Penrith Selective High School

2016

Higher School Certificate
Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A separate reference sheet is to be provided for this examination paper
- In Questions 11-16, show relevant mathematical reasoning and/or calculations
- All diagrams are not to scale
- Multiple choice answer sheet is on page 14 of this paper

Total Marks – 100

Section I Pages 2–5

10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II Pages 6–13

90 marks

- Attempt Questions 11–16
- Allow about 2 hour 45 minutes for this section

Student Number: ______________________________

Students are advised that this is a trial examination only and cannot in any way guarantee the content or format of the 2016 Higher School Certificate Examination.

Section I:

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple choice answer sheet provided on page 14 for Questions 1–10.

Q1. Which expression is a correct factorisation of $x^3 - 8$

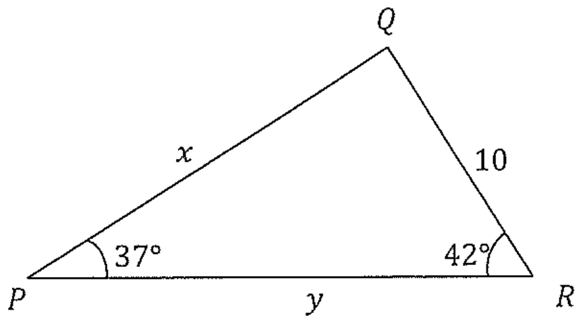
(A) $(x-2)(x^2-2x+4)$

(B) $(x-2)(x^2-4x+4)$

(C) $(x-2)(x^2+2x+4)$

(D) $(x-2)(x^2+4x+4)$

Q2. ΔPQR has side lengths x, y and 10 as shown. $\angle RPQ = 37°$ and $\angle QRP = 42°$.



Which of the following is correct for ΔPQR?

(A) $x = 10 \times \dfrac{\sin 42°}{\sin 37°}$

(B) $y = 10 \times \dfrac{\sin 37°}{\sin 101°}$

(C) $x = \dfrac{10}{\sin 37°}$

(D) $y = \dfrac{10}{\tan 37°}$

Q3. Which of the following values of m make the points $(4,\ -3)$, $(0,\ m)$ and $(-2,\ 5)$ collinear?

(A) $m = 1$

(B) $m = \frac{7}{3}$

(C) $m = 4$

(D) $m = -\frac{1}{2}$

Q4. The condition for the quadratic equation $3x^2 - 12x + k = 0$ to have real roots is

(A) $k \leq 36$

(B) $k \geq 36$

(C) $k \leq 12$

(D) $k \geq 12$

Q5. What is the centre and radius of the circle with the equation

$$x^2 + y^2 + 6x - 8y - 11 = 0?$$

(A) Centre $(-3,\ -4)$ and radius 36

(B) Centre $(-3,\ 4)$ and radius 36

(C) Centre $(-3,\ -4)$ and radius 6

(D) Centre $(-3,\ 4)$ and radius 6

Q6. What is the equation of the normal to the curve $y = x^2 - 4x$ at $(1,\ -3)$?

(A) $x + 2y - 7 = 0$

(B) $x - 2y - 7 = 0$

(C) $2x - y - 5 = 0$

(D) $2x + y + 5 = 0$

Q7. If $a > b$, which of the following is always true?

(A) $a^2 > b^2$

(B) $\frac{1}{a} > \frac{1}{b}$

(C) $-a > -b$

(D) $2^a > 2^b$

Q8. If $\tan 2x = \sqrt{3}$ in the domain $-180° \leq x \leq 180°$, the value of x is:

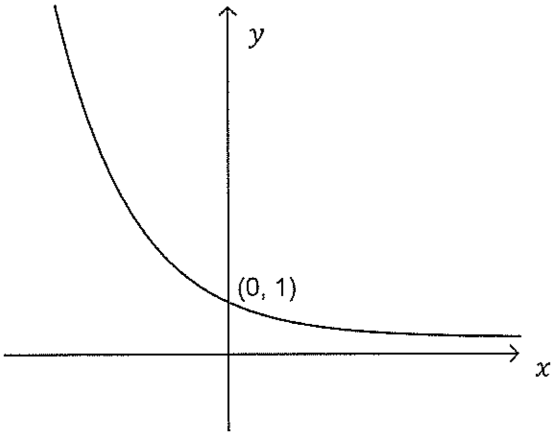
(A) $30°,\ 210°$

(B) $-150°,\ -330°$

(C) A and B

(D) None of above

Q9. The graph illustrated could be:



(A) $y = 2^x$

(B) $y = (-2)^x$

(C) $y = \left(\frac{1}{2}\right)^x$

(D) $y = \left(-\frac{1}{2}\right)^x$

Q10. The sum of the interior angles of a regular polygon is 4140°. What is the size of each interior angle?

(A) 157.5°

(B) 160°

(C) 162°

(D) 165.6°

Section II

90 Marks
Attempt Questions 11–16
Allow about 2 hour and 45 minutes for this section

Answer each question on a SEPARATE page.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) **Start this question on a new writing page**

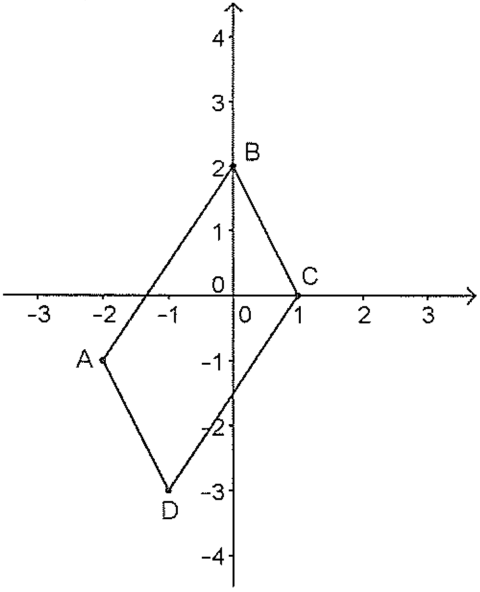
a) Solve $|4x - 5| = 3(x + 1)$. **3**

b) Express $\dfrac{\sqrt{8}}{\sqrt{8} - \sqrt{7}}$ in the form $a + b\sqrt{14}$, where a and b are integers. **2**

c) Simplify $\dfrac{3^{m+1} - 3^m}{3^{2m+1} - 3^{2m}}$ **2**

Question 11 continues on page 7

d) In the diagram below, $A(-2,\ -1)$, $B(0,2)$, $C(1,0)$ and $D(-1,-3)$ are the vertices of a parallelogram $ABCD$.



i) Find the midpoint of CD. **1**

ii) Find the gradient of CD. **1**

iii) Show that the equation of CD is $3x - 2y - 3 = 0$. **1**

iv) Find the exact length of CD. **2**

v) Find the exact perpendicular distance from A to CD. **2**

vi) Hence find the area of parallelogram $ABCD$. **1**

End of Question 11

Question 12 (15 marks) **Start this question on a new writing page**

a) Show that $\cot\theta + \tan\theta = \operatorname{cosec}\theta \sec\theta$ **2**

b) State the domain and range of $y = \sqrt{9 - x^2}$ **2**

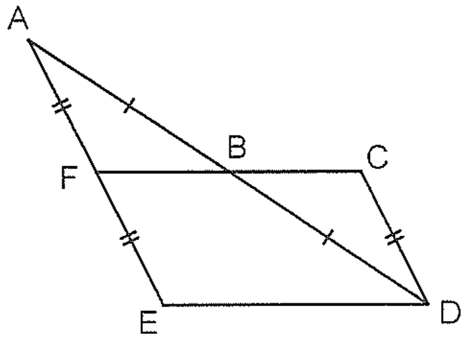
c) For the parabola: $(y-2)^2 = 12(x+3)$

i) State the coordinates of the vertex and the focus. **2**

ii) Write down the equation of the axis of symmetry. **1**

iii) Sketch this parabola showing all above information. **1**

d) In the diagram, the line FC bisects AE at F and AD at B. The line AE is parallel to CD.



