Chapter 10
Sample HSC Examinations

Sample HSC Examination 1

Time allowed: 3 hours
Total marks = 100

Section I

Objective-response questions

1 \( \log_3 3 = \)

A -1  B \( \frac{1}{2} \)  C \( \frac{1}{3} \)  D \( -\frac{1}{3} \)

2 To three decimal places, the value of \( \sin \frac{3\pi}{8} \) is

A 0.206  B 0.021  C 0.924  D 1.178

3 A tank is being emptied. The volume of water, in litres, remaining in the tank \( t \) minutes after it started to be emptied is given by \( V = t^2 - 375t + 35100 \). At what rate (in litres/minute) is the water flowing from the tank after 30 minutes?

A 315  B 351  C 357  D 375

4 What is the period of the graph of \( y = 3 \cos \frac{\pi t}{2} \)?

A 3  B \( \frac{1}{2} \)  C \( \frac{2}{3} \)  D 4

5 The diagram shows the graph of the derivative \( f'(x) \) of a function. The function \( f(x) \) has a minimum turning point at \( x = \)

A 0  B 2  C 4  D 5

6 \( \sin \theta + \cos \theta \cot \theta = \)

A \( \tan \theta \)  B \( \sec \theta \)  C cosec \( \theta \)  D 1

7 When \( x = 1 \), the curve \( y = 5x^3 - 6x^2 + 7x - 8 \) is

A increasing  B decreasing  C stationary  D undefined

8 The displacement of a moving particle is given by \( x = 2e^t - 3t - 4 \). The initial displacement is

A 0  B -2  C -4  D -5
Sample HSC Examination 1

9 The probability that a person selected at random has a gene linked with a particular disease is 0.12. Of people with that gene, the probability that they will get that disease in their lifetime is 0.4. What is the probability that a person selected at random has the gene but will not get the disease?

A 0.6  B 0.48  C 0.72  D 0.072

10 The volume of the solid formed by rotating the section of the curve \( y = x^2 \) between \( y = 0 \) and \( y = 3 \) about the \( y \)-axis is given by

A \( \int_0^3 \pi x^4 \, dx \)  B \( \int_0^3 \pi y \, dy \)

C \( \int_0^3 \pi y^2 \, dy \)  D \( \int_0^3 \pi y^4 \, dy \)

Total for Section I: 10 marks

Section II

Question 11

a Find a primitive of \( e^x - x \)  2 marks

b Find the exact length of the arc PQ.  2 marks

c Find the second derivative of \( y = (3x - 4)^7 \)  2 marks

d Differentiate the following functions:

i \( \log_e(2x - 1) \)  2 marks

ii \( x \sin x \)  2 marks
Sample HSC Examination 1

Question 11

e A particle is moving in a straight line from a fixed point O. At time $t$ seconds its displacement from 0, $x$ m, is given by $x = 18t^2 - t^3$

i At what times is the particle stationary?  

ii Find the displacement at the time when the acceleration is zero.

Total for question 11: 15 marks

Question 12

a A box contains 10 unlabelled CDs, 2 of which are blank and the remainder used. Jack chooses one at random, puts it aside and then chooses another.

i Draw a tree diagram to show the possible outcomes.  

ii Find the probability that both CDs are blank.

iii Find the probability that at least one CD is blank.

iv Jack plays one of the CDs and finds that it is blank. What is the probability that the other CD is also blank?

Total for question 11: 15 marks
Sample HSC Examination 1

Question 12

b  Find the exact value of:

i  \[ \int_{\frac{\pi}{4}}^{\pi} \sec^2 x \, dx \]  3 marks

ii  \[ \int_{1}^{e} \frac{dx}{x} \]  3 marks


c  Find all solutions over the domain \( 0 \leq x \leq 2\pi \) for which \( 2 \cos x + \sqrt{3} = 0 \)  4 marks

Total for question 12: 15 marks

Question 13

a  Use Simpson’s rule with the table of values to estimate \( \int_{3}^{5} f(x) \, dx \)  3 marks

