SERIES

1) Find the next 3 terms in the series 8 + 5 + 2 + ...
2) Find the next 3 terms in the series 3 + 12 + 48 + ...
3) Find the 20th term of the series with nth term given by $T_n = 5n - 4$
4) Is 102 a term of the series with nth term $T_n = 3n + 1$?
5) If $3 + x + 19 + ...$ form an arithmetic series, find the value of $x$.
6) Find the 16th term of the series 6 + 10 + 14 + ...
7) Which term of $2 + 5 + 8 + ...$ is equal to 320?
8) How many terms are there in the series $3 + 10 + 17 + ... + 157$?
9) Find the first value of $n$ for which the series $98 + 93 + 88 + ...$ becomes negative.
10) The 1st term of an arithmetic series is 7 and the 5th term is 23. Find the 20th term of the series.
11) The 5th term of an arithmetic series is 14 and the 10th term is 59. Find the first term and the common difference of the series.
12) Find the sum of the first 25 terms of the series 6 + 10 + 14 + ...
13) Evaluate $9 + 14 + 19 + ... + 494$.
14) Find the sum of the first 50 terms of the series 100 + 97 + 94 + ...
15) How many terms of the series $6 + 8 + 10 + ...$ give a sum of 2064?
16) How many terms of the series $53 + 49 + 45 + ...$ give a sum of 378?
17) The sum of the first 5 terms of an arithmetic series is 20 and the 8th term is 19.
   (a) Find the values of $a$ and $d$.
   (b) Find the sum of the first 50 terms of the series.
18) Evaluate $\sum_{n=1}^{25} 6n - 5$
19) The positive multiples of 9 are 9, 18, 27, 36,...
   (a) What is the largest multiple of 9 less than 500?
   (b) Find the sum of all the multiples of 9 that are less than 500.
20) The series $5 + y + 20 + ...$ is geometric. Find the value of $y$.
21) Find the series that is not geometric:
   (a) $\frac{3}{5} + \frac{2}{5} + \frac{4}{15} + ...$
   (b) $\frac{7}{8} - \frac{21}{32} + \frac{63}{128} + ...$
   (c) $\frac{6}{7} + \frac{4}{7} + \frac{10}{21} + ...$
   (d) $\frac{5}{6} + \frac{2}{3} + \frac{8}{15} + ...$
22) Find the 10th term of the series 3 + 6 + 12 + ...
23) Find the 50th term of the series 3 + 9 + 27 + ... and leave your answer in index form.
24) Which term of 5 - 15 + 45 - ... is equal to -98 415?
25) Which term of the series $\frac{7}{8} + \frac{1}{4} + \frac{1}{14} + \ldots$ is equal to $\frac{16}{117649}$?

26) The second term of a geometric series is 10 and the 5th term is 1250. Find the common ratio of the series.

27) If the 3rd term of a geometric series is 2 and the 7th term is 32, find the 10th term of the series.

28) Find the sum of the first 12 terms of the series $\frac{1}{6} + \frac{1}{2} + \frac{1}{2} + \ldots$.

29) Evaluate $6 + 12 + 24 + \ldots + 768$.

30) Find the number of terms of the series $3 - 12 + 48 - \ldots$ that give a sum of 615.

31) Evaluate $\sum_{k=1}^{12} 3.2^k$

32) Find the limiting sum (sum to infinity) of the series $12 + 6 + 3 + \ldots$.

33) (a) In the series $y + y^2 + y^3 + \ldots$, for what values of $y$ does a limiting sum exist?
   (b) If $y = \frac{2}{3}$, find the limiting sum.

34) The sum to infinity of the series $x + \frac{2x}{3} + \frac{4x}{9} + \ldots$ is 15. Evaluate $x$.

35) Evaluate $\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n$

36) Sarah earns $21,000 in her first year of work. Her salary increases by $800 per year.
   (a) Calculate how much she earns in the 9th year.
   (b) Find her total earnings over the first 5 years.

1) A tall cupboard has 12 shelves, each 20 mm thick and 200 mm apart. The top and bottom of the cupboard is 50 mm thick.
   (a) Find the distance from the bottom of the cupboard to the top of the lowest shelf from the bottom.
   (b) Find the distance from the bottom of the cupboard to the top of the second lowest shelf.
   (c) Explain briefly why the distance of the top of each shelf from the bottom of the cupboard is given by the arithmetic sequence 270, 490, 710,...
   (d) Find the distance from the bottom of the cupboard to the top of the 9th shelf up from the bottom.
   (e) How tall is the cupboard?
In a game, Jill runs 10 metres from the starting point and picks up a ball, then runs back to the start and places the ball in a bucket. She then runs to the next ball which is 2 metres further than the first ball. She picks that ball up and runs back to the start. She then runs and collects the next ball which is 2 metres further than the previous one, and so on until 6 balls are collected and taken back to the start. What is the total distance she will run?

