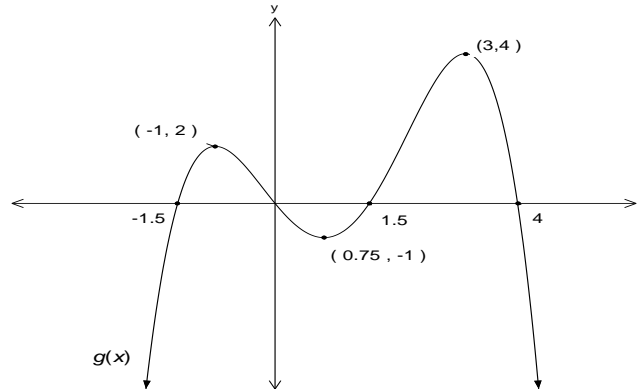
(e) Let  $z$  be a complex number of modulus 3 and  $\omega$  be a complex number of modulus 1.

Show that  $|z - \omega|^2 = 10 - (\overline{z\omega} + \bar{z}\omega)$  (2)

**Question 3** (Answer in a separate booklet)

**15**

(a) The diagram below shows the graph of  $g(x)$ .



Using this information, sketch the graphs:

(i)  $k(x) = g(|x|)$  (1)

(ii)  $t(x) = \frac{1}{g(x)}$  (1)

(iii) Graph  $f(x)$  given  $f(x) = \left(x - \frac{\pi}{2}\right) \cos x$  (3)

(b) Given the point  $P(6\cos\theta, 2\sin\theta)$  lies on an ellipse, determine the following

(i) The eccentricity. (1)

(ii) Coordinates of the foci. (1)

(iii) Equations to the directrices. (1)

(iv) Determine the gradient to the ellipse when  $\theta = \frac{2\pi}{3}$ . (2)

(c) Given the polynomial  $P(x) = 2x^3 + 3x^2 - x + 1$  has roots  $\alpha$ ,  $\beta$  and  $\gamma$ :

(i) Find the polynomial whose roots are  $\alpha^2$ ,  $\beta^2$  and  $\gamma^2$ . (2)

(ii) Determine the value of  $\alpha^3 + \beta^3 + \gamma^3$ . (3)

**Question 4** (*Answer in a separate booklet*)

**15**

(a) Factorise the polynomial  $P(x) = 3x^3 - 7x^2 + 8x - 2 = 0$  over  $\mathbb{C}$ . (2)

(b) Using the method of taking slices parallel to the  $x$ -axis, calculate the volume of the solid of revolution when the region bounded by the curve  $y = \frac{1}{2}\sqrt{x-2}$ , the  $x$ -axis and the line  $x = 6$  is rotated around the line  $x = 6$ . (4)

(c) The area enclosed between the curves  $y = (x-4)^2$  and  $y = x+2$  is rotated about the  $y$ -axis.

(i) Draw a diagram to show the area. (1)

(ii) By taking slices of the area parallel to the axis of rotation, show that the volume of the solid formed is given by  $2\pi \int_2^7 9x^2 - x^3 - 14x \, dx$  (2)

(iii) Find the volume of the solid formed. (1)

(d) A solid has as its base the region bounded by the curves  $y = x$  and  $x = 2y - \frac{y^2}{2}$ .

Cross sections parallel to the  $x$  axis are equilateral triangles with a side in the base.

Determine the volume of this solid. (5)

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**Question 5** (Answer in a separate booklet)

**15**

(a) (i) If  $I_n = \int_0^1 (1-x^2)^{\frac{n}{2}} dx$  where  $n$  is a positive integer, show that (4)

$$I_n = \frac{n}{(n+1)} I_{n-2}.$$

(ii) Hence evaluate  $I_5$ . (2)

(b) (i) The polynomial  $f(x) = x^4 - 6x^3 + 13x^2 - ax - b$  has two double zeros  $\alpha$  and  $\beta$ .  
Find the values of  $a$  and  $b$ . (4)

(ii) Hence determine, with full explanation, the equation of the line which touches the curve

$$y = x^4 - 6x^3 + 13x^2$$

at two distinct points. (1)

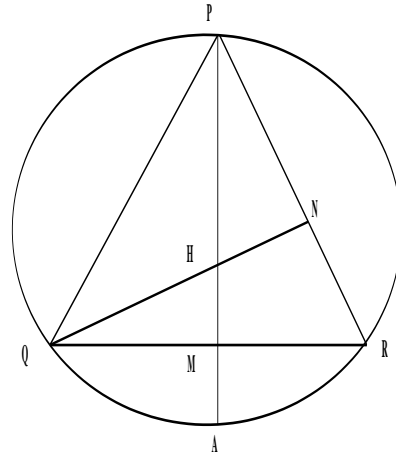
(c) If  $x + \frac{1}{x} = t$ , find  $x^6 + \frac{1}{x^6}$  in terms of  $t$ . (4)

**Question 6 (Answer in a separate booklet)**

**15**

- (a) The points P, Q and R lie on the circumference of a circle and form an acute angled triangle.

The altitudes PM and QN meet at H (which is not the centre of the circle)



(i) Prove that  $\hat{RQA} = \hat{RQN}$  (3)

(ii) Prove that  $HM = MA$  (1)

(iii) Prove that RH produced meets PQ at right angles. (2)

(b) (i) Show that  $a^2 + b^2 \geq 2ab$  where a and b are distinct positive real numbers. (1)

(ii) Hence show that  $a^2 + b^2 + c^2 \geq ab + ac + bc$ . (1)

(iii) Hence show that  $\sin^2\alpha + \cos^2\alpha \geq \sin 2\alpha$ . (2)

(iv) Hence show that  $\sin^2\alpha + \cos^2\alpha + \tan^2\alpha \geq \sin \alpha - \cos \alpha + \sec \alpha + \frac{1}{2} \sin 2\alpha$ . (2)

(c) One of the roots of the equation  $kxe^{-x} - 4 = 0$  is a double root.  
 Find the value of k. (3)

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**Marks**  
**15**

**Question 7** (Answer in a separate booklet)

(a) Determine the equation to the tangent to the curve  $x^3 + y^3 - 3x^2y^2 = 1$  at the point P (1,3). (3)

(b) A railway line has been constructed around a circular curve of radius 800metres, and is banked by raising the outer rail to a certain level above the inner rail.

