



Name: \_\_\_\_\_

Teacher: \_\_\_\_\_

Class: \_\_\_\_\_

**FORT STREET HIGH SCHOOL**

**2010**

HIGHER SCHOOL CERTIFICATE COURSE

**ASSESSMENT TASK 3: TRIAL HSC**

**Mathematics**

**TIME ALLOWED: 3 HOURS**

**(PLUS 5 MINUTES READING TIME)**

Outcomes Assessed	Questions	Marks
Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1, 2, 3	
Manipulates algebraic expressions to solve problems from topic areas such as functions, quadratics, trigonometry, probability and logarithms	4, 5, 7, 8	
Demonstrates skills in the processes of differential and integral calculus and applies them appropriately	6, 9	
Synthesises mathematical solutions to harder problems and communicates them in appropriate form	10	

Question	1	2	3	4	5	6	7	8	9	10	Total	%
Marks	/12	/12	/12	/12	/12	/12	/12	/12	/12	/12	/120	

**Directions to candidates:**

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started in a new booklet

## STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left( x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left( x + \sqrt{x^2 + a^2} \right)$$

NOTE :  $\ln x = \log_e x, \quad x > 0$

**Question One: (Start a NEW BOOKLET)**

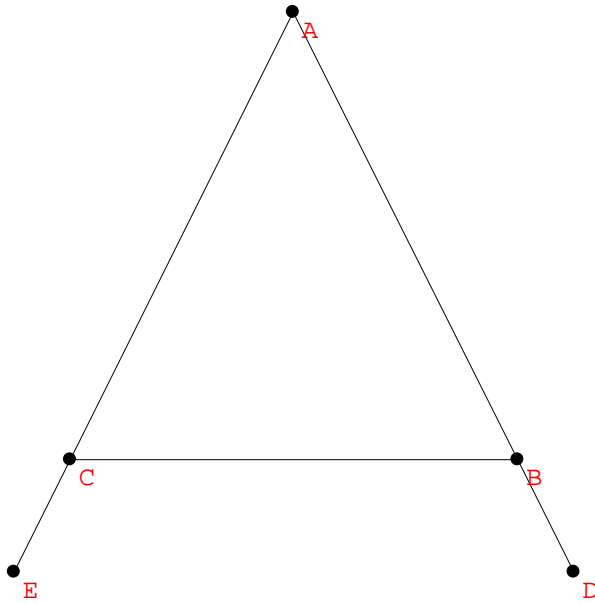
- a) Evaluate  $\frac{\pi}{\sqrt{e^2-1}}$  correct to 3 decimal places. [2]
- b) Solve  $\frac{3-2x}{x} = 4$ . [2]
- c) Rationalise the denominator of  $\frac{2}{1+\sqrt{3}}$ . [2]
- d) Factorise  $4+11x-3x^2$ . [2]
- e) Sketch the graph of  $x+2y-6=0$ , showing the intercepts on both axes. [2]
- f) Find the equation (in General Form) of the line perpendicular to  $4x-3y-1=0$  that passes through the point  $(2,-3)$ . [2]

**Question Two: (Start a NEW BOOKLET)**

- a) Differentiate with respect to  $x$ :
- i.  $x^2e^x$  [2]
- ii.  $(1+\tan x)^2$  [2]
- b) Find
- i.  $\int 4x - \sin x \, dx$  [2]
- ii.  $\int_1^3 \frac{1}{x^2} \, dx$  [3]
- c) Find the equation of the tangent to the curve  $y = x - \frac{1}{x}$  at the point  $(-1,0)$ . [3]

**Question Three: (Start a NEW BOOKLET)**

- a) Triangle ABC is isosceles with  $AB=AC$ . AB and AC are extended to D and E respectively, with  $BD=CE$ , as shown in the diagram below.



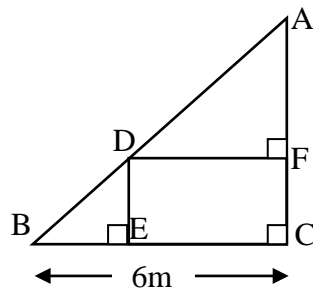
- i. Copy the diagram into your answer booklet showing the information given. [1]
- ii. Prove that  $\triangle ABE \cong \triangle ACD$ . [4]
- b) Using Simpson's Rule with five function values to find an approximate value for the integral  $\int_0^2 e^{x^2} dx$ , to 2 decimal places. [3]
- c) A geometric series has a 3<sup>rd</sup> term of  $\frac{1}{12}$  and an eighth term of  $\frac{-1}{384}$ . For this series:
- i. Find an expression for  $T_n$ . [2]
- ii. Find the sum of the first 8 terms. [1]
- iii. Find the limiting sum. [1]

**Question Four: (Start a NEW BOOKLET)**

- a) Solve  $2 \cos x = \sqrt{3}$  for  $-\pi \leq x \leq \pi$ . [2]
- b) For the parabola  $x^2 = 6(y+1)$ :
- Write down the coordinates of the vertex. [1]
  - Find the coordinates of the focus. [1]
  - Draw a neat sketch of the parabola. [1]
  - Calculate the area bounded by the parabola and the line  $y = 5$ . [3]
- c) Two ordinary dice are rolled and the score is the sum of the numbers on the top faces.
- What is the probability that the score is 5? [2]
  - What is the probability that the score is not 5? [1]
  - What is the probability that the dice show “doubles” (i.e. that both numbers on the top faces are the same)? [1]

**Question Five: (Start a NEW BOOKLET)**

- a) A 10m long ladder (AB) rests against a wall, with its foot (B) 6m from the base (C) of the wall (AC) as shown in the diagram below (Not drawn to Scale). D is a point on the ladder AB.



