GEOMETRICAL APPLICATIONS OF CALCULUS

1) Find all values of $x$ for which the curve $y = 2x^2 - 8x + 3 = 0$ is increasing.
2) Find the domain over which the function $f(x) = 4 - x^3$ is decreasing.
3) Find the stationary point on the curve $y = 3x^2 + 12x - 11$.
4) Show that $f(x) = \frac{2}{x-1}$ has no stationary points.
5) The curve $y = ax^2 + bx - 1$ has a stationary point at $(1,-5)$. Find values of $a$ and $b$.
6) Find the stationary point on the curve $y = x^2 - 4x + 3$ and show that it is a minimum turning point.
7) Find the stationary point on the curve $y = x^3 - 1$ and determine its nature.
8) Find the stationary points on the curve $y = x^3 + 3x^2 - 9x + 4$ and determine their nature.
9) Find the first and second derivatives of $2x^5 - 7x^3 + 2x - 1$.
10) Find $f^1(x)$ and $f^{11}(x)$ if $f(x) = 4x^3 - 9$.
11) Find $f^1(2)$ and $f^{11}(2)$ if $f(x) = 3x^4 - x^2 + 2x - 3$.
12) Find all values of $x$ for which the curve $y = x^3 - 3x^2 + x - 4$ is concave downwards.
13) For the curve $y = f(x)$, $f^1(a) = 0$ and $f^{11}(a) > 0$. Describe the shape of the curve at the point where $x = a$.
14) The number, $N$, of people with flu is increasing over time $t$. Also, the rate at which people are catching flu is also increasing.
   (a) Describe the sign of $\frac{dN}{dt}$ and $\frac{d^2N}{dt^2}$.
   (b) Sketch a graph that describes the information.
15) The population $P$, of birds in a certain area is increasing over time $t$, but the rate of population growth is slowing. Describe the sign of $\frac{dP}{dt}$ and $\frac{d^2P}{dt^2}$.
16) Find the stationary point on the curve $y = x^3$ and show that it is a point of inflexion.
17) Find the point of inflexion on the curve $y = 2x^3 - 12x^2 + 5$.
18) Consider the curve $y = x^3 - 3x^2 + 1$.
   (a) Find any stationary points on the curve.
   (b) Determine their nature.
   (c) Find any points of inflexion on the curve.
   (d) Hence sketch the curve in the domain $-2 \leq x \leq 3$
   (e) Find the minimum value of the curve in this domain.
19) A closed box with a square base is to be made so that its volume is 100 cm$^3$.
   (a) Show that the surface area of the box is given by $A = 2x^2 + \frac{400}{x}$.
   (b) Find the minimum possible surface area of the box, to one decimal place.
20) A rectangle with sides $x$ and $y$ is cut out of a circle with diameter 50 cm.
(a) Show that the area of the rectangle is given by $A = x\sqrt{2500 - x^2}$
(b) Find the maximum possible area of the rectangle.

21) Fixed points $M(3, 9)$ and $N(-2, 4)$ lie on the parabola $y = x^2$, and $P(p, p^2)$ is a variable point on the parabola.

(a) Find the exact perpendicular distance from $P$ to the line $MN$.
(b) If $P$ always lies under the line $MN$, show that the area of triangle $MNP$ is given by $A = \frac{5(p - p^2 + 6)}{2}$.
(c) Hence find the maximum area of triangle $MNP$.

22) Find the primitive function of $2x - 3$.

23) Find the primitive function of $5x^3 + 2x^2 - x^2 + 1$.

24) If $\frac{dy}{dx} = 4x + 1$ and $y = 2$ when $x = 3$, find $y$ when $x = 5$.

25) Given $f'(x) = 6x^2 - 1$, and $f(0) = 4$, find $f(1)$.

26) If a curve has $\frac{d^2y}{dx^2} = 12x - 12$ and $\frac{dy}{dx} = 2$ and $y = 1$ when $x = 1$, find the equation of the curve.

27) The tangent to a curve at point $N$ has equation $5x - y - 1 = 0$.
(a) If $\frac{dy}{dx} = 4x - 3$, find the coordinates of $N$.
(b) Find the equation of the curve.
ANSWERS

1) $x > 2$
2) $x > 0$
3) (-2,-23)
4) $\frac{-2}{(x - 1)^2} \neq 0$
5) $a = 4, b = -8$
6) (2,-1); LHS $\frac{dy}{dx} < 0$, RHS $\frac{dy}{dx} > 0$.
7) (0,-1); point of inflexion
8) (-3,31) maximum, (1,-1) minimum
9) $y' = 10x^4 - 21x^2 + 2; y'' = 40x^3 - 42x$
10) $f'(x) = 12x^2; f''(x) = 24x$
11) $f'(2) = 94; f''(2) = 142$
12) $x < 1$
13) There is a minimum turning point where $x = a$.
14) (a) $\frac{dN}{dt} > 0, \frac{d^2N}{dt^2} > 0$
      (b) \[\begin{array}{c}
      \end{array}\]
      \[\begin{array}{c}
      \end{array}\]
15) $\frac{dP}{dt} > 0, \frac{d^2P}{dt^2} < 0$
16) (0,0); At (0,0) $\frac{d^2y}{dx^2} = 0$. On LHS $\frac{d^2y}{dx^2} < 0$, on RHS $\frac{d^2y}{dx^2} > 0$.
17) (2,-27)
18) (a) (0,1), (2,-3)
      (b) (0,1) maximum, (2,-3) minimum
      (c) (1,-1)
      (d)
19) (a) \(x^2y = 100\)

So \(y = \frac{100}{x^2}\)

\[A = 2x^2 + 4xy\]

\[= 2x^2 + 4x \cdot \frac{100}{x^2}\]

\[= 2x^2 + \frac{400}{x}\]

(b) 129.3 cm²

20) (a) \(x^2 + y^2 = 50^2\)

So \(y^2 = 2500 - x^2\)

Now \(A = xy\)

\[= x\sqrt{2500 - x^2}\]

(b) 1250 cm²

21) (a) \(\frac{p - p^2 + 6}{\sqrt{2}}\)

(b) \(MN = \sqrt{(3 - -2)^2 + (9 - 4)^2} = \sqrt{50}\)

\[A = \frac{1}{2}bh\]

\[= \frac{1}{2} \sqrt{50} \times \frac{p - p^2 + 6}{\sqrt{2}}\]

\[= \frac{5(p - p^2 + 6)}{2}\]

(c) 15.625 units²
22) \( x^2 - 3x + C \)
23) \( x^5 + \frac{x^4}{2} - \frac{x^3}{3} + x + C \)
24) \( y = 36 \)
25) \( f(1) = 5 \)
26) \( y = 2x^3 - 6x^2 + 8x - 3 \)
27) (a) \( N = (2,9) \)  (b) \( 2x^2 - 3x + 7 \)