<table>
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<tr>
<th>( x )</th>
<th>3</th>
<th>3.5</th>
<th>4</th>
<th>4.5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>3.0</td>
<td>2.6</td>
<td>2.3</td>
<td>2.0</td>
<td>1.8</td>
</tr>
</tbody>
</table>
Sample HSC Examination 1

Question 13

b The first term of an arithmetic series is 17 and the common difference is \(-3\). Find:

i the ninth term 1 mark

ii the sum of the first nine terms 2 marks


c i Find the equation of the tangent to the curve \(y = 2e^x + 1\) at the point where \(x = 1\). 3 marks

ii At what point does the tangent in part i cut the \(y\)-axis? 1 mark

d The diagram shows the graph of \(y = \sin x\) and \(y = -\cos x\).

i Show that \(x = \frac{3\pi}{4}\) is a solution of the equation \(\sin x + \cos x = 0\) 1 mark

ii Find the area shaded in the diagram. 4 marks

Total for question 13: 15 marks
Sample HSC Examination 1

Question 14

a Solve $2 \ln x = \ln (6 + x)$  

b The mass, $M$, in grams of a radioactive substance is expressed as $M = 175e^{-kt}$ where $k$ is a positive constant and $t$ the time in days. The mass of the substance halved in 6 days.

i Find the value of $k$ correct to 5 decimal places.  

ii Find the mass of the substance remaining after 10 days.  

iii At what rate is the mass disintegrating after 10 days?  

c In the diagram, AB is parallel to DC and AB = DC.

i By proving triangle ABC congruent to triangle CDA, prove that AD = BC.  

ii If CA bisects $\angle BAD$ show that ABCD is a rhombus.  

Total for question 14: 15 marks
Sample HSC Examination 1

Question 15

a  Find the limiting sum of the series $10 + 5 + 2.5 + 1.25 + ...$  

b  For what values of $x$ is the curve $y = x^3 - 5x^2 + 10x + 7$ concave up?

c  A and B are the points (3, –1) and (6, 5) respectively.

i  Find the midpoint of AB.  

ii  Find the gradient of AB.  

iii  Find the equation of the perpendicular bisector of AB.  

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At the beginning of each year, Jennifer invests $3000 into an account earning 5% p.a. interest compounded annually.

i  Find, to the nearest dollar, the amount to which the first $3000 accumulated at the end of twenty years. (Assume the last amount of interest has just been paid.)  

ii  Find the total value of the investment at the end of 20 years.
Sample HSC Examination 1

Question 15

d  At the beginning of each year, Jennifer invests $3000 into an account earning 5% p.a. interest compounded annually.

i  Find, to the nearest dollar, the amount to which the first $3000 accumulated at the end of twenty years. (Assume the last amount of interest has just been paid.)  
   1 mark

ii  Find the total value of the investment at the end of 20 years.  
   4 marks

Total for question 15:  15 marks

Question 16

a  The graph of \( y = f(x) \) passes through the point \((2, -3)\). \( f'(x) = 6x - 1 \). Find \( f(x) \).  
   3 marks
Sample HSC Examination 1

Question 16

b. \( \triangle ABC \) is right-angled at B. DEFG is a rectangle.
AB = 120 m.
BC = 160 m.
FG = \( x \) m, GD = \( y \) m, BG = \( z \) m.

i Find the length of AC.

ii Show that \( \triangle FBG \) is similar to \( \triangle ABC \).

iii Find an expression for \( z \) in terms of \( x \).

iv Show that \( \triangle GDC \) is similar to \( \triangle ABC \).

v Find an expression for \( y \) in terms of \( z \).

vi Show that \( y = 96 - \frac{12x}{25} \)

vii Find the area of the largest possible rectangle DEFG.