- How far up the wall does the ladder reach? [1]
  - Explain why  $DF \parallel BC$ . [1]
  - Prove  $\triangle ADF \parallel \triangle ABC$ . [2]
  - Felix climbs the ladder to point D so that he is 3m directly above the ground (E). How far along the ladder (BD) has he climbed? [2]
- b) Find the equation of the normal to the curve  $y = x(x-2)$  when  $x = 2$ . [2]
- c) Given  $g(x) = ax^2 + bx + c$  and that  $g(0) = 4, g(1) = 23, g(-1) = 1$ , determine the values of  $a, b$  and  $c$ . [2]

**Question Six: (Start a NEW BOOKLET)**

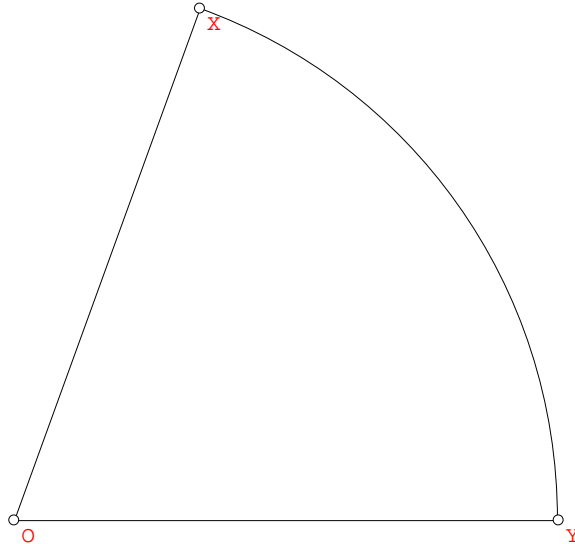
- a) For the function  $y = x^3 - 3x^2 - 9x + 6$  :
- i. At what point does this curve cut the y-axis? [1]
  - ii. Find the coordinates of any stationary points and determine their nature. [3]
  - iii. Find the coordinates of any points of inflection. [1]
  - iv. For what values of x is the curve concave up? [1]
  - v. Sketch the curve, showing the information above. [2]
- b) If  $\sin \theta = -\frac{2}{3}$  and  $\cos \theta > 0$ , find the value of  $\tan \theta$  (in surd form). [2]
- c) Show that the derivative of  $xe^x$  is  $e^x + xe^x$ , and hence find  $\int xe^x dx$ . [2]

**Question Seven: (Start a NEW BOOKLET)**

- a) If  $\alpha$  and  $\beta$  are the roots of  $4x^2 + 8x - 1 = 0$ , find the value of
- i.  $\alpha + \beta$  [1]
  - ii.  $\alpha\beta$  [1]
  - iii.  $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$  [2]
- b) For the curve  $y = 3 \sin 2x$  :
- i. State the amplitude. [1]
  - ii. Find the period. [1]
  - iii. Draw a neat sketch of the curve for  $0 \leq x \leq \pi$ . [2]
- c) Find the value of:
- i.  $\log_2 45$ , given that  $\log_2 3 = 1.585$  and  $\log_2 5 = 2.322$ , without using the change of base law. [1]
  - ii.  $\log_7 0.3$ , using change of base law. [1]
- d) Find  $p$  so that  $9x^2 - 3x + p = 0$  has only one root. [2]

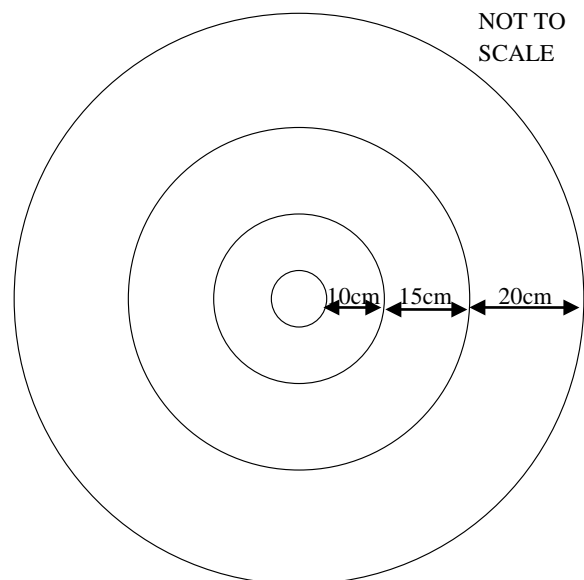
**Question Eight: (Start a NEW BOOKLET)**

- a) In the diagram,  $XY$  is an arc of a circle with centre  $O$  and radius  $12\text{cm}$ . The length of the arc  $XY$  is  $4\pi$  cm.