i) Prove that ΔABF is similar to ΔADE. **2**

ii) Hence explain why $ED = 2BF$. **1**

Question 12 continues on page 9

e) If α and β are the roots of $5x^2 + 3x - 4 = 0$. Find the values of:

i) $\alpha + \beta$ **1**

ii) $\alpha\beta$ **1**

iii) $\frac{2}{\alpha} + \frac{2}{\beta}$ **1**

iv) $\alpha^2 + \beta^2$ **1**

End of Question 12

Question 13 (15 marks) **Start this question on a new writing page**

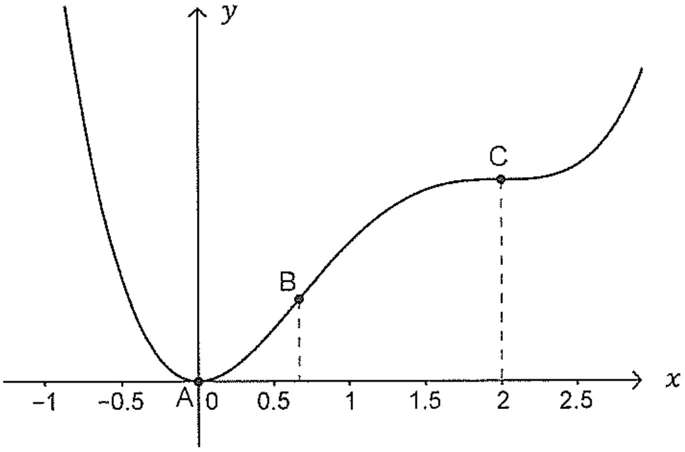
a) Given that $f(x) = (x^3 - 2)^5$, evaluate:

i) $f'(1)$ **2**

ii) $f''(1)$ **2**

b) The diagram shows the graph of a function $f(x)$. **3**

The graph has a minimum turning point at A, a point of inflexion at B and a horizontal point of inflexion at C.



Sketch the graph of the derivative $f'(x)$.

c) Given that $y = x^2 - x$, show that $\frac{dy}{dx} - \frac{d^2y}{dx^2} = \frac{2y - x}{x}$ **3**

d) Consider the curve given by $y = 3 + 21x - 9x^2 - x^3$

i) Find any turning points and determine their nature. **3**

ii) Find any points of inflexion. **2**

End of Question 13

Question 14 (15 marks) **Start this question on a new writing page**

a) Find the 17th term of the sequence $5, 9, 13, 17, \ldots$ **1**

b) The sum of the first four terms of an arithmetic series equals to the 11th term. The sum of the 6th and the 7th term is -123.

i) Find the first term and the common difference. **2**

ii) Find the sum of the first 10 terms. **1**

c) i) In a geometric series $T_1 = 64$ and $T_4 = 1$. Find the common ratio. **1**

ii) Explain why this series has a limiting sum. **1**

iii) Find its limiting sum. **1**

d) Mario borrows \$650 000 to buy a house. An interest rate of 7.2% per annum, compounded monthly is charged on the outstanding balance. The loan is to be repaid in equal monthly instalments (M) over 30 year period. If A_n is the amount owing at the end of the nth month:

i) Write down an expression for A_1. **1**

ii) Show that the amount owing after three months is: **2**

$$A_3 = 650000(1.006)^3 - M(1 + 1.006 + 1.006^2)$$

iii) Explain why $A_{360} = 0$. **1**

iv) Find the value of each instalment M to the nearest cent. **3**

v) Suppose now that Mario elects to pay \$5000 per month instead of the amount calculated in part iii). Show that he can pay off the loan 106 months earlier. **1**