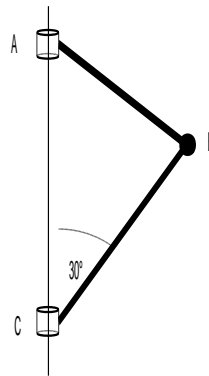
(i) When the train travels at 20m/s, the lateral thrust F, is on the outer rail. Show that

$$F = m\left(\frac{1}{2} \cos\theta - g\sin\theta\right) \text{ where } \theta \text{ is the angle of inclination.} \quad (2)$$

(ii) When the train travels at 10m/s, the lateral thrust on the inner rail is the same as the lateral thrust on the outer rail at a speed of 20m/s.

a) Find the angle of the banking. (2)

b) Find the speed of the train when there is no lateral thrust exerted on the rails. Use  $g = 9.8ms^{-2}$  (2)



(c) The above diagram shows a mass of 10 kilograms at B connected by light rods (at right angles) to sleeves A and C which revolve freely about the vertical axis AC but do not move vertically. The angle between the vertical axis AC and the light rod BC is  $30^\circ$ . The acceleration due to gravity is  $9.8 \text{ m/sec}^2$ .

(i) Given AC is 2 metres, show that the radius of the circular path of rotation of B is  $\frac{\sqrt{3}}{2}$  metres. (1)

(ii) Find the tensions in the rods AB and BC when the mass makes 90 revolutions per minute about the vertical axis. (5)



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**Question 8 (Answer in a separate booklet)**

**15**

(a) Given the ellipse  $\frac{x^2}{225} + \frac{y^2}{144} = 1$ , prove that the section of the tangent between the point of contact and its point of intersection with the directrix subtends a right angle with the corresponding focus.

(nb. The equation to the tangent to the ellipse is  $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$  and does need to be proved.)

(3)

(b) The hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  with eccentricity  $e$ , has one focus  $S$  on the positive  $x$ -axis and the corresponding directrix  $d$  cuts the asymptotes to the hyperbola at points  $P$  and  $Q$  in the first and fourth quadrants respectively.

(i) Show that  $PS$  is perpendicular to the asymptote through  $P$  and that  $PS=b$ .

(3)

(ii) A circle with centre  $S$  touches the asymptotes of the hyperbola.

Deduce that the points of contact are the points  $P$  and  $Q$ .

(1)

(iii) The circle with centre  $S$  which touches the asymptotes of the hyperbola cuts the hyperbola at points  $R$  and  $T$ . If  $b=a$ , show that  $RT$  is a diameter.

(2)

(c) When  $(1 + ax)^5 + (1 + bx)^5$  is expanded in ascending powers of  $x$ , the expansion begins

$$2 + 30x + 220x^2 + \dots$$

(i) Prove that  $(a + b) = 6$  and  $(a^2 + b^2) = 22$

(2)

(ii) Deduce the value of  $ab$ .

(2)

(iii) Determine the value of the coefficient of  $x^3$ .

(2)

**END OF EXAMINATION**

**STANDARD INTEGRALS**

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

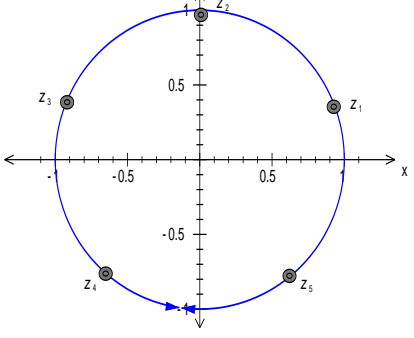
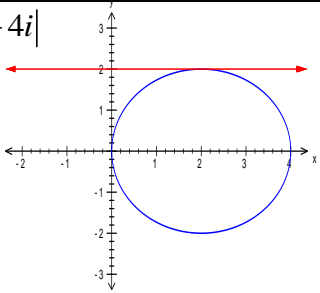
NOTE :  $\ln x = \log_e x, \quad x > 0$

**Question 1:**

a)	$\int x \sec^2(x^2).dx$ <p style="margin-left: 20px;">Let <math>u = x^2</math>  <math>\therefore du = 2x.dx</math>  <math>\therefore I_0 = \frac{1}{2} \int \sec^2 u .du</math>  <math>= \frac{1}{2} \tan u + c = \frac{1}{2} \tan x^2 + c</math></p>	1 mark – correct answer
b)	$\int \frac{dx}{x(1+x^2)}$ <p style="margin-left: 20px;">Let <math>\frac{1}{x(1+x^2)} = \frac{A}{x} + \frac{Bx+C}{1+x^2}</math>  <math>\therefore 1 = A(1+x^2) + x(Bx+C)</math>  Sub <math>x=0</math>: <math>1 = A+0 \rightarrow A=1</math>  Equate <math>x^2</math>: <math>0 = A+B \rightarrow B=-1</math>  Equate <math>x</math>: <math>0 = C \rightarrow C=0</math>  <math>\therefore \int \frac{dx}{x(1+x^2)} = \int \left( \frac{1}{x} - \frac{x}{x^2+1} \right).dx</math>  <math>= \ln x - \frac{1}{2} \ln(x^2+1) + c</math></p>	<p>1 mark for setting up partial fractions</p> <p>1 mark for finding constants</p> <p>1 mark for answer</p> <p><i>Comment: Many students gained the first 2 marks but left the x out of the final integral</i></p>
c)	$\int_0^{\frac{\pi}{2}} \frac{dx}{5+3\sin x+4\cos x}$ <p style="margin-left: 20px;">Let <math>t = \tan \frac{x}{2}</math>                      At <math>x=0</math>, <math>t=0</math>  <math>\therefore x = 2 \tan^{-1} t</math>,                      <math>x = \frac{\pi}{2}</math>, <math>t=1</math>  and <math>dx = \frac{2dt}{1+t^2}</math>, <math>\sin x = \frac{2t}{1+t^2}</math>, <math>\cos x = \frac{1-t^2}{1+t^2}</math>  <math>\therefore I = \int_0^1 \frac{\frac{2dt}{1+t^2}}{5+3\left(\frac{2t}{1+t^2}\right)+4\left(\frac{1-t^2}{1+t^2}\right)}</math>  <math>= \int_0^1 \frac{2dt}{5+5t^2+6t+4-4t^2}</math>  <math>= \int_0^1 \frac{2dt}{(t+3)^2}</math>  <math>= -2 \left[ \frac{1}{t+3} \right]_0^1 = 2 \left[ \frac{1}{t+3} \right]_1^0 = 2 \left( \frac{1}{3} - \frac{1}{4} \right) = \frac{1}{6}</math></p>	<p>1 mark – sub in correct expressions</p> <p>1 mark – correct integral</p> <p>1 mark – integration</p> <p>1 mark - substitution</p> <p><i>Comment: many students lost the 2 next to the dt</i></p>