- i. Find the exact size of  $\theta$  in radians. [1]
  - ii. Find the area of the sector  $OXY$  [1]
- b) The region bounded by the curve  $y = e^x + e^{-x}$ , the  $x$ -axis and the lines  $x = 0$  and  $x = 2$  is rotated about the  $x$ -axis. Find the volume of the solid formed. (Answer in terms of  $e$ ). [3]

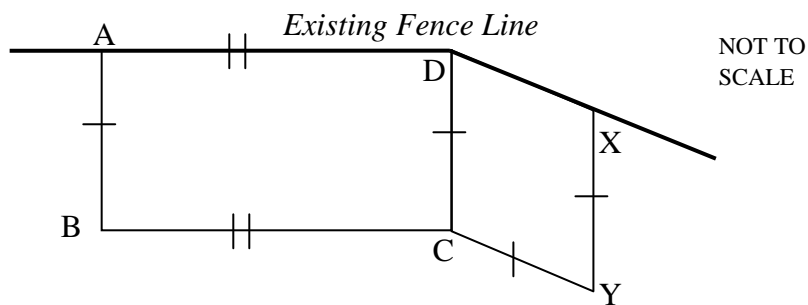
- c) Beginning with a circular piece of fabric of radius  $5\text{cm}$ , Lynn sewed together circular strips of different coloured fabrics which increased in width to make a circular tablecloth. The finished width of the first strip was  $10\text{cm}$ , the second was  $15\text{cm}$ , the third  $20\text{cm}$ , and so on, as shown opposite.



- i. Show that the width of the  $10^{\text{th}}$  strip was  $55\text{cm}$ . [2]
  - ii. The radius of the Tablecloth was  $455\text{cm}$ . How many strips were sewn onto the edge of the first circular piece? [3]
- d) Solve for  $x$ :  $4e^{2x} - e^x = 0$ . [2]

**Question Nine: (Start a NEW BOOKLET)**

- a) For the inequality  $y \leq 4 - x^2$ :
- Shade the region bounded simultaneously by the inequality above, and the inequalities  $x \geq 0$  and  $y \geq 3x$ . [2]
  - Find the volume of the solid of revolution formed when the region defined in (i) above is rotated about the  $y$ -axis. [4]
- b) A farmer needs to construct two holding paddocks, one rectangular (ABCD) and one a rhombus (CDXY) for horses and cattle respectively. The diagram below shows an aerial view of the paddocks, including the use of an existing fence as part of the boundary.



The farmer has only 700m of fencing. We also know that  $\angle CDX = 30^\circ$ .

- By letting  $AB = x$ , show that the area  $A$  of the two paddocks is given by  $A = 700x - \frac{7x^2}{2}$ . [2]
- Hence find the maximum area that can be enclosed. [3]
- Calculate the dimensions for the rectangular paddock, when the overall area is a maximum. [1]



**Question Ten: (Start a NEW BOOKLET)**

- a) Jerry joins a Superannuation fund, investing \$P at the beginning of every year at 9% p.a. compounding annually.
- i. Write an expression for the value of his investment  $A_1$  at the end of the first year. [1]
  - ii. Write an expression for the value of his investment  $A_2$  at the end of the second year. [1]
  - iii. Show that, after  $n$  years, the value of his investment  $A_n$  is given by  
$$A_n = \frac{109P}{9}(1.09^n - 1)$$
 [2]
  - iv. If, after 30 year, he wishes to collect \$1,000,000, calculate the value of \$P to the nearest dollar. [1]
- b) A triangle is right-angled at  $B$ .  $D$  is the point on  $AC$  such that  $BD$  is perpendicular to  $AC$ . Let  $\angle BAC = \theta$ .
- i. Draw a diagram showing this information. [1]
- You are given that  $6AD + BC = 5AC$  :
- ii. Show that  $6 \cos \theta + \tan \theta = 5 \sec \theta$  [2]
  - iii. Deduce that  $6 \sin^2 \theta - \sin \theta - 1 = 0$  [2]
  - iv. Find  $\theta$ . [2]

**Question One:**

a)  $1.242886646$   
 $= 1.243$  (to 3dp)

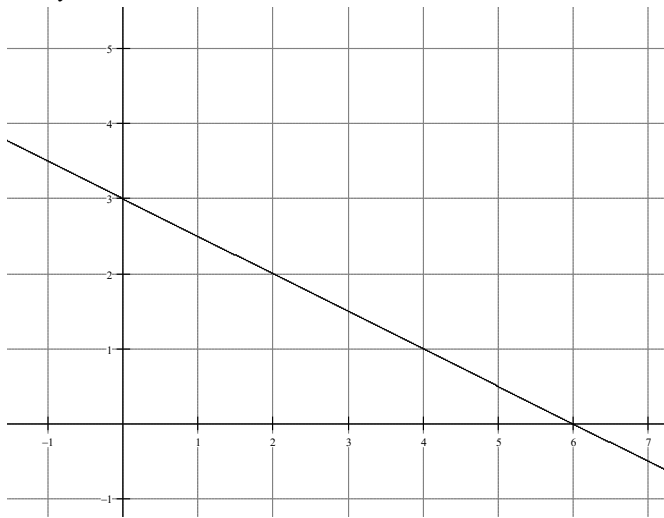
b)  $3 - 2x = 4x$   
 $3 = 6x$   
 $x = \frac{1}{2}$