End of Question 14

Question 15 (15 marks) **Start this question on a new writing page**

a) Find

i) $\int \left(3x^2 - \frac{1}{x^2}\right) dx$ **2**

ii) $\int \left(\sqrt{x} + 1\right)^2 dx$ **2**

b) Find the area enclosed between the curve $y = x^2 - x - 6$, the x-axis and the lines $x = 1$ and $x = 4$. **2**

c) Find the values of k if $\int_1^k (3x^2 - 25)dx = 24$ **3**

d) Use Simpson's rule with 5 function values to find the approximate area enclosed between the curve $f(x) = \frac{x}{1 + x^2}dx$, the x-axis and the line $x = 1$ and $x = 2$. Round your answer to 3 decimal places. **2**

e) The region bounded by the curve $y = 2 - \sqrt{x}$ and the x-axis between $x = 0$ and $x = 4$ is rotated about the x-axis to form a solid.

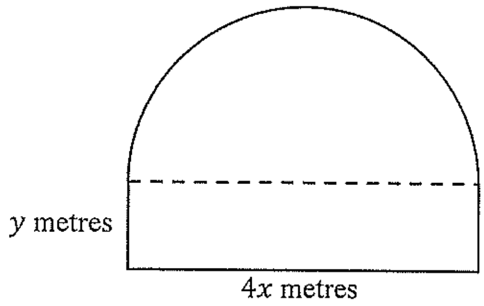
i) Sketch the above region, showing all important features. **1**

ii) Hence or otherwise, find the exact volume formed. **3**

End of Question 15

Question 16 (15 marks) **Start this question on a new writing page**

a) Luigi has designed a garden bed which consists of a rectangle of width y metres and length $4x$ metres, and a semi-circle as shown in the diagram.



If the perimeter of the garden bed is to be 50 metres.

i) Show that the perimeter of this garden bed can be expressed as **1**

$$2x\pi + 2y + 4x = 50$$

ii) Rearrange the above perimeter to express y in terms of x. **1**

iii) Show that the area of the garden bed can be given by the formula **2**

$$A = 100x - 8x^2 - 2x^2\pi$$

iv) Find the value of x that gives the maximum area. Correct your answer to 2 decimal places. **3**

b) Yoshi is on a paddle board in the ocean 3 km from the nearest point O on a straight beach. He needs to paddle to a point A east along the beach and walk further east the rest of the distance to meet his friend Peach who is 6 km along the beach from O. Yoshi is able to paddle at a rate of 4 km/h and walk at a rate of 5 km/h.

i) Draw a diagram to represent this information. **1**

ii) Let x be the distance between point O and point A. Show that the total time $T(x)$ hours, for Yoshi to reach Peach is given by: **2**

$$T(x) = \frac{\sqrt{x^2 + 9}}{4} + \frac{6 - x}{5}$$

iii) Find value of x that gives the minimum time for Yoshi to reach Peach on the beach. **4**

iv) Find the minimum time for Yoshi to reach Peach on the beach. **1**

End of Paper

Name:____Antone________ Teacher:_____Antone_____

Maths Trial 2016

4 Options Multiple Choice Answer Sheet for Qns 1 To 10

1) [✓ A] B C D
2) A B C [✓ D]
3) A [✓ B] C D
4) A B C [✓ D]
5) A B [✓ C] D

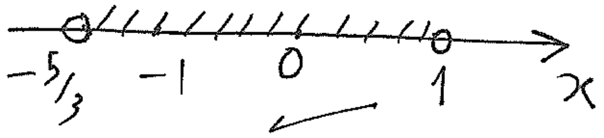
6) A [✓ B] C D
7) A [✓ B] C D
8) A B C [✓ D]
9) A [✓ B] C D
10) ~~A~~ B C [✓ D]