d)	$\int \frac{dx}{\sqrt{x+5}\sqrt{4-x}} \quad \text{Let } u^2 = 4-x \rightarrow 2u \cdot du = -dx$ $\text{and } x = 4 - u^2$ $\therefore I = \int \frac{-2du}{\sqrt{9-u^2} \cdot \sqrt{u^2}} = -\int \frac{2du}{\sqrt{9-u^2}}$ $= -2 \sin^{-1}\left(\frac{u}{3}\right) + c = -2 \sin^{-1}\left(\frac{\sqrt{4-x}}{3}\right) + c$ <p>Or <math display="block">= 2 \cos^{-1}\left(\frac{u}{3}\right) + c = 2 \cos^{-1}\left(\frac{\sqrt{4-x}}{3}\right) + c</math></p>	<p><i>1 mark – correct substitution</i></p> <p><i>1 mark – set up correct integral</i></p> <p><i>1 mark – correct answer</i></p> <p><i>Comment: there was mixing up of u's and x's</i></p>
e)	$\int \sin(\ln x) dx \quad \text{Let } u = \ln x \rightarrow x = e^u \text{ and } dx = e^u du$ $\therefore I = \int \sin(u) \cdot e^u \cdot du$ <p>Let <math>U = e^u</math>, <math>dV = \sin u \cdot du</math></p> $\therefore dU = e^u du, \quad V = -\cos u$ $\therefore \int \sin(u) \cdot e^u \cdot du = -e^u \cos u + \int e^u \cos u \cdot du$ <p>Let <math>U = e^u</math>, <math>dV = \cos u \cdot du</math></p> $\therefore dU = e^u du, \quad V = \sin u$ $\therefore \int \sin(u) \cdot e^u \cdot du = -e^u \cos u + e^u \sin u - \int e^u \sin u \cdot du$ $\therefore 2 \int e^u \sin u \cdot du = -e^u \cos u + e^u \sin u$ $\therefore \int e^u \sin u \cdot du = \frac{1}{2}(e^u \sin u - e^u \cos u) + c$ <p>i.e <math display="block">\int \sin(\ln x) dx = \frac{x}{2}(\sin(\ln x) - \cos(\ln x)) + c</math></p>	<p><i>1 mark for correct substitution</i></p> <p><i>1 mark for correct use in integration by parts</i></p> <p><i>1 mark for Int by Parts correct answer</i></p> <p><i>1 marks for final answer</i></p> <p><i>Comment: Setting out was often not clear</i></p>

Question 2

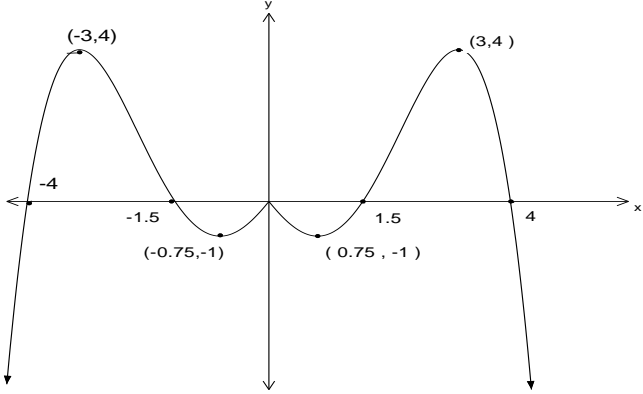
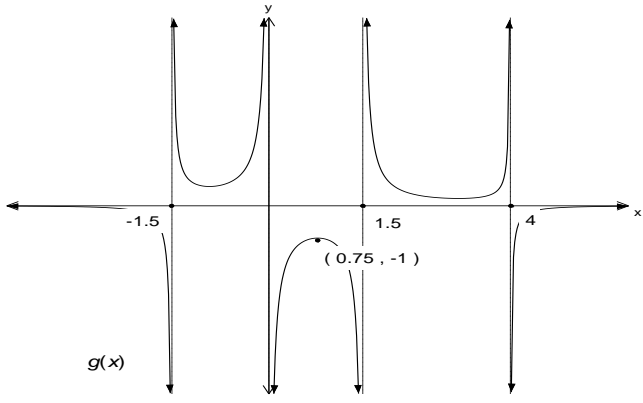
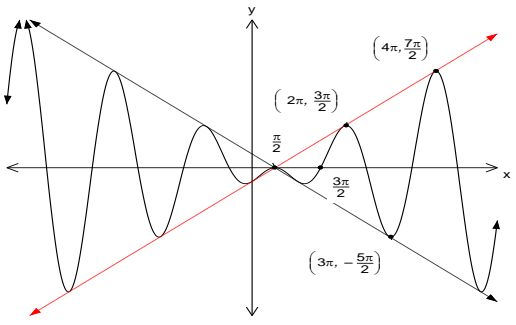
2ai)	$(4+3i)^2 = 16+24i-9=7+24i$	1 mark for correct answer
ii)	$\frac{7-2i}{3+i} = \frac{7-2i}{3+i} \times \frac{3-i}{3-i} = \frac{21-6i-7i-2}{9+1} = \frac{19-13i}{10}$	1 mark for correct answer
iii)	$2cis\frac{\pi}{3} \times 3cis\frac{\pi}{6} = 6cis\frac{\pi}{2} = 6i$	1 mark for correct answer
b)	<p><math>z^5 - i = 0 \rightarrow z^5 = i</math>          Let <math>z = \cos\theta + i\sin\theta</math>  <math>\therefore z^5 = \cos 5\theta + i\sin 5\theta = i</math>          Equate real and imaginary parts:  <math>\cos 5\theta = 0</math> and <math>\sin 5\theta = 1</math>  <math>\therefore 5\theta = \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}, \frac{17\pi}{2}</math>          and <math>\theta = \frac{\pi}{10}, \frac{\pi}{2}, \frac{9\pi}{10}, \frac{13\pi}{10}, \frac{17\pi}{10}</math>  <math>\therefore z_1 = cis\frac{\pi}{10}, z_2 = i, z_3 = cis\frac{9\pi}{10}, z_4 = cis\frac{13\pi}{10}, z_5 = cis\frac{17\pi}{10}</math></p> 	<p>1 mark for 1 correct value of z           1 mark for 5 correct values of z           1 mark for plotting the values, showing equal moduli and equal imaginary parts for <math>z_1, z_3</math> and <math>z_4, z_5</math>           Comment: This question was marked very generously</p>
ci)	<p><math> z-2  = 2,  z  =  z-4i </math></p> 	1 mark for each graph
ii)	The line and curve intersect at $2+2i$	1 mark for correct answer
di)	<p><math>y = i(1-i\sqrt{3})(\sqrt{3}+i)</math>  <math>= (i+\sqrt{3})(\sqrt{3}+i) = 2+2i\sqrt{3} = 4cis\frac{\pi}{3}</math></p>	<p>1 mark for correct expansion          1 mark for correct answer</p>