c)  $\frac{2}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}}$   
 $= \frac{2(1-\sqrt{3})}{1-3}$   
 $= \frac{2(1-\sqrt{3})}{-2}$   
 $= \sqrt{3} - 1$

d)  $= 4 + 12x - x - 3x^2$   
 $= 4(1+3x) - x(1+3x)$   
 $= (1+3x)(4-x)$

e)  $2y = 6 - x$   
 $y = 3 - \frac{1}{2}x$

f)  $x = 0: y = 0:$   
 $y = 3 \quad x = 6$



g)  $3y = 4x - 1$

$$y = \frac{4}{3}x - \frac{1}{3} \Rightarrow m_1 = \frac{4}{3}, \text{ so perp. gives } m_2 = -\frac{3}{4}$$

So through  $(2, -3)$ :

$$y - (-3) = -\frac{3}{4}(x - 2)$$

$$4y + 12 = -3x + 6$$

$$3x + 4y + 6 = 0$$

**Marking**

- ① answer
- ① dp's
- ① algebra

- ① answer

- ①  $\times$  conjugate

- ① answer
- ① resolves pairs

- ① answer

- ① intercepts

**Comments**

In simplest form!

Some had trouble finding these!

- ① graph

- ① perp. gradient

- ① eqn (in GF) Again, simplify!

**Question Two:**

a) i)  $u = x^2 \quad v = e^x$   
 $du = 2x \quad dv = e^x$

$$\therefore \frac{d(x^2 e^x)}{dx}$$

$$= x^2 e^x + 2xe^x$$

$$= xe^x(x+2)$$

ii)  $\frac{d(1 + \tan x)^2}{dx}$

$$= 2(1 + \tan x)^1 \times \frac{d(\tan x)}{dx}$$

$$= 2 \sec^2 x (1 + \tan x)$$

b) i)  $\int 4x - \sin x \, dx$

$$= 2x^2 + \cos x + c$$

ii)  $\int_1^3 \frac{1}{x^2} \, dx$

$$= \left[ -\frac{1}{x} \right]_1^3$$

$$= \frac{-1}{3} - \frac{-1}{1}$$

$$= \frac{2}{3}$$

c)  $\frac{dy}{dx} = 1 + \frac{1}{x^2}$ ,

so when  $x = -1 \quad \frac{dy}{dx} = 2$

Hence  $y - 0 = 2(x + 1)$

or  $y = 2x + 2$

**Marking****Comments**

① product rule    Mostly good

① answer

① chain rule

Some struggled with structure of the Chain Rule.

① answer

Some did not know the derivative of tanx.

① answer

Quite a few integration errors, and some forgot "+c"

① for '+c'

① int & limits

Many 2 Unit candidates got a log or differentiated.

① subst

Many also made sign errors in substitution.

① answer

① derivative

Some could not find derivative.

① gradient

Many errors to find "m=2"

① answer

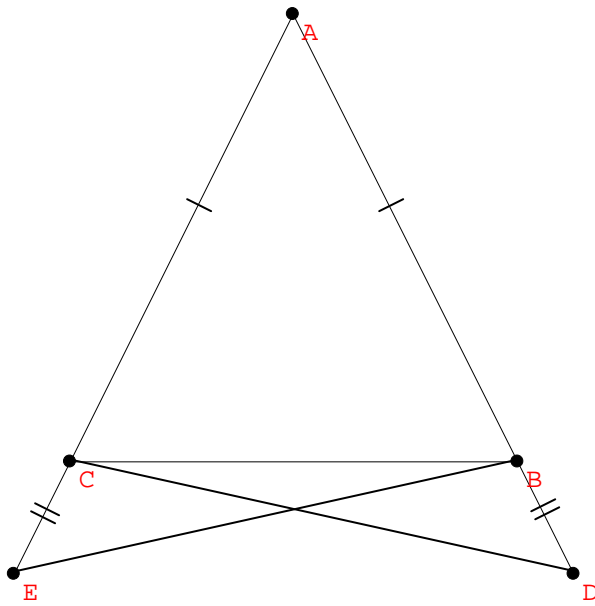
("m=0" was popular). More seriously, not finding an "m" value but using algebra:

$$y = \left(1 + \frac{1}{x^2}\right)(x+1)$$

is meaningless!