Total for question 16: 15 marks
Total for Section II: 90 marks
Total marks: 100
Sample HSC Examination 2

Time allowed: 3 hours
Total marks = 100

SECTION I

Objective-response questions

1 Correct to two decimal places, the value of \( \frac{\sqrt{9.6 + 2.8}}{3.1 + 1.7} \) is
A 0.73 B 1.61 C 4.94 D 11.84

2 Which number line shows the values of \( x \) for which \( |x - 2| \geq 3 \)?

A

\[ -7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \]

B

\[ -7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \]

C

\[ -7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \]

D

\[ -7 \quad -6 \quad -5 \quad -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \]

3 \( \frac{3x}{x^2 - 9} - \frac{1}{x + 3} = \)
A \( \frac{3}{x - 3} \)
B \( \frac{3}{x + 3} \)
C \( \frac{2x - 3}{x^2 - 9} \)
D \( \frac{2x + 3}{x^2 - 9} \)

4 A bag contains 5 green, 7 yellow and 8 pink pegs. Two pegs are chosen at random. What is the probability that both pegs are green?
A \( \frac{5}{76} \)
B \( \frac{1}{19} \)
C \( \frac{1}{16} \)
D \( \frac{2}{21} \)

5 A car was bought for $24 950 five years ago. If it depreciates at 15% p.a., which is the best approximation to its value now?
A $11 100 B $13 900 C $18 700 D $6 200

6 \( \frac{5\pi}{9} \) radians =
A 75° B 100° C 150° D 200°

7 \( 3\log_{4}2 - \log_{2}2 \) =
A \( \log_{6} \) B \( \log_{10} \) C \( \log_{32} \) D \( \log_{62} \)
8 ABDE is a square. Triangle BCD is equilateral.

What is the size of ∠EAC?

A 60°  B 65°  C 70°  D 75°

9 \( x = 4 \cos 2t \)  \( \dot{x} = \)

A \( \cos 2t \)  B \( -\cos 2t \)  C \( 16 \cos 2t \)  D \( -16 \cos 2t \)

10 For a particular curve \( y = f(x) \) it is known that \( f'(a) < 0 \) and \( f''(a) > 0 \).

At \( x = a \) the curve is

A increasing and concave up  B increasing and concave down
C decreasing and concave up  D decreasing and concave down

**Total for Section I: 10 marks**

**SECTION II**

**Question 11**

a Differentiate with respect to \( x \)

\[ i \quad e^x \ln x \]  \quad 2 marks

\[ ii \quad \frac{\sin x}{x} \]  \quad 2 marks

\[ \text{____________________________} \]  \quad \text{____________________________} \]

\[ \text{____________________________} \]  \quad \text{____________________________} \]

\[ \text{____________________________} \]  \quad \text{____________________________} \]

b i Find the coordinates of Q, the point of intersection of \( k: y = 3x + 9 \) and \( l: x + 3y - 17 = 0 \)  \quad 2 marks

\[ \text{____________________________} \]  \quad \text{____________________________} \]

\[ \text{____________________________} \]  \quad \text{____________________________} \]

\[ \text{____________________________} \]  \quad \text{____________________________} \]

\[ \text{____________________________} \]  \quad \text{____________________________} \]

\[ \text{____________________________} \]  \quad \text{____________________________} \]
Question 11

b ii If R is the point where $k$ cuts the $x$-axis, find the coordinates of R.  

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iii Find the coordinates of $M$, the midpoint of $Q$ and $S(3, -2)$.  

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________

iv Find the equation of the line joining $P(5, 4)$ and $R$.  

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________________________________________________________________________
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(c) Leah opens a bank account and deposits $10 in the first month, $30 in the second month and $50 in the third month. If she can continue to increase the amount of her deposit by $20 each month, how much will she have saved by the end of eighteen months?  

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________________________________________________________________________
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Sample HSC Examination 2

Question 12

a  AB ⊥ BD, AE ⊥ ED, AB = DE = 4 cm, AE = 8 cm.

b  Find the exact value of:

i  \( \int_1^2 (3x - 2)^5 \, dx \)

ii  \( \int_0^1 e^{4x} \, dx \)

Question 13

a  i  Solve \(-x^2 + 13x - 36 = 0\) 

ii  Find the equation of the tangent to the parabola \(y = -x^2 + 13x - 36\) at the point where \(x = 6\).
Sample HSC Examination 2

Question 12

a  Find the exact value of:

i  

\[
\int_2^5 \frac{1}{2^x} \, dx
\]