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ii)	$y^3 + y^{-3} = \left(4cis \frac{\pi}{3}\right)^3 + \left(4cis \frac{\pi}{3}\right)^{-3}$ $= 64cis\pi + \frac{1}{64}cis(-\pi)$ $= -(64 + \frac{1}{64})$	<p>1 mark for correct use of De Moivre's theorem</p> <p>1 mark for correct answer</p>
e)	<p>Since <math>z\bar{z} =  z ^2</math>, <math> z-w ^2 = (z-w)\overline{(z-w)}</math></p> $= (z-w)(\bar{z}-\bar{w})$ $= z\bar{z} - z\bar{w} - w\bar{z} + w\bar{w}$ $= 9 - (z\bar{w} + w\bar{z}) + 1 = 10 - (z\bar{w} + w\bar{z})$	<p>1 mark for knowledge of conjugates</p> <p>1 mark for correct answer</p> <p>Comment: There was only one correct response</p>

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**Question 3:**

<p>(a)i</p>		<p><i>1 mark – correct answer clearly showing symmetry.</i></p>
<p>(a)ii</p>		<p><i>1 mark – correct answer including showing asymptotes</i></p>
<p>(a)iii</p>		<p><i>3 marks – correct answer clearly showing shape (eg. Domain approx. <math>-4\pi \leq x \leq 4\pi</math> or greater) and upper and lower asymptotes or some indication of max/min values</i></p> <p><i>2 marks – clearly showing shape (eg. Domain approx. <math>-4\pi \leq x \leq 4\pi</math> or greater)</i></p> <p><i>1 mark – significant process towards demonstrating shape.</i></p>

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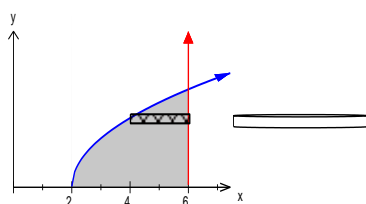
<b>b-(i)</b>	$P(6\cos\theta, 2\sin\theta) \therefore \frac{x^2}{36} + \frac{y^2}{4} = 1$ $b^2 = a^2(1 - e^2)$ $e = \sqrt{1 - \frac{4}{36}}$ $= \sqrt{\frac{32}{36}} = \frac{2\sqrt{2}}{3}$	1 mark – correct answer
<b>b-(ii)</b>	$\text{Foci} = (\pm ae, 0) = (\pm 4\sqrt{2}, 0)$	1 mark – correct answer
<b>b-(iii)</b>	$\text{Directrices } x = \pm \frac{a}{e}$ $x = \pm \frac{6 \times 3}{2\sqrt{2}} = \pm \frac{9\sqrt{2}}{2}$	1 mark – correct answer
<b>b-(iv)</b>	$\frac{x^2}{36} + \frac{y^2}{4} = 1$ $\frac{2x}{36} + \frac{2y}{4} \frac{dy}{dx} = 0$ $\frac{dy}{dx} = -\frac{x}{9y}$ $\theta = \frac{2\pi}{3}$ $x = -3 \quad y = \sqrt{3}$ $m = \frac{1}{3\sqrt{3}} = \frac{\sqrt{3}}{9}$ $x = 6\cos\theta \quad y = 2\sin\theta$ $\frac{dx}{d\theta} = -6\sin\theta \quad \frac{dy}{d\theta} = 2\cos\theta$ $\frac{dy}{dx} = -\frac{1}{3}\cot\theta$ $\frac{dy}{dx} = \frac{1}{3\sqrt{3}} = \frac{\sqrt{3}}{9}$	2 mark – correct answer  1 mark – correct equation for $\frac{dy}{dx}$
<b>(c)</b> <b>ii</b>	$P(x) = 2x^3 + 3x^2 - x + 1$ $Q(x) = 2(\sqrt{x})^3 + 3(\sqrt{x})^2 - \sqrt{x} + 1$ $0 = 2(\sqrt{x})^3 + 3(\sqrt{x})^2 - \sqrt{x} + 1$ $2(\sqrt{x})^3 - \sqrt{x} = -3x - 1$ $\sqrt{x}(2x - 1) = -(3x + 1)$ $x(2x - 1)^2 = (3x + 1)^2$ $4x^3 - 4x^2 + x = 9x^2 + 6x + 1$ $Q(x) = 4x^3 - 13x^2 - 5x - 1$	2 marks – correct answer  1 mark – correct substitution of $\sqrt{x}$ .



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<p>(d) Iii</p>	$P(\alpha) = 2\alpha^3 + 3\alpha^2 - \alpha + 1 = 0$ $P(\beta) = 2\beta^3 + 3\beta^2 - \beta + 1 = 0$ $P(\gamma) = 2\gamma^3 + 3\gamma^2 - \gamma + 1 = 0$ $2(\alpha^3 + \beta^3 + \gamma^3) = -3(\alpha^2 + \beta^2 + \gamma^2) + (\alpha + \beta + \gamma) - 3$ $\alpha^2 + \beta^2 + \gamma^2 = -\frac{b}{a} = \frac{13}{4} \quad \text{from part (i)}$ $\alpha + \beta + \gamma = -\frac{3}{2}$ $2(\alpha^3 + \beta^3 + \gamma^3) = -3 \times \frac{13}{4} - \frac{3}{2} - 3$ $\alpha^3 + \beta^3 + \gamma^3 = -\frac{57}{8}$	
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Question 4