**Question Three:**

a) i)



**Marking**

**Comments**

ii)  $AB=AC$  and  $BE=CD$  (given)  
 $AD=AB+BD$ ,  $AE=AC+CE$  (st. lns), so  
 $AD=AC+CE$ , hence  
 $AD=AE$

In  $\Delta$ 's  $ABE, ACD$  ←

- i)  $AB = AC$  (given)
  - ii)  $\angle EAB = \angle DAC$  (common angle)
  - iii)  $AD=AE$  (shown above)
- $\therefore \Delta ABE \equiv \Delta ACE$  (SAS)

b) For  $y = e^{x^2}$ , with  $h = 0.5$ :

$x$	0	0.5	1	1.5	2
$y_i$	1	$e^{0.25}$	$e$	$e^{2.25}$	$e^4$

Simpsons Rule:

$$A = \frac{h}{3} [(y_0 + y_4) + 2(y_1 + y_3) + 4y_2]$$

$$= \frac{0.5}{3} [(1 + e^4) + 4(e + e^{2.25}) + 2e]$$

$$\doteq 17.35362645$$

$$\doteq 17.35$$

c) With  $T_3 = \frac{1}{12}, T_8 = \frac{-1}{384}$ :

i.  $ar^2 = \frac{1}{12}$  and  $ar^7 = \frac{-1}{384}$ , hence

1 markings

1 reason

1 reason

1 reason

1 conclusion

1 values

1 subst

1 ans to 2dp

Some students failed to give all the information required.

Some did not show this (or equivalent);  
 Need to state the triangles;

Some did not state the test used.

Some students had the 4 and the 2 the wrong way around in the formula.

$$\frac{T_8}{T_3} = \frac{-1}{384} \div \frac{1}{12}$$

$$\frac{ar^7}{ar^2} = \frac{-1}{384} \times \frac{12}{1}$$

$$r^5 = \frac{-1}{32}$$

$$r = \frac{-1}{2}$$

$$a \cdot \left(\frac{-1}{2}\right)^2 = \frac{1}{12}$$

$$a = \frac{1}{3}$$

$$T_n = ar^n$$

$$= \frac{1}{3} \cdot \left(\frac{-1}{2}\right)^n$$

$$\text{ii. } S_n = \frac{a(1-r^n)}{1-r}$$

$$S_8 = \frac{\frac{1}{3} \left(1 - \left(\frac{-1}{2}\right)^8\right)}{1 - \frac{-1}{2}}$$

$$= \frac{\frac{1}{3} \left(1 - \frac{1}{256}\right)}{\frac{3}{2}}$$

$$= \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{255}{256}$$

$$= \frac{85}{384}$$

$$\text{iii. } S_\infty = \frac{a}{1-r}$$

$$= \frac{\frac{1}{3}}{1 - \frac{-1}{2}}$$

$$= \frac{1}{3} \cdot \frac{2}{3}$$

$$= \frac{2}{9}$$

This part done well provided the correct ratio of

$\frac{-1}{2}$  found (some

used  $\frac{\pm 1}{2}$  or  $\frac{1}{2}$ ).

①  $a, r$  values

①  $T_n$  correct

①  $S_8$  correct

This answer best left as a fraction.

①  $S_\infty$  correct

**Question Four:**

a)  $\cos x = \frac{\sqrt{3}}{2}$

$$x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\pi}{6}$$

$\cos x$  positive in Q1 & Q4, but with  $-\pi \leq x \leq \pi$ ,

$$x = \frac{\pi}{6}, \frac{-\pi}{6}$$

b)

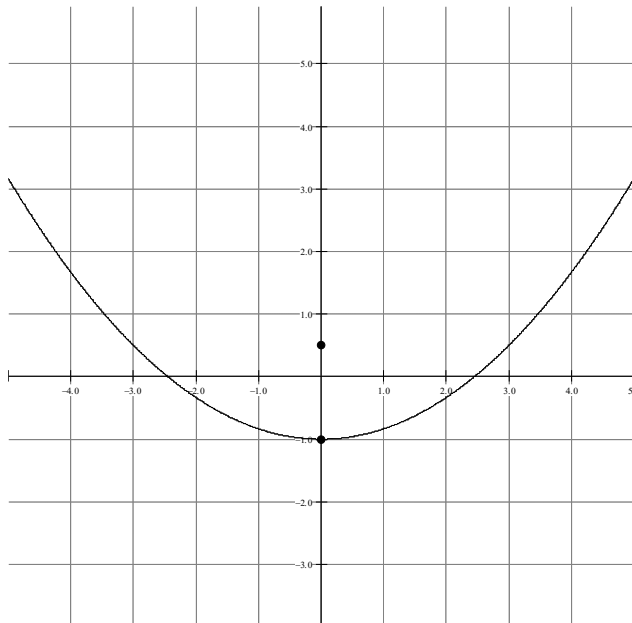
i. Vertex is  $(0, -1)$

ii.  $4a = 6$

Hence  $a = \frac{3}{2}$ , so focus is  $(0, -1 + a)$  or

$$\left(0, \frac{1}{2}\right)$$

iii.



iv.  $y = 5$  and  $x^2 = 6(y + 1)$

$$\begin{aligned} x^2 &= 6(5 + 1) \\ &= 36 \end{aligned}$$

$$y + 1 = \frac{x^2}{6}$$

$$y = \frac{x^2}{6} - 1$$

$x = \pm 6$ , hence

**Marking**

**Comments**

1 base angle

Many did not read the requirements of the question:

1 answers

$-\pi \leq x \leq \pi$  also implies radians!

1 answer

1 answer

Sketch was poorly done by many – must show vertex, focus, intercepts.

Use a ruler to draw axes!

1 graph

1 eqn for Int

Poorly done. Students need to review area between two curves.

