APPLICATIONS OF CALCULUS TO THE PHYSICAL WORLD

1) The rate of change in the number of koalas in a certain area is given by 
\[ R = 54 - 2t \] 
where \( t \) is time in months. If there are initially 640 koalas in the area, 
(a) how many koalas there are after a year 
(b) when there are no koalas left. 

2) The surface area of a disc in mm\(^2\) expands as it heats up according to the formula 
\[ A = 5 + t^2 \] 
where \( t \) is time in seconds. Find the rate at which the surface area is expanding after 5 seconds. 

3) The mass of a chemical is given by 
\[ M = 100e^{0.023t} \] 
where \( M \) is its mass in grams and \( t \) is time in hours. 
(a) What is the initial mass? 
(b) What is the growth rate as a percentage? 
(c) What is the mass after 7 hours, correct to one decimal place? 
(d) What is the rate of increase in the mass after 7 hours, to two significant figures? 
(e) Find the time, to the nearest hour, taken for the chemical to reach a mass of 500 g. 

4) The population of a certain city is given by the formula 
\[ P = Ae^{kt} \] 
where \( t \) is the number of years. Initially the population is 55 000, and after 5 years it has increased to 68 000. 
(a) Find the values of \( A \) and \( k \). (Give \( k \) to three significant figures) 
(b) Find the population after 10 years. 
(c) Find the rate of population growth after 10 years. 
(d) Find the time it takes for the population to increase to 1 000 000, to the nearest year. 

5) A mass of uranium decays from 300 g to 260 g in 7 years. Find, to one decimal place, 
(a) its half life (the time it takes to reach half its initial mass) 
(b) how long it takes to decay to 50 g. 

6) In summer, the number of red algae in a river satisfies the equation 
\[ N(t) = Ae^{0.12t} \] 
where \( t \) is measured in days. 
(a) Show that the number of algae increases at a rate proportional to the number present. 
(b) Find how long it takes for the number of algae to increase by 20% (to one decimal place). 
(c) After 5 days, the number of algae is estimated as \( 2.3 \times 10^{11} \). Evaluate \( A \) to two significant figures. 
(d) The number of algae doubles every \( n \) days. Find the value of \( n \) to one decimal place. 

7) The population of a certain bird colony is given by 
\[ P(t) = P_0e^{kt} \] 
If the population decreases by 15% after 2 years, find
(a) the value of $k$ to three significant figures.
(b) when the population is decreased by 40%.

8)

The graph shows the displacement $x$ metres of a particle at time $t$ seconds.
(a) When is the particle at rest?
(b) When is the particle at the origin?
(c) How far does the particle travel in the first 6 seconds?
(d) Draw a rough sketch showing the velocity of the particle.

9)

The velocity of a particle is shown above as it moves along a straight line.
(a) At what times is the acceleration of the particle zero?
(b) Between what times is the acceleration at its greatest?
(c) At what times is the particle at rest?

10) A car starts and drives off, increasing its speed at a constant rate until it reaches 30 m/s after 10 seconds. It travels at this speed for 5 seconds, then it slows down at a constant speed for 5 seconds until it is travelling at 20 m/s. It travels at this speed for 15 seconds.
(a) Sketch the speed \( v \) as a function of time \( t \).
(b) Graph the distance travelled by the car over time \( t \) on a separate diagram.

11) The displacement of a particle in cm is given by \( x = 2t^3 - 21t^2 + 60t \) at time \( t \) seconds. Find
(a) the displacement after 3 seconds
(b) the initial velocity
(c) the acceleration after 5 seconds
(d) the times when the particle is at rest
(e) the total displacement over the first 3 seconds.

12) The height of a ball in metres over time \( t \) seconds is given by \( h = 1 + 15t - 5t^2 \).
(a) Find the maximum height reached.
(b) Show that the acceleration is constant.
(c) Find the time, to one decimal place, that the ball reaches the ground.