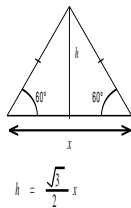
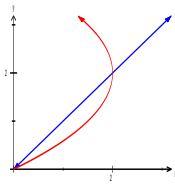
(a)	$P(x) = 3x^3 - 7x^2 + 8x - 2 = 0$ $\text{Test } \pm 1, 2, \frac{1}{3}, \frac{2}{3}$ $P\left(\frac{1}{3}\right) = 0 \therefore (3x - 1) \text{ is a factor}$ $P(x) = (3x - 1)(x^2 - 2x + 2) \text{ by inspection \&/or long division.}$ $= (3x - 1)[(x - 1)^2 + 1]$ $= (3x - 1)(x - 1 - i)(x - 1 + i)$	<p>2 marks – correct answer factorised over C.</p> <p>1 mark – correctly factorised over R.</p>
(b) (i)	 <p><b>Volume of single cylinder</b></p> $V = \pi r^2 \cdot h \quad r = 6 - x \quad h = \delta y$ $\delta V = \pi(6 - x)^2 \cdot \delta y$ $y = \frac{1}{2}\sqrt{x - 2}$ $x = 4y^2 + 2$ $\delta V = \pi(4 - 4y^2)^2 \cdot \delta y$ $= 16\pi(1 - y^2)^2 \cdot \delta y$ <p><b>Approximate volume of shape.</b></p> $V \cong \lim_{\delta y \rightarrow 0} \sum_0^1 16\pi(1 - y^2)^2 \cdot \delta y$ $V = 16\pi \int_0^1 (1 - 2y^2 + y^4) \cdot dy$ $= 16\pi \left[ y - \frac{2y^3}{3} + \frac{y^5}{5} \right]_0^1$ $= \frac{128}{12} \pi \text{ units}^3$	<p>4 marks – correct answer fully explained.</p> <p>3 marks – Correct integration for correct volume expression</p> <p>2 mark – approximate volume showing <math>\Sigma</math> notation</p> <p><u>(nb <math>\Sigma</math> notation was specifically required to be demonstrated in this solution.)</u></p> <p>1 mark – formula for volume of single cylinder ie. <math>\delta V</math></p>

**Question 4 (continued)**

<p>c-(i)</p>		<p>1 mark – correct diagram including intercepts.</p>
<p>c-(ii)</p>	$\delta V = 2\pi r \cdot h \cdot \delta x \quad r = x \quad h = (x + 2) - (x - 4)^2$ $\therefore \delta V = 2\pi \cdot x \cdot [(x + 2) - (x - 4)^2] \cdot \delta x$ $= 2\pi x(9x - x^2 - 14) \cdot \delta x$ $= 2\pi(9x^2 - x^3 - 14x) \cdot \delta x$ <p>Intercepts</p> $x + 2 = (x - 4)^2$ $0 = x^2 - 9x + 14$ $= (x - 2)(x - 7)$	<p>2 marks – correctly demonstrated derivation of formula</p> <p>1 mark – correct process with single error.</p>
<p>c-(iii)</p>	$V \cong \lim_{x \rightarrow 0} \sum_2^7 2\pi(9x^2 - x^3 - 14x) \cdot \delta x$ $V = 2\pi \int_2^7 (9x^2 - x^3 - 14x) \cdot dx$ $= 2\pi \left[ 3x^3 - \frac{x^4}{4} - 7x^2 \right]_2^7$ $= \frac{375\pi}{2} \text{ units}^3$	<p>1 mark – correct answer</p>

Question 4 (continued)

(d)



Intercepts

$$y = 2x - \frac{y^2}{2}$$

$$y^2 - 2y = 0$$

$$y = 0 \quad y = 2$$

Length of side =  $x_2 - x_1$

$$= 2y - \frac{y^2}{2} - y = y - \frac{y^2}{2}$$

Area of Triangle =  $\frac{1}{2} ab \sin 60^\circ$

$$= \frac{1}{2} \left( y - \frac{y^2}{2} \right)^2 \cdot \frac{\sqrt{3}}{2}$$

Volume of single triangular prism

$$V = \frac{\sqrt{3}}{4} \cdot \left( y - \frac{y^2}{2} \right)^2 \cdot \delta y$$

Volume of shape.

$$V \approx \lim_{\delta y \rightarrow 0} \sum_{y=0}^2 \frac{\sqrt{3}}{4} \left( y - \frac{y^2}{2} \right)^2 \delta y$$

$$= \frac{\sqrt{3}}{4} \int_0^2 \left( y^2 - y^3 + \frac{y^4}{4} \right) dy$$

$$= \frac{\sqrt{3}}{4} \left[ \frac{y^3}{3} - \frac{y^4}{4} + \frac{y^5}{20} \right]_0^2$$

$$= \frac{\sqrt{3}}{15} u^3$$

5 marks – correct answer clearly demonstrated.

4 marks – Correct to final integration

3 marks – correct derivation of  $\delta V$

2 marks – correct formula in terms of  $y$  for Area of triangle.

1 mark – length of side in terms of  $y$ .

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Question 5:

(a) (i)	$I_n = \int_0^1 (1-x^2)^{\frac{n}{2}} dx$ $= \left[ x(1-x^2)^{\frac{n}{2}} \right]_0^1 + n \int_0^1 x^2(1-x^2)^{\frac{n}{2}-1} dx$ $= 0 - n \int_0^1 (1-x^2+1)(1-x^2)^{\frac{n}{2}-1} dx$ $= -n \int_0^1 (1-x^2)^{\frac{n}{2}} dx + n \int_0^1 \left(\frac{1}{x}\right)^{\frac{n}{2}-1} dx$ $= -nI_n + nI_{n-2}$ $= \frac{nI_{n-2}}{n+1}$	<p>4 marks – correct proof</p> <p>3 marks – error in manipulation of separate parts</p> <p>2 marks – incorporation of transformation in step 2</p> <p>1 mark – correct integration by parts</p>
(ii)	$I_5 = \frac{5}{6} I_3$ $I_3 = \frac{3}{4} I_1$ $I_1 = \int_0^1 \sqrt{1-x^2} dx$ $= \frac{\pi}{4}$ $\therefore I_3 = \frac{3}{4} \times \frac{\pi}{4}$ $I_5 = \frac{5}{6} \times \frac{3}{4} \times \frac{\pi}{4} = \frac{5\pi}{32}$	<p>2 marks – correct value determined</p> <p>1 mark – correct process but error</p>
(b) (i)	$f(x) = x^4 - 6x^3 + 13x^2 - ax - b$ $f'(x) = 4x^3 - 18x^2 + 26x - a$ <p>Roots are <math>\alpha \alpha \beta \beta</math></p> $\therefore 2(\alpha + \beta) = 6 \text{ so } \alpha + \beta = 3$ $\alpha^2 + \beta^2 + 4\alpha\beta = 13$ $(\alpha + \beta)^2 + 2\alpha\beta = 13$ $9 + 2\alpha(3 - \alpha) = 13$ $\alpha^2 - 3\alpha + 2 = 0$ <p><math>\alpha = 1</math> or <math>2</math> so <math>b = 2</math> or <math>1</math></p> <p>So <math>2(\alpha\alpha\beta + \alpha\beta\beta) = -a</math></p> <p><math>a = 12</math></p> <p><math>b = -4</math></p>	<p>4 marks – both values correct</p> <p>3 marks – correct values for <math>\alpha</math> and <math>\beta</math></p> <p>2 marks – both equations with <math>\alpha</math> and <math>\beta</math> correct</p> <p>1 mark – one equation with <math>\alpha</math> and <math>\beta</math> correct</p>
(ii)	<p>Equation is <math>y = 12x - 4</math> as solving the equations together gives the equation in (i) and the double zero gives the two points of contact.</p>	<p>1 mark – correct integration by parts</p>