1 boundaries

$$\begin{aligned}
 A &= \int_{-6}^6 5 - \left( \frac{x^2}{6} - 1 \right) dx \\
 &= 2 \int_0^6 5 - \left( \frac{x^2}{6} - 1 \right) dx \\
 &= 2 \left[ 6x - \frac{x^3}{18} \right]_0^6 \\
 &= 2 \left[ \left( 6^2 - \frac{6^3}{18} \right) - 0 \right] \\
 &= 48
 \end{aligned}$$

1 answer

c)

- i. First die 1 to 4 must correspond to 2<sup>nd</sup> die 4 to 1, so 4 outcomes give a total of 5

1 reason for 4

$$\begin{aligned}
 P(5) &= \frac{4}{36} \\
 &= \frac{1}{9}
 \end{aligned}$$

1 answer

- ii.  $P(\text{not } 5) = 1 - P(5)$

$$\begin{aligned}
 &= 1 - \frac{1}{9} \\
 &= \frac{8}{9}
 \end{aligned}$$

1 answer

- iii.  $P(\text{doubles}) = \frac{1}{6}$

1 answer

**Question Five:**

a)

i. By Pythagoras:  $AC^2 = 10^2 - 6^2$   
 $= 64$

$$AC = 8$$

ii.  $\angle DFA = \angle BCA = 90^\circ$

For  $BC$  and  $DF$ ,  $\angle DFA$  and  $\angle BCA$  are in a corresponding position and equal.

Hence  $BC \parallel DF$ .

iii. In  $\Delta$ 's  $ADF, ABC$

$$\angle DFA = \angle BCA = 90^\circ \text{ (given in diagram)}$$

$\angle A$  is common.

Hence all angles are equal so

$$\triangle AFD \parallel \triangle ABC$$

iv. Similarly,  $\triangle BDE \parallel \triangle ABC$ , hence

$$\frac{DE}{AC} = \frac{DB}{AB}$$

$$\frac{3}{8} = \frac{DB}{10}$$

$$DB = 3.75$$

b)  $y = x^2 - 2x$ ;  $x = 2, y = 0$

$$\frac{dy}{dx} = 2x - 2; x = 2, \frac{dy}{dx} = 2, \text{ so normal gradient is}$$

$$\frac{-1}{2}$$

$$y - 0 = \frac{-1}{2}(x - 2)$$

$$y = 1 - \frac{x}{2}$$

$$0 = x + 2y - 2$$

c)  $g(0) = 4:4 = a \cdot 0^2 + b \cdot 0 + c$

$$g(1) = 23:23 = a \cdot 1^2 + b \cdot 1 + c$$

$$g(-1) = 1:1 = a \cdot (-1)^2 + b \cdot (-1) + c, \text{ giving the eqns}$$

$$4 = c \quad \langle 1 \rangle$$

$$23 = a + b + c \quad \langle 2 \rangle$$

$$1 = a - b + c \quad \langle 3 \rangle$$

$\langle 1 \rangle$  in  $\langle 2 \rangle$  and  $\langle 3 \rangle$  gives:

$$a + b = 19$$

$a - b = -3$ , then adding gives:

$$2a = 16; \text{ back-substitution gives } 23 = 8 + b + 4$$

$$a = 8$$

$$b = 11$$

Hence  $a = 8, b = 11, c = 4$

**Marking****Comments**

1 answer

1 reasoning

1 reasons

1 conclusion

1 subst

1 answer

1 gradient

1 answer

1 set-up

1 answers

Some students failed to recognise corresponding angles, or failed to write that fact down!

Parts b) and c) generally well done.



**Question Six:**

a)

i. y-intercept at (0,6).

ii.  $y' = 3x^2 - 6x - 9$

$y'' = 6x - 6$

Stat Pts when  $y' = 0$ :

$0 = 3x^2 - 6x - 9$

$= x^2 - 2x - 3$

$= (x-3)(x+1)$

$\therefore x = -1, 3$

$x = -1$

$x = 3$

$y = (-1)^3 - 3(-1)^2 - 9(-1) + 6$      $y = (3)^3 - 3(3)^2 - 9(3) + 6$

$= 11$

$= -21$

Pts are (-1,11) and (3,-21)

$x = -1$      $x = 3$

$y'' = -12$      $y'' = 12$

$\Rightarrow ccd$      $\Rightarrow ccu$

$\therefore (-1,11)$  is a max t.p.

and  $(3,-21)$  is a min t.p.

iii.  $y'' = 0: 0 = 6x - 6$ , hence possible

inflection pt when  $x = 1$ .

when  $x < 1, y'' < 0 \Rightarrow ccd$

when  $x > 1, y'' > 0 \Rightarrow ccu$

hence concavity changes, so  $(1,-5)$  is a point of inflexion.

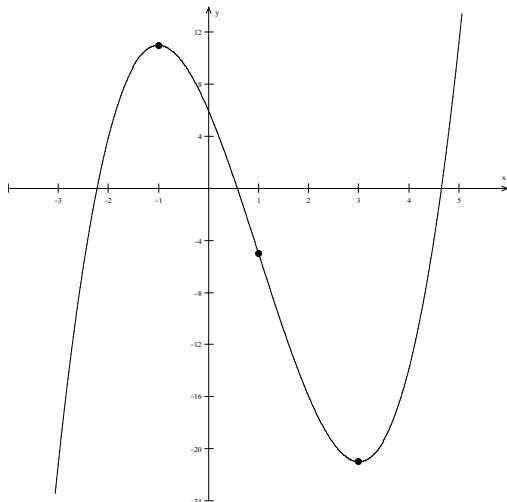
iv. Concave up when  $y'' > 0$

$6x - 6 > 0$

$6x > 6$

i.e. when  $x > 1$

v.