Suggested Solutions	Marker's Comments
a) $2x^2-9x-5=(2x+1)(x-5)$ ✓ ✓	

b) $|3x+1|<4$

$-4<3x+1<4 \Rightarrow -5<3x<3$

$\Rightarrow -5/3<x<1$ ✓ ✓



✓

c) $y=x^2+3x$, at $x=2$, $y=10$

$y'=2x+3$, at $x=2$, $m=7$ ✓

Equation of tangent at $(2, 10)$:

$y-10=7(x-2)$

$\Rightarrow y=7x-4$ or $7x-y-4=0$ ✓

d) $f(x)=x\sin 2x$

$f'(x)=2x\cos 2x+\sin 2x$ ✓ ✓

e) $x^2=8(y-3)$

$a=2$, Focus $(0, 5)$ ✓ ✓

f) $\theta^{r}=20$ ✓

$l=2\times 20=40$ cm ✓

g) $\int_1^4 e^{2x}\,dx=\frac{1}{4}[e^{2x}]_1^4$ ✓

$=\frac{1}{4}(e^8-e^2)$ ✓

D. Antone

Exam 2U trial exam MATHEMATICS : Question 12

Suggested Solutions	Marker's Comments
i). $\frac{d}{dx}\left(e^{4x}+2x\right)^3$ $= 3\left(e^{4x}+2x\right)^2\left(4e^{4x}+2\right)$	Some students did deduct 1 from the power
ii) $\frac{d}{dx}\left(\frac{\cos x}{x+1}\right)$ $= \frac{-\sin x(x+1)-\cos x}{(x+1)^2}$	Most students did this correctly
b) $\int \frac{6x}{x^2+3}\,dx = 3\int \frac{2x}{x^2+3}\,dx$ $= 3\ln\left(x^2+3\right)+C$	Most students did this correctly
c) i) $T_{15} = a+(n-1)d$ $= 6+(15-1)5$ $= 76$	
ii) $S_{15} = \frac{15}{2}\left[2\times 6+(15-1)5\right]$ $= \frac{15}{2}(12+70)$ $= 615$	Some students incorrectly stated the formula for T_n and not S_n.
iii) $350 = \frac{n}{2}(2\times 6+$ $700 = n(12+5n-5)$ $700 = 5n^2+7n$ $5n^2+7n-700=0$ $n = \frac{-7\pm\sqrt{7^2-4\times 5\times -700}}{2\times 5}$ $= \frac{-7\pm\sqrt{49+14000}}{10}$ $n = -12.55$ or $11.153..$ $n>0$ and must be an integer $\therefore n = 11$	Some students did not solve for n.
d) i) $A = \frac{h}{2}\left(0+2\times(15+12+17)+0\right)$ where $h=5$ $= 220\ m^2$	Students did not get all y values. Some students did not know the trapezoid rule.
ii) $V = 1.2\times 220$ $= 264\ m^3$	

13 a) (i) A(0, 4) C(6, 1)

$m = \frac{1-4}{6-0} = -\frac{3}{6} = \frac{-1}{2}$

$y - 4 = -\frac{1}{2}(x - 0)$

$2y - 8 = -x$

$x + 2y - 8 = 0$

(ii) $d = \frac{|1(1) + 2(-4) - 8|}{\sqrt{1^2 + 2^2}}$ B(1, -4)

$= \frac{|1 - 8 - 8|}{\sqrt{5}} = \frac{15}{\sqrt{5}}$

most common error mixing +ve and -ve

(iii) $m = 2$, B(1, -4)

$y + 4 = 2(x \mp 1)$

$y - 4 = 2x \mp 2$

$2x - y \mp 4 = 0$

Solving simultaneously

$2x - y \mp 6 = 0$ ... (1)

$x + 2y - 8 = 0$ ... (2)

$y = 2x \mp 6$ ... (3)

Sub (3) into (2)

$x + 2(2x \mp 6) - 8 = 0$

$x + 4x + 12 - 8 = 0$

$x + 4x = -10$

$5x = -20$

$x = 4$

Sub into (1)

$-4 - y + 6 = 0$

$2 = y$

$\therefore (4, 2)$

Many students did not use simultaneous equations but tried to use perpendicular distance.