**Question 5 (continued)**

<p>(c)</p> $\left(x + \frac{1}{x}\right)^2 = x^6 + \frac{6x^5}{x} + \frac{15x^4}{x^2} + \frac{20x^3}{x^3} + \frac{15x^2}{x^4} + \frac{6x}{x^5} + \frac{1}{x^6}$ $= \left(x^6 + \frac{1}{x^6}\right) + 6\left(x^4 + \frac{1}{x^4}\right) + 15\left(x^2 + \frac{1}{x^2}\right) + 20$ $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = t^2 - 2$ $\left(x^4 + \frac{1}{x^4}\right) = \left(x^2 + \frac{1}{x^2}\right)^2 - 2 = (t^2 - 2)^2 - 2$ $= t^4 - 4t^2 + 2$ $t^6 = \left(x^6 + \frac{1}{x^6}\right) + 6\left(t^4 - 4t^2 + 2\right) + 15(t^2 - 2) + 20$ $= t^6 - 6t^4 + 9t^2 - 2$	<p><i>4 marks – correct expression for <math>t^6</math></i></p> <p><i>3 marks – correct substitution for <math>t^6</math> (2<sup>nd</sup> last line)</i></p> <p><i>2 marks – correct expression for <math>x^4 + \frac{1}{x^4}</math></i></p> <p><i>1 mark – correct expression for <math>x^2 + \frac{1}{x^2}</math></i></p>
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**Question 6**

(a)(i)	<p>Join QA and AR</p> $\hat{P}RQ = \hat{P}AQ = \alpha \text{ (angle at circumference subtended by PQ)}$ $\hat{A}QR = \hat{A}PR = \beta \text{ (as above subtended by AR)}$ <p>But in <math>\triangle MPR</math> <math>\alpha + \beta = 90^\circ</math></p> $\therefore \text{ in } \triangle RQN \hat{R}QN + \hat{N}RQ = 90^\circ$ $\therefore \hat{R}QN = 90^\circ - \alpha = \beta = \hat{A}QR$ $\therefore \hat{R}QA = \hat{R}QN$	<p>3 marks – correct proof with full reasoning</p> <p>2 marks – correct approach continued for a second angle</p> <p>1 mark – approach using angles at circumference properly demonstrated</p>
(ii)	<p>In <math>\triangle MQH</math> and <math>\triangle MQA</math></p> <p>QM is common</p> $\hat{Q}MH = \hat{Q}MA = 90^\circ \text{ (given)}$ $\hat{R}QA = \hat{R}QN \text{ (proved above)}$ $\therefore \triangle MQH \equiv \triangle MQA \text{ (ASA)}$ $\therefore AM = MA \text{ (opposite corresponding angles)}$	<p>1 mark – correct proof of congruence</p>
(iii)	<p>In <math>\triangle MHR</math> and <math>\triangle MAR</math></p> <p>MH = MA (proved above)</p> <p>MR is common</p> $\hat{H}MR = \hat{A}MR = 90^\circ \text{ (given)}$ $\therefore \triangle MHR \equiv \triangle MAR$ $\therefore \hat{M}AR = \hat{M}HR$ <p>But <math>\hat{A}RQ = \hat{A}PQ</math> (angles at circumference subtended by AQ)</p> <p>RH produced meets QP at X</p> $\hat{M}HR = \hat{X}HP \text{ (vertically opposite angles)}$ $\therefore \hat{X}PQ + \hat{X}HP = 90^\circ$ $\therefore \hat{P}XH = 90^\circ$ $\therefore \text{RH produced meets PQ at right angles}$	<p>2 marks – correct proof with reasons clearly stated</p> <p>1 mark – suitable approach</p>

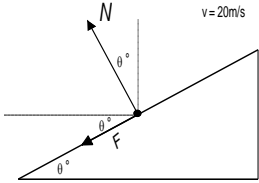
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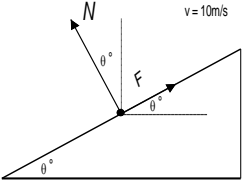
**Question 6 (continued)**

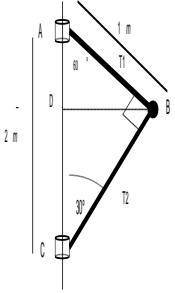
(b) (i)	$(a-b)^2 \geq 0$ $a^2 + b^2 > + 2ab$	1 mark – correct proof of inequality
(ii)	$a^2 + b^2 > + 2ab$ $a^2 + c^2 > + 2ac$ $b^2 + c^2 > + 2bc$ $2(a^2 + b^2 + c^2) \geq 2(ab + ac + bc)$ $a^2 + b^2 + c^2 \geq ab + ac + bc$	1 mark – correct proof of inequality
(iii)	Let $a = \sin \alpha$ and $b = \cos \alpha$ $\therefore \sin^2 \alpha + \cos^2 \alpha \geq 2 \sin \alpha \cos \alpha$ (using (i)) $\therefore \sin^2 \alpha + \cos^2 \alpha \geq \sin 2\alpha$	1 mark – correct proof of congruence
(iv)	$\sin^2 \alpha + \cos^2 \alpha + \tan^2 \alpha \geq \sin \alpha \cos \alpha + \sin \alpha \tan \alpha + \cos \alpha \tan \alpha$ $\text{RHS} = \frac{1}{2} \sin 2\alpha + \frac{\sin^2 \alpha}{\cos \alpha} + \sin \alpha$ $= \frac{1}{2} \sin 2\alpha + \sin \alpha + \frac{1 - \cos^2 \alpha}{\cos \alpha}$ $= \frac{1}{2} \sin 2\alpha + \sin \alpha + \frac{1}{\cos \alpha} - \frac{\cos^2 \alpha}{\cos \alpha}$ $= \frac{1}{2} \sin 2\alpha + \sin \alpha + \sec \alpha - \cos \alpha$	2 marks – correct proof with full logic shown  1 mark – correct approach to simplification of RHS
(c)	$kxe^{-x} - 4 = 0$ $\therefore ke^{-x} \times 1 - kxe^{-x} = 0$ $ke^{-x}(1-x) = 0$ $e^{-x} \neq 0 \therefore x = 1$ $\therefore k \times 1 \times e^{-1} = 4$ $k = 4e$	3 marks - correct value for k  2 marks – correct value for x  1 mark – correct differentiation