2 marks

ii  

\[
\int_4^0 1 \, dx
\]

2 marks

Total for question 12: 15 marks

Question 13

a  i  Solve \(-x^2 + 13x - 36 = 0\)

1 mark

ii  Find the equation of the tangent to the parabola \(y = -x^2 + 13x - 36\) at the point where \(x = 6\). 3 marks
Sample HSC Examination 2

Question 13

a iii Draw a diagram showing the parabola and the tangent. Shade the region bounded by the parabola, the tangent and the x-axis.  

iv Find the area shaded in the diagram.

b i Factorise \(2m^2 - 3m - 2\)  

ii Find all values of \(x\), \(0 \leq x \leq 2\pi\), for which \(2\cos^2x - 3\cos x = 2\)
Sample HSC Examination 2

Question 13

c  For what values of $m$ does the series $1 - \frac{m^2}{4} + \frac{m^4}{16} - \frac{m^6}{64} + \ldots$ have a limiting sum?  

Total for question 13: 15 marks

Question 14

a  The temperature $T(\degree C)$ of a cooling object at time $t$ minutes ($t \geq 0$) is given by $T = 120e^{-0.009t} - 20$

i  What was the initial temperature of the object?  

ii  After approximately how many minutes will the temperature of the object be 50$\degree$?  

b  A curve, $y = f(x)$ has a turning point at (0, 3). If $f''(x) = e^x + e^{-x}$ find the equation of the curve.

Total for question 14: 15 marks
Sample HSC Examination 2

Question 14

c  A circular sector OABC has radius \( r \) m and angle AOC measures \( \theta \) radians.

If the length of the chord AC is \( x \) m:

i  Show that \( x^2 = 2r^2(1 - \cos \theta) \)  

ii  If \( \theta = \frac{2\pi}{3} \) and \( x = 6\sqrt{3} \) find the exact length of the arc ABC.  


d  A bag holds 20 red and 30 green apples. Two apples are drawn at random from the bag. What is the probability that:

i  both apples are red?  

ii  one apple is red and one is green?  

Total for question 14: 15 marks
Question 15

a. Consider the function \( f(x) = -x^4 + 4x^3 - 16 \)
   
   i. Find the stationary points of the curve and determine their nature. \( 4 \text{ marks} \)
   
   ii. Find any points of inflection. \( 2 \text{ marks} \)
   
   iii. What value does \( y \) approach as \( x \to \infty \)? \( 1 \text{ mark} \)
   
   iv. What value does \( y \) approach as \( x \to -\infty \)? \( 1 \text{ mark} \)
   
   v. Sketch the curve \( y = -x^4 + 4x^3 - 16 \) \( 2 \text{ marks} \)
   
   vi. What is the range of the function? \( 1 \text{ mark} \)
Sample HSC Examination 2

Question 15

b i Use Simpson’s rule, with three function values, to give an estimate (in terms of \( \pi \)) of \( \int_{0}^{\pi/2} \sin^2 x \, dx \) 2 marks

ii Use the result from (i) to estimate the volume when the curve \( y = \sin x \), between \( x = 0 \) and \( x = \pi/2 \) is rotated about the \( x \)-axis. 2 marks

Total for question 15: 15 marks

Question 16

a Two particles, A and B, are moving on the \( x \)-axis. The position, \( x \), of particle A at time \( t \) seconds is given by \( x = t - \log_e(t + 1) \) and the position, \( X \), of particle B at time \( t \) seconds is \( X = t + 1 + \frac{1}{1 + t} \) \((t \geq 0)\)

i Find expressions for the velocities of the two particles. 2 marks
Sample HSC Examination 2

Question 15

b

i Use Simpson's rule, with three function values, to give an estimate (in terms of \( p \)) of

\[
\int_{2}^{\pi} \sin^2 x \, dx
\]

2 marks

ii Use the result from (i) to estimate the volume when the curve \( y = \sin x \), between \( x = 0 \) and \( x = \frac{\pi}{2} \) is rotated about the \( x \)-axis.