**Marking**

**Comments**

1 answer

1 x values

1 test

1 points

1 point & test

1 answer

1 t.p's

1 intercepts

Make sure co-ordinates are stated when the question asks for them

Must test the nature of Stat. Pts and Inflexion Pts!

Sketch very poorly done.

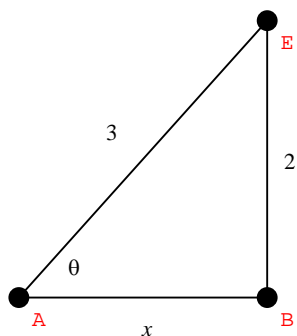
Use:

- a ruler for axes
- a suitable scale

Show the information requested:

- turning points
- inflexion point
- intercepts (x and y)

$$\text{b) } \sin \theta = \frac{-2}{3}$$



$$\text{Hence } x^2 = 3^2 - 2^2, \text{ or } x = \sqrt{5}$$

$$\sin \theta < 0 \Rightarrow Q_3, Q_4$$

$$\cos \theta > 0 \Rightarrow Q_1, Q_4 \Rightarrow \tan \theta \text{ in } Q_4; \tan \theta < 0$$

$$\text{Hence } \tan \theta = \frac{-2}{\sqrt{5}}$$

$$\text{c) } \frac{d(xe^x)}{dx} \quad \begin{array}{l} u = x \quad v = e^x \\ u' = 1 \quad v' = e^x \end{array}$$

$$= xe^x + 1 \cdot e^x$$

$$= e^x + xe^x \text{ as reqd.}$$

$$\text{Hence } \frac{d(xe^x)}{dx} = e^x + xe^x, \text{ so integrating gives:}$$

$$xe^x = \int (e^x + xe^x) dx$$

$$= \int e^x dx + \int xe^x dx$$

$$\therefore \int xe^x dx = xe^x - \int e^x dx$$

$$= xe^x - e^x + c$$

$$= e^x(x-1) + c$$

Some students did not know the ASTC results.

1 quadrant

1 answer

1 product rule Showing a result – you must clearly demonstrate the link for each step.

1 answer

**Question Seven:**

a)  $4x^2 + 8x - 1 = 0$ :

i.  $\alpha + \beta = \frac{-8}{4}$   
 $= -2$

ii.  $\alpha\beta = \frac{-1}{4}$

iii.  $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$   
 $= \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2}$   
 $= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$   
 $= \frac{(-2)^2 - 2 \cdot \frac{-1}{4}}{\left(\frac{-1}{4}\right)^2}$   
 $= 16\left(4 + \frac{1}{2}\right)$   
 $= 72$

b)  $y = 3\sin 2x$

i. Amplitude is 3

ii. Period is  $\frac{2\pi}{2}$ , or  $\pi$

iii. 1 amplitude, 1 period/shape

**Marking**

**Comments**

1 answer

1 answer

1 resolves

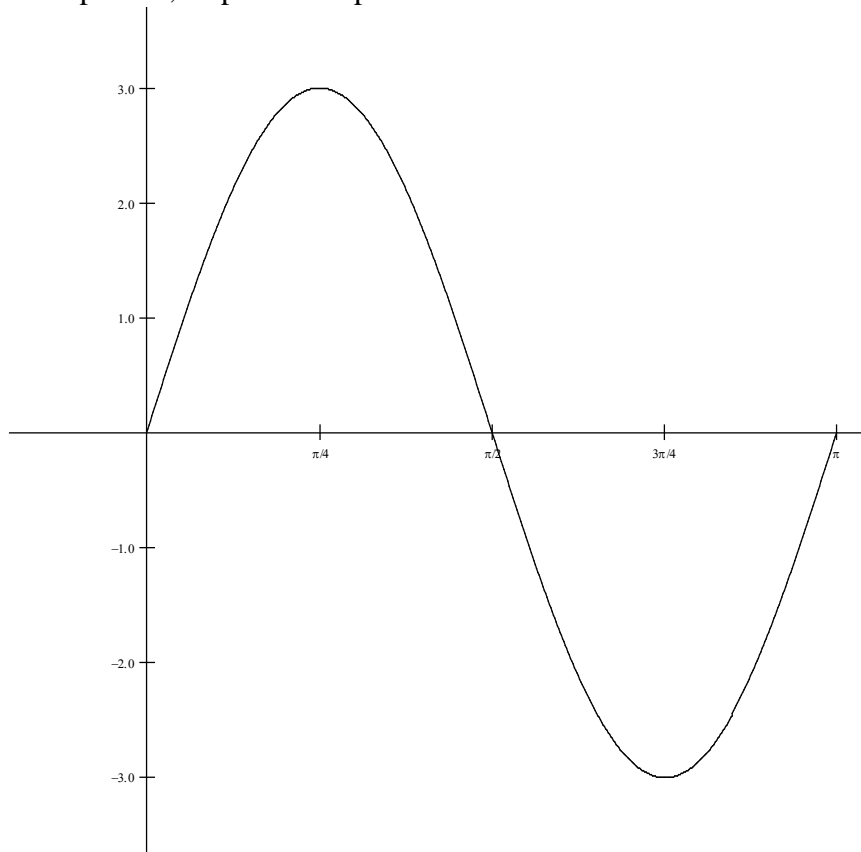
1 answer

1 answer

1 answer

Many had problems with this identity

Some drew Cos!