13) b) (i) when $y=0$

$6x - x^2 = 0$

$x(6-x) = 0$

$x=0$ and $x=6$

Students tended to integrate either $A = \int_0^7$

or $A = \int_0^6 + \int_6^7$.

(ii) $A = \left|\int_6^7 (6x - x^2)\,dx\right|$

$= \left|\left[\frac{6x^2}{2} - \frac{x^2}{3}\right]_6^7\right|$

$= \left|\left[3x^2 - \frac{x^3}{3}\right]_6^7\right|$

$= \left|\left[3(7)^2 - \frac{(7)^3}{3}\right] - \left[3(6)^2 - \frac{(6)^3}{3}\right]\right|$

$= \left|\left[149 - 114\tfrac{1}{3}\right] - \left[108 - 72\right]\right|$

$= 8\frac{2}{3}$ units

c) (i) $P(RR) = \frac{5}{7} \times \frac{4}{5} = \frac{20}{35} = \frac{4}{7}$

(ii) $P(\text{at least } 1W) = 1 - P(RR)$

$= 1 - \frac{4}{7}$

$= \frac{3}{7}$

(iii) $P(\text{Both same}) = \left(\frac{5}{7} \times \frac{4}{5}\right) + \left(\frac{2}{7} \times \frac{1}{5}\right)$

$= \frac{22}{35}$

14) a) (i) $y = x^3 + 3x^2 - 9x$

$\frac{dy}{dx} = 3x^2 + 6x - 9$

when $dy/dx = 0$

$3(x^2 + 2x - 3) = 0$

$3(x-1)(x+3) = 0$

$x = 1$ and $x = -3$.

$\frac{d^2y}{dx^2} = 6x + 6$

at $x = 1$, $d^2y/dx^2 = 12$

$y = -5$

∴ min turning point at $(1, -5)$

at $x = -3$, $y = 27$

and $d^2y/dx^2 = -12$

∴ max at $(-3, 27)$

*Full coordinate was not given.
* Concave up/down was used in conclusion instead of max/min.

(ii) $\frac{d^2y}{dx^2} = 6x + 6$

when $d^2/y/dx^2 = 0$

$6x + 6 = 0$

$6x = -6$

$x = -1$

* many students did not test concavity

x	-2	-1	0
d^2y/dx^2	-6	0	6

(change in concavity)

∴ point of inflexion at $(-1, 11)$

(iii) when $\frac{d^2y}{dx^2} < 0$

$\therefore \quad x < -1$

(iv) $-6 \le x \le 4$

$x \doteq 6 , \quad y = -34$

$x = 4 , \quad y = 76$

Some students used max from $x = -3$ in part (i)

∴ max value at 76

b) $V = \int_a^b \pi \ y^2 \ dx$

$= \pi \int_1^3 \frac{2x}{3x^2-1} \, dx$

$= \frac{\pi}{3} \int_1^2 \frac{6x}{3x^2-1} \, dx$

Some students made careless substitution errors.