3) At a timber shop, a certain type of timber posts come in different lengths from 1.2 metres to 5.4 metres. The lengths of the posts increase at a constant rate. The shop displays one post of each length, and the total length of these posts is 49.5 metres. 
   (a) Find the number of posts displayed.  
   (b) Find the difference in length between each adjacent post.

4) A girl on a pogo stick jumps 0.5 metre on the first jump, then 0.25 on the second, 0.125 on the third and so on. What is the total distance that she travels?

5) Jo bounces a ball, dropping it from a height of 1 metre on the first bounce. It then rises up to \( \frac{2}{5} \) of its height on each bounce. Find the distance through which the ball travels.

6) Mary invests $2 000 in a bank account that pays 4% compound interest p.a., paid quarterly. How much money does Mary have in her account after 5 years?

7) The Bank of Saturn offers interest at 5.8% p.a. paid monthly, while the Bank of Neptune pays interest at 6%, paid twice a year. If I have $5 000 to invest, which bank should I use, and how much extra money will I have after 7 years?

8) Mark invests $1 000 in a bank account that pays interest compounded quarterly. After four years he has $1219.89 in the bank. What is the interest rate?

9) (a) Grandma wants to invest a certain amount of money for her grandchild so that she will have $5 000 in ten years time. If the bank account pays 5% p.a. interest, paid annually, how much will she need to invest now? 
   (b) Instead of investing just one amount now, Grandma decides to bank $500 at the beginning of each year for ten years. If the money earns 5% p.a., how much money will she have after ten years?

10) Mark starts work and invests $200 in a superannuation fund at the beginning of each year until he retires 30 years later. If the money earns 9% p.a., how much will Mark have when he retires?

11) Maria applies for a loan of $15 000 to be paid back in monthly instalments over 5 years. The interest on the loan is 16% p.a. 
   (a) How much does Maria owe after one month? 
   (b) What is the amount of each monthly instalment? 
   (c) How much money will Maria pay on the loan altogether?
12) Farmer Brown buys a harvester for $250 000 and pays it off over 3 years in equal yearly instalments. If the interest rate on the harvester is 14.5% p.a., find the amount of each yearly instalment.

13) Furniture Galore has a special deal where you can buy a lounge on hire purchase and pay it off over two years. No repayments are needed for the first three months. Hire purchase interest is 18% p.a. If George buys a $3 000 lounge, find
   (a) the amount he owes after three months
   (b) the amount of each monthly repayment.
ANSWERS

1) -1 - 4 - 7
2) 192 + 768 + 3072
3) 96
4) No
5) \( x = 11 \)
6) 66
7) 107th
8) 23
9) \( n = 21 \)
10) 83
11) \( a = -22, d = 9 \)
12) 1350
13) 24 647
14) 1325
15) 43
16) 14
17) (a) \( a = -2, d = 3 \)  (b) 3575
18) 1804
19) (a) 495  (b) 13 860
20) \( y = \pm 10 \)
21) (c)
22) 1536
23) \( 3^{50} \)
24) 10th
25) 8th
26) \( r = 5 \)
27) \( \pm 256 \)
28) 44286 \( \frac{2}{3} \)
29) 1530
30) 5
31) 24 570
32) 24
33) (a) \( |y| < 1 \)  (b) 2
34) \( x = 5 \)
35) \( \frac{1}{4} \)
36) (a) $27 400  (b) $113 000
37) (a) 270 mm  (b) 490 mm
    (c) Add 220 mm for each shelf, so \( d = 220 \)
    (d) 2030 mm  (e) 2940 mm
38) 180 metres
39) (a) 15  (b) 0.3 m
40) 1 metre
41) $2 \frac{1}{3}$ metres
42) $2440.38$
43) Neptune; $66.27$ extra
44) 5%
45) (a) $3069.57$  (b) $6603.39$
46) $29715.04$
47) (a) $15200$  (b) $364.77$  (c) $21886.25$
48) $108587.44$
49) (a) $3137.04$  (b) $175.25$