**Question 7:**

(a)	$x^3 + y^3 - 3x^2y^2 = 1$ $3x^2 + 3y^2 \frac{dy}{dx} - 3\left(2xy^2 + 2x^2y \frac{dy}{dx}\right) = 0$ $(3x^2 - 6xy^2) + (3y^2 - 6x^2y) \frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{3(2xy^2 - x^2)}{3(y^2 - 2x^2y)}$ <p style="text-align: center;">at <math>x = 1, y = 3</math></p> $\frac{dy}{dx} = \frac{18 - 1}{9 - 6} = \frac{17}{3}$ $(y - y_1) = m(x - x_1)$ $y - 3 = \frac{17}{3}(x - 1)$ $17x - 3y - 8 = 0$	<p>3 marks – correct answer</p> <p>2 marks – correct expression for <math>\frac{dy}{dx}</math> (gradient function).</p> <p>1 mark – correct differentiation</p> <p>(nb – equation formed from incorrect implicit differentiation not considered.)</p>
(b)	<div style="text-align: center;">  </div> $N \cos \theta - F \sin \theta = mg \quad \text{①}$ $N \sin \theta + F \cos \theta = \frac{mv^2}{r} \quad \text{②}$ <p>① <math>\times \sin \theta</math> &amp; ② <math>\times \cos \theta</math></p> $N \cos \theta \sin \theta - F \sin^2 \theta = mg \sin \theta \quad \text{①}$ $N \cos \theta \sin \theta + F \cos^2 \theta = \frac{mv^2}{r} \cos \theta \quad \text{②}$ $F(\cos^2 \theta + \sin^2 \theta) = \frac{mv^2}{r} \cos \theta - mg \sin \theta$ $F = m \left( \frac{400}{800} \cos \theta - g \sin \theta \right)$ $F = m \left( \frac{1}{2} \cos \theta - g \sin \theta \right)$	<p>2 marks – correct answer fully demonstrated.</p> <p>1 mark – correct resolution of forces horizontally and vertically.</p>

b-(ii)	<p>Train travelling at 10m/s</p>  $N \cos \theta + F \sin \theta = mg \quad \text{①}$ $N \sin \theta - F \cos \theta = \frac{mv^2}{r} \quad \text{②}$ <p>① <math>\times \sin \theta</math> &amp; ② <math>\times \cos \theta</math></p> $N \cos \theta \sin \theta + F \sin^2 \theta = mg \sin \theta \quad \text{③}$ $N \cos \theta \sin \theta - F \cos^2 \theta = \frac{mv^2}{r} \cos \theta \quad \text{④}$ $F(\cos^2 \theta + \sin^2 \theta) = mg \sin \theta - \frac{mv^2}{r} \cos \theta$ $F = m \left( g \sin \theta - \frac{100}{800} \cos \theta \right)$ $F = m \left( g \sin \theta - \frac{1}{8} \cos \theta \right)$ $m \left( g \sin \theta - \frac{1}{8} \cos \theta \right) = m \left( \frac{1}{2} \cos \theta - g \sin \theta \right)$ $\frac{5}{8} \cos \theta = 2g \sin \theta$ $\tan \theta = \frac{5}{16g}$ $\theta = 1^\circ 50'$	<p>2 marks – correct answer fully demonstrated.</p> <p>1 mark – correct equation for F for 10m/s</p>
v	$N \cos \theta = mg$ $N \sin \theta = \frac{mv^2}{r}$ $\tan \theta = \frac{v^2}{gr}$ $v = \sqrt{gr \tan \theta}$ $= \sqrt{g \times 800 \times \frac{5}{16g}}$ $= 5\sqrt{10} \text{ m/s}$	<p>2 marks – correct answer fully demonstrated.</p> <p>1 mark – correct derivation of <math>\tan \theta</math></p>

vi	 <p>Given triangle ABC is 60/30 triangle then side AB = 1</p> <p>Triangle ABD is also 60/30 therefore side DB = <math>\frac{\sqrt{3}}{2}</math></p>	1 mark – correct demonstration of length of DB
vii	$f = 90 \text{ rev/min} \quad f = \frac{\omega}{2\pi} \text{ rev/sec}$ $\omega = \frac{180\pi}{60} = 3\pi \text{ s}^{-1}$ <p>Resolving forces vertically.</p> $T_1 \cos 60 - T_2 \sin 60 = mg = 10g$ $\frac{T_1}{2} - \frac{\sqrt{3} T_2}{2} = 98 \quad \textcircled{1}$ <p>Resolving forces horizontally</p> $T_1 \sin 60 + T_2 \cos 60 = m r \omega^2$ $\frac{\sqrt{3} T_1}{2} + \frac{T_2}{2} = 10 \times \frac{\sqrt{3}}{2} \times (3\pi)^2 = 45\sqrt{3} \pi^2$ $\frac{\sqrt{3} T_1}{2} + \frac{T_2}{2} = 45\sqrt{3} \pi^2 \quad \textcircled{2}$ $\textcircled{2} \times \sqrt{3}$ $\frac{3T_1}{2} + \frac{\sqrt{3} T_2}{2} = 135\pi^2$	<p>5 marks – correct answer showing all steps involved.</p> <p>4 marks – one correct tension determined.</p> <p>3 marks – forces resolved correctly</p> <p>2 marks – forces resolved correctly – either horizontally or vertically</p> <p>1 mark – determination for <math>\omega</math> – angular velocity</p>

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$$\frac{T_1}{2} - \frac{\sqrt{3} T_2}{2} = 98$$