$= \frac{\pi}{3} \left[\ln(3x^2-1) \right]_1^3$

$= \frac{\pi}{3} \left[\ln 26 - \ln 2 \right]$

$\doteqdot 2.69$ units^3

c) (i) $M_0 = 100$

$60 = 100e^{-35k}$

$\frac{60}{100} = e^{-35k}$

$\ln(0\cdot6) = -35k$

$\therefore k = \frac{\ln(\frac{6}{10})}{-35}$

Many students did not leave in exact form

(ii) $M = M_0 e^{-kt}$

$m' = -k M_0 e^{-kt}$

where $m = M_0 e^{-kt}$

Hence $m' = -km$

$m = 50g$

$m' = -\left(\frac{-\ln \frac{6}{10}}{35}\right) \times 50$

$= -0\cdot7297\ldots$

$\therefore m' = 0\cdot73$ g/m

(iii) $5 = 100e^{-kt}$

$\frac{1}{20} = e^{-kt}$

$\ln(0\cdot05) = -kt$

$t = \frac{\ln(0.05)}{-k} = 205\cdot26$

$\therefore t = 206$ min

Exam 2016 Trial MATHEMATICS : Question...... 15

Suggested Solutions

a i. $\frac{T_3}{T_2} = \frac{T_2}{T_1} = \frac{2x}{3}$ $\therefore$ geometric

ii. $|r|<1$ $\left|\frac{2x}{3}\right| < 1$ $-1 < \frac{2x}{3} < 1$

$-3 < 2x < 3$

$-\frac{3}{2} < x < \frac{3}{2}$

iii. $S_\infty = \frac{a}{1-r} = \frac{x}{3} \div \left(1 - \frac{2x}{3}\right)$

$= \frac{x}{3} \times \frac{3}{3-2x}$

$= \frac{x}{3-2x}$

bi. $v(t) = \frac{6t(4+t^3) - 3t^2(3t^2)}{(4+t^3)^2}$

$= \frac{24t - 3t^4}{(4+t^3)^2}$

ii. rest when $v(t) = 0$

$0 = \frac{24t - 3t^4}{(4+t^3)^2}$

$0 = 3t(8-t^3)$

$3t = 0$ $t^3 = 8$

$t = 0$ $t = 2$

(iii) $x(1) = \frac{3(1)^2}{4+(1)^3} = \frac{3}{5}$ $x(2+2\sqrt{2}) = \frac{3(2+2\sqrt{2})^2}{4+(2+2\sqrt{2})^2} = \frac{3}{5}$

$\therefore$ particle is in the same position at t_1 and t_2.

(iv) the particle moves away from $x = \frac{3}{5}$ and then returns

(v) test

t	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{3}{2}$	2	$\frac{5}{2}$
$x'(t)$	-0.8	0	0.69	0.38	0	-0.14
	⤡	↔	⤢	⤢	↔	⤡

$\therefore$ max displacement when $x = 2$

$x(2) = \frac{3(2)^2}{4+2^3} = 1$

Marker's Comments

i. not enough to find $r = \frac{2x}{3}$ only

ii. some students found limiting sum instead of $|r|<1$

iii. many students found sum of GP instead of limiting sum

bi. first line is enough for full marks without simplifying

(iii) 'Show that' question → show substitution

(v) always test for max or min when it is mentioned

Exam MATHEMATICS : Question...... 15

Suggested Solutions	Marker's Comments
ci. $A_1 = 20000 \times 1\cdot0075 - P$ $A_2 = 20000 \times 1\cdot0075^2 - 1\cdot0075P - P$	
ii. $A_n = 20000 \times 1\cdot0075^n - P(1 + 1\cdot0075 + \ldots + 1\cdot0075^{n-1})$ $A_n = 0$ $P = \dfrac{20000 \times 1\cdot0075^n}{1 + 1\cdot0075 + \cdots + 1\cdot0075^{n-1}}$ $= \dfrac{20000 \times 1\cdot0075^n}{\frac{1\cdot0075^{60} - 1}{0\cdot0075}} = \$415\cdot167$ $= \$415$	ii. show the calculation ie \$415.167
iii. $415 \times 60 = \$24910$ $24910 - 20000 = 4900$ saved	iii. show your calculation ie subtracting 20000

Suggested Solutions | Marker's Comments

a-i) $\left(\frac{a+b}{5}\right)^2 = \frac{(a^2+b^2)+2ab}{25} = \frac{23ab+2ab}{25}$