2 marks

Total for question 15: 15 marks

Question 16

b

An amount of $10 000 is borrowed and an interest rate of 1% per month is charged monthly. An amount \( M \) is repaid every month.

i If \( A_n \) is the amount owing after \( n \) months, show that \( A_n = 10000(1.01)^n - M \left( \frac{1.01^n - 1}{0.01} \right) \)

4 marks
Sample HSC Examination 2

ii Find the value of \( M \), to the nearest cent, if the loan is repaid at the end of 5 years.  

iii How much extra, in total, will be repaid if the loan is taken over 7 years?  

Total for question 16: 15 marks  
Total for Section II: 90 marks  
Total marks: 100
Answers

**Pages 204–212**  1 B  2 C  3 A  4 D  5 D  6 C  7 A  8 B  9 D  10 B

11 a $e^t - \frac{x^2}{2} + (C)$  b $10\pi \cdot \frac{cm}{3}$  c $378(3x - 4)^4$  d $i \frac{2}{2x - 1}$  e $0 \leq s \leq 12 \text{ s} \text{ ii} 432 \text{ m}$

12 a $i$ (see diagram right)  ii $\frac{1}{45}$  iii $\frac{17}{45}$  iv $\frac{1}{9}$  b $i \sqrt{3} - 1$  ii $1 \frac{c}{x} = \frac{5\pi}{6}$ or $x = \frac{7\pi}{6}$  12 a $i$

13 a $4.63$ [2 d.p.]  b $i - 7$  ii $45$  c $i y = 2ex + 1$  ii $(0, 1)$  d $ii \left(2 - \sqrt{2}\right)$ units²

14 a $x = 3$  b $k = 0.11552$  ii $55 \text{ g}$ [nearest g]  iii $6.4 \text{ g/day}$ [1 d.p.]  c $i$ (SAS)

15 a $20$  b $x > \frac{2}{3}$  c $i (4.5, 2)$  ii $2$  iii $2x + 4y - 17 = 0$  d $i \$7960$  ii $\$104 \text{158}$ [nearest $]

16 a $f(x) = 3x^2 - x - 13$  b $i 200 \text{ m}$  ii (equiangular)  iii $z = \frac{4x}{5}$  iv (equiangular)  v $y = 96 - \frac{3z}{5}$

vii $4800 \text{ m}^2$

**Pages 213–224**  1 A  2 C  3 D  4 B  5 A  6 B  7 C  8 D  9 D  10 C

11 a $i \frac{e^t}{x(1 + x \ln x)}$  ii $\frac{x \cos x - \sin x}{x^2}$  b $i (-1, 6)$  ii $(-3, 0)$  iii $(1, 2)$  iv $x - 2y + 3 = 0$  v Yes, the coordinates of M satisfy the equation of PR.  c $\$3240$

12 a $i$ (AAS)  ii $5 \text{ cm}$  b $i \frac{227}{2}$  ii $\frac{e^t - 1}{4}$  c $i 55°$  ii $172 \text{ km}^2$  d $x = 2.170$

13 a $i x = 4$ or $x = 9$  ii $y = x$  iii (see diagram right)  iv $10 \frac{2}{3}$ units²

b $i (2m + 1) (m - 2)$  ii $x = \frac{2\pi}{3}$ or $x = \frac{4\pi}{3}$  c $-2 < m < 2$

14 a $100° \text{C}$  b $\text{60 minutes}$  c $\text{B} y = e^x + e^{-x} + 1$  d $\text{i} \frac{38}{245}$  ii $\frac{24}{49}$

15 a $i$ horizontal point of inflection at $(0, -16)$, maximum at $(3, 11)$  ii point of inflection at $(2, 0)$ (and horizontal point of inflection at $(0, -16)$)  iii $\infty$

iv $\infty$  v (see diagram right)  vi $y = 11$  b $i \frac{\pi}{4}$  ii $\frac{\pi^2}{4}$ units²

16 a $i x = 1 - \frac{1}{t + 1}$  ;  $\dot{x} = 1 - \frac{1}{(1 + t)^2}$  iii $(t \geq 0)$  b $\$222.44$  ii $\$1482.12$