$$\frac{3T_1}{2} + \frac{\sqrt{3} T_2}{2} = 135\pi^2$$

$$2T_1 = 98 + 135\pi^2$$

$$T_1 = \frac{98 + 135\pi^2}{2} = 715.2N$$

$$T_1 - \sqrt{3} T_2 = 196$$

$$\sqrt{3} T_2 = \frac{98 + 135\pi^2}{2} - 196$$

$$T_2 = \frac{98 + 135\pi^2 - 392}{2\sqrt{3}} = 299.76N$$

**Question 8:**

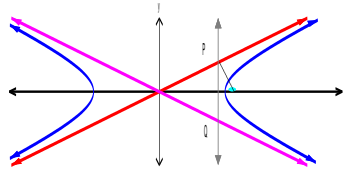
(a) (i)	<div style="text-align: center;"> </div> <p>Equation to tangent</p> $\frac{x x_1}{a^2} + \frac{y y_1}{b^2} = 1 \quad \text{or} \quad \frac{x \cos \theta}{a} + \frac{y \sin \theta}{b} = 1$ $b^2 = a^2(1 - e^2)$ $e^2 = 1 - \frac{144}{225}$ $e = \frac{3}{5}$ $\text{Focus} = ae = 15 \times \frac{3}{5} = 9$ $\text{Directrix } x = \frac{a}{e} = \frac{15 \times 5}{3} = 25$ <p>Point of intersection of tangent and directrix.</p> $\frac{x \cos \theta}{15} + \frac{y \sin \theta}{12} = 1 \quad x = 25$ $\frac{25 \cos \theta}{15} + \frac{y \sin \theta}{12} = 1$ $y = \frac{12 - 20 \cos \theta}{\sin \theta}$ <p>Gradient – Focus to P</p> $m_1 = \frac{12 \sin \theta}{15 \cos \theta - 9} = \frac{4 \sin \theta}{5 \cos \theta - 3}$ <p>Gradient – Focus to Directrix intercept</p> $m_2 = \frac{\frac{12 - 20 \cos \theta}{\sin \theta}}{25 - 9} = \frac{12 - 20 \cos \theta}{16 \sin \theta} = \frac{3 - 5 \cos \theta}{4 \sin \theta}$ $m_1 \times m_2 = \frac{4 \sin \theta}{5 \cos \theta - 3} \times \frac{3 - 5 \cos \theta}{4 \sin \theta} = -1$ <p>Therefore right angle subtended at focus.</p>	<p>3 marks – correct answer fully demonstrated</p> <p>2 marks – correct gradient for either PS or SQ</p> <p>1 mark – focus and directrix correctly determined.</p>
(ii)		

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Focus  $(ae, 0)$

Directrix  $x = \frac{a}{e}$

Asymptotes  $y = \pm \frac{b}{a}x$



Point P

$$y = \frac{b}{a} \cdot \frac{a}{e} = \frac{b}{e} \text{ therefore } P\left(\frac{a}{e}, \frac{b}{e}\right)$$

Gradient PS  $m = \frac{\frac{b}{e}}{\frac{a}{e} - ae}$

$$= \frac{\frac{b}{e}}{\frac{a - ae^2}{e}}$$

$$= \frac{b}{a(1 - e^2)}$$

$$= \frac{b}{a} \times -\frac{a^2}{b^2}$$

$$= -\frac{a}{b}$$

$$m_1 \times m_2 = \frac{b}{a} \times \frac{-a}{b} = -1$$

Therefore PS is perpendicular to asymptote.

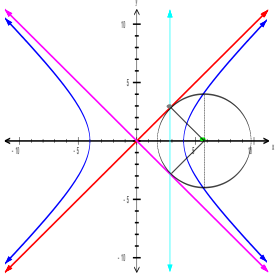
*Demonstrating perpendicular.*

*2 marks – correct answer fully demonstrated*

*1 mark – correct approach and significant progress made towards reaching correct answer.*

*Determining Length*

*1 mark – correct answer determined by correct method.*

	<p>Length PS</p> $  \begin{aligned}  PS &= \sqrt{\left(\frac{a}{e} - ae\right)^2 + \frac{b^2}{e^2}} \\  &= \sqrt{\left(\frac{a(e^2 - 1)}{e}\right)^2 + \frac{b^2}{e^2}} \\  &= \sqrt{\frac{a^2(e^2 - 1)(e^2 - 1) + b^2}{e^2}} \\  &= \sqrt{\frac{b^2(e^2 - 1 + 1)}{e^2}} \\  &= b  \end{aligned}  $	
<p>(iii)</p>	<p>As PS and PQ are perpendicular to asymptotes and equal in length. Also radii are perpendicular to tangents at point of contact. Therefore circle with centre at S would have points of contact with asymptotes at Points P and Q.</p>	
<p>(b)</p>		<p><i>2marks – correct answer fully demonstrated</i></p> <p><i>1 mark – correct approach and significant progress made towards reaching correct answer.</i></p>

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<p><i>Focus (ae, 0)      a = b</i></p> <p><i>Circle with centre at focus and touching at P and Q</i></p> $(x - ae)^2 + y^2 = b^2 = a^2$ $y^2 = a^2 - (x - ae)^2$ <p><i>Hyperbola with a=b</i></p> $x^2 - y^2 = a^2$ <p><math>\therefore</math></p> $y^2 = x^2 - a^2$ <p><math>\therefore</math></p> $x^2 - a^2 = a^2 - (x - ae)^2$ $x^2 - a^2 = a^2 - x^2 + 2aex - a^2 e^2$ $x^2 - a^2 = a^2(1 - e^2) - x^2 + 2aex$ $x^2 - a^2 = -b^2 - x^2 + 2aex$ $x^2 - a^2 = -a^2 - x^2 + 2aex \quad \text{as } b = a$ $2x^2 - 2aex = 0$ $x = ae \quad \text{as } x \neq 0$ <p>Therefore <math>x</math> coordinate of <math>R</math>, <math>T</math> and <math>S</math> is the same for each point ie. <math>x = ae</math>. Therefore are collinear and single line through centre meets circle and hyperbola at same points, therefore <math>RT</math> must be a diameter.</p>	
<p>Using binomial expansion gives to determine coefficient of <math>x</math> and <math>x^2</math></p> ${}^5C_1 ax + {}^5C_1 bx = {}^5C_1(a + b)x = 30x$ <p><math>\therefore</math></p> $5(a + b) = 30$ $a + b = 6$ ${}^5C_2 a^2 x^2 + {}^5C_2 a^2 x^2 = {}^5C_2(a^2 + b^2)x^2 = 220x^2$ $10(a^2 + b^2) = 220$ $a^2 + b^2 = 22$	<p><b>2 marks – one mark per correct answer.</b></p>



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Question 8 (continued)

	$(a + b)^2 = a^2 + 2ab + b^2$ $ab = \frac{(a + b)^2 - (a^2 + b^2)}{2}$ $ab = \frac{36 - 22}{2} = 7$	<p><b>2marks – correct answer fully demonstrated</b></p> <p><b>1 mark – correct approach and significant progress made towards reaching correct answer.</b></p>
	${}^5C_3 a^3 x^3 + {}^5C_3 b^3 x^3 = {}^5C_3 (a^3 + b^3) x^3$ $\text{Coefficient} = {}^5C_3 (a^3 + b^3) = 10(a^3 + b^3) = 900$ $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$ $= (a + b)(a^2 + b^2 - ab)$ $= 6 \times (22 - 5) = 90$ <p>or</p> $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$ $a^3 + b^3 = 216 - 3 \times 7 \times 6 = 90$ <p><math>\therefore</math> Coefficient of <math>x = 40 \times 90 = 3600</math></p>	<p><b>2 marks – correct answer fully demonstrated.</b></p> <p><b>1 mark – correct process with arithmetic error.</b></p>