13) The displacement of a spring is given by \( s = \cos 2t \) cm at time \( t \) seconds. Find
(a) the initial velocity and acceleration
(b) the times when the spring is at rest
(c) the displacement of the spring at these times.
(d) Show that the acceleration of the spring is given by \( \ddot{s} = -4s \)

14) A particle moves in a straight line so that its displacement is given by \( x = 2e^{3t} \) cm at time \( t \) seconds. Find
(a) the exact value of the velocity after 9 seconds
(b) the exact time when the acceleration is 36 cms\(^{-2}\).

15) A particle moves along a straight line and its displacement from a fixed point \( O \) is given by \( x = 5 - 3t + 7\ln(t + 1) \) metres, where \( t \) is time measured in seconds.
(a) Find the initial position of the particle.
(b) Find the velocity of the particle after 3 seconds.
(c) Find the acceleration of the particle at time \( t \) seconds.

16)

\[ v \]

\[ 15 \]

\[ 10 \]

\[ 5 \]

\[ 1 \] \[ 2 \] \[ 3 \] \[ 4 \] \[ 5 \] \[ 6 \]

\( t \)
The graph above shows the velocity of a particle in cm/second. Find the total displacement of the particle in the first 6 seconds.

17) The velocity of a particle is \( v = 3t^2 - 12t \text{ m/s} \). If the initial displacement is 3 m, find the displacement of the particle after 5 seconds.

18) The acceleration of a particle is given by \( a = 18e^{3t} \text{ cm/s}^2 \). If the particle is at rest at the origin after 1 second, find the exact displacement of the particle after 3 seconds.

19) Two particles M and N start moving along the x-axis at time \( t = 0 \). Particle M is initially at \( x = 4 \) and its velocity at time \( t \) is given by \( v = 4t - 6 \). Particle N has its displacement given by \( x = 2t - 2 \).
   (a) Find the equation of displacement for particle M.
   (b) When do the two particles meet?

20) The velocity of a particle after \( t \) seconds is \( v = 3\cos 3t \text{ m/s} \). The particle is initially 2 m to the left of the origin. Find
   (a) the acceleration of the particle after \( \frac{\pi}{6} \) seconds
   (b) the displacement of the particle after \( \frac{\pi}{6} \) seconds.
ANSWERS

1) (a) 1 144 (b) 64 months
2) 10 mm³/s
3) (a) 100 g (b) 2.3% (c) 117.5 g (d) 2.7 g/hour (e) 70 hours
4) (a) A = 55 000, k = 0.0424 (b) 84 073 (c) 3568 people/year (d) 68 years
5) (a) 33.9 years (b) 87.6 years
6) (a) N(t) = Ae^{0.12t}
   N(1) = 0.12Ae^{0.12t}
   = 0.12N(t)
   So rate is proportional to the number of algae.
   (b) 1.5 days (c) A = 1.3 \times 10^{11} (d) n = 5.8
7) (a) k = -0.0813 (b) 6.3 years
8) (a) t = 1, 4 s (b) t = 2, 6 s (c) 12 metres
   (d)
9) (a) t = 3, 5 (b) 3 < t < 5 (c) 0 \leq t \leq 1, t \geq 6
10) 
(a) 
\[ v \]
\[ t \]

(b) 
\[ v \]
\[ t \]

11) 
(a) 45 cm  
(b) 60 cm/s  
(c) 18 cm/s²  
(d) \( t = 2, 5 \) s  
(e) 59 cm

12) 
(a) 12.25 m  
(b) \( a = -10 \) ms⁻²  
(c) After 3.1 s

13) 
(a) \( v = 0 \) cm/s, \( a = -4 \) cm/s²  
(b) \( t = 0, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots \) s  
(c) ±1 cm

(d) \[ s = \cos 2t \]
\[ s = -2 \sin 2t \]
\[ s = -4 \cos 2t \]
\[ = -4s \]

14) 
(a) \( 6e^{27} \) cm/s  
(b) \[ \frac{\log_e 2}{3} \] s
15) (a) 5 m  (b) -1.25 ms$^{-1}$  (c) $\frac{7}{(t+1)^2}$ ms$^2$

16) 47.5 cm

17) -22 m

18) $2e^3(e^n - 7)$ cm

19) (a) $x = 2t^2 - 6t + 4$  (b) $t = 1, 3$ s

20) (a) -9 ms$^{-2}$  (b) -1 m