$= \frac{25ab}{25} = ab$ ✓

a-ii) Since $\left(\frac{a+b}{5}\right)^2 = ab$

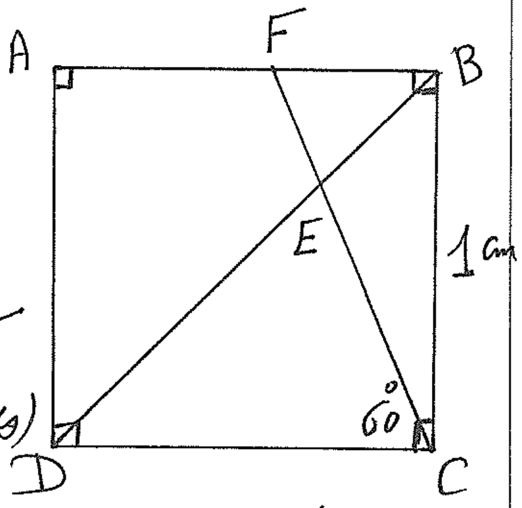
$\Rightarrow \log\left(\frac{a+b}{5}\right)^2 = \log(ab)$

$\Rightarrow 2\log\frac{a+b}{5} = \log a + \log b$ ✓

$\Rightarrow \log\frac{a+b}{5} = \frac{1}{2}(\log a + \log b)$

may students could not see these simple relations.

b-i)



∠CED = ∠BEF (vertically opposite angles)

∠DCE = ∠BFE ✓ (Alternate angles on parallel lines)

∴ ΔBEF ||| ΔDEF (equi-angular) ✓

b-ii) $\frac{FB}{BC} = \tan 30 \Rightarrow FB = \frac{1}{\sqrt{3}}$ ✓ ✓

most students could not see this simple relation.

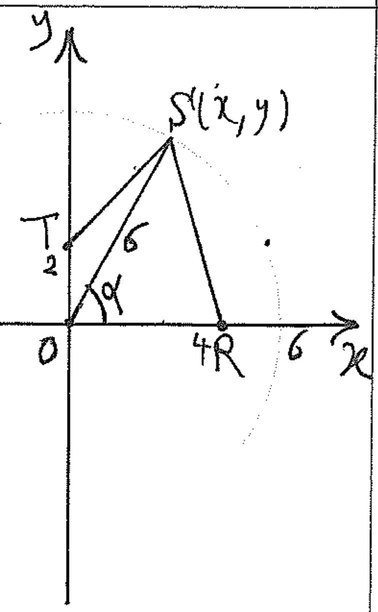
b-iii) ΔDEC : ΔBEF

$= (FB)^2 : (CD)^2$ (corresponding sides in similar Δs)

$= \frac{1}{3} : 1 = 1:3$ ✓

Qn 16 – Continued next page

D. Antone



c-i) $\Delta SOR = \frac{1}{2}\times 4\times 6\sin\alpha$

$= 12\sin\alpha$ ✓✓

c-ii) $\Delta S'OT = \frac{1}{2}\times 2\times 6\sin(90-\alpha)$

$= 6\cos\alpha$ ✓✓

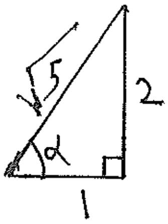
$A = 12\sin\alpha + 6\cos\alpha$

c-iii) $\frac{dA}{d\alpha} = 12\cos\alpha - 6\sin\alpha$

$\frac{dA}{d\alpha} = 0 \Longrightarrow 12\cos\alpha - 6\sin\alpha = 0 \Rightarrow \tan\alpha = 2$ ✓

$\frac{d^2A}{d\alpha^2} = -12\sin\alpha - 6\cos\alpha$

When $\tan\alpha = 2$



$\frac{d^2A}{d\alpha^2} = -12\times\frac{2}{\sqrt{5}} - 6\times\frac{1}{\sqrt{5}} < 0$

∴ There is a max value at $\alpha = \tan^{-1}2$. ✓

c-iv) For S(x, y):

$x = 6\cos\alpha = \frac{6}{\sqrt{5}}$ ✓

$y = 6\sin\alpha = \frac{12}{\sqrt{5}}$ ✓

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