c)

$$\begin{aligned}
 \text{i. } \log_2 45 & \\
 &= \log_2(5 \times 9) \\
 &= \log_2 5 + \log_2 3^2 \\
 &= \log_2 5 + 2\log_2 3 \\
 &= 2.322 + 2 \times 1.585 \\
 &= 5.492
 \end{aligned}$$

Many multiplied the logs, instead of adding them.

1 answer

$$\begin{aligned}
 \text{ii. } \log_7 0.3 & \\
 &= \frac{\ln 0.3}{\ln 7} \\
 &= -0.6187196284 \\
 &\approx -0.619
 \end{aligned}$$

1 answer

$$\text{d) } 9x^2 - 3x + p = 0$$

For only one root,  $\Delta = 0$

Too many had  $\Delta > 0$  for one real root!

$$\begin{aligned}
 \therefore b^2 - 4ac &= 0 \\
 0 &= 9 - 4 \cdot 9 \cdot p \\
 36p &= 9 \\
 p &= \frac{1}{4}
 \end{aligned}$$

1 set-up

1 answer

This was also successfully resolved by sums and products of roots by many:

Alternate marking for (d):

$$\alpha + \beta = -\frac{-3}{9}, \alpha\beta = \frac{p}{9} \quad \alpha = \beta, \text{ giving:}$$

$$\begin{aligned}
 2\alpha &= \frac{1}{3} \\
 \alpha &= \frac{1}{6} (= \beta)
 \end{aligned}$$

1 uses = roots

$$\left(\frac{1}{6}\right)^2 = \frac{p}{9}$$

$$p = \frac{9}{36}$$

$$= \frac{1}{4}$$

1 answer

**Question Eight:**

a)

$$\text{i. } l = r\theta$$

$$4\pi = 12\theta$$

$$\therefore \theta = \frac{\pi}{3}$$

$$\text{ii. } A = \frac{1}{2}r^2\theta$$

$$= \frac{1}{2} \cdot 12^2 \cdot \frac{\pi}{3}$$

$$= 24\pi \text{ cm}^2$$

$$\text{b) } y = e^x + e^{-x}, \text{ so}$$

$$y^2 = (e^x + e^{-x})^2$$

$$= e^{2x} + 2e^x e^{-x} + e^{-2x}$$

$$= e^{2x} + e^{-2x} + 2$$

Volume is given by:

$$v = \int_0^2 \pi y^2 dx$$

$$= \pi \int_0^2 (e^{2x} + e^{-2x} + 2) dx$$

$$= \pi \left[ \frac{1}{2}e^{2x} - \frac{1}{2}e^{-2x} + 2x \right]_0^2$$

$$= \pi \left[ \left( \frac{1}{2}e^4 - \frac{1}{2}e^{-4} + 4 \right) - \left( \frac{1}{2}e^0 - \frac{1}{2}e^0 + 0 \right) \right]$$

$$= \frac{\pi}{2}(e^4 - e^{-4} + 8) \text{ cu. units}$$

c)

$$\text{i. } T_1 = 10, T_2 = 15, T_3 = 20 \dots$$

$$T_2 - T_1 = 5; T_3 - T_2 = 5, \text{ so this is an AP}$$

$$\text{with } a = 10, d = 5, \text{ hence}$$

$$T_n = 10 + 5(n-1)$$

$$= 5 + 5n$$

When  $n = 10$ 

$$T_n = 5 + 5 \cdot 10$$

$$= 55$$

$$\text{ii. } S_n = \frac{n}{2}(2a + (n-1)d), \text{ so radius will be}$$

$$r = 5 + S_n, \text{ so}$$

$$S_n = 455 - 5$$

$$= 450$$

**Marking****Comments**

Well done.

1 answer

Mostly good. A few tried to find the area of a segment or triangle.

1 answer

Students who could expand  $y^2$  correctly generally got full marks.1  $y^2$  correct

A few made horrible integration attempts ☹:

$$\int e^{2x} dx = \left[ \frac{e^{x^2}}{x^2} \right]$$

1 int &amp; limits

Some mistakes substituting:  
 $e^0 - e^0$ 

1 answer

i) Was poorly set

out – when a question says “Show that...”

you must show your thinking, theory used and calculations!

1 justifies AP then shows subst

For full marks, students needed to show that they

1 to get answer

recognised an AP and then substituted.

1 radius correct

A good answer would have proved an AP (as shown in solns).

$$450 = \frac{n}{2}(2 \times 10 + 5(n-1))$$

$$900 = n(15 + 5n)$$

$$0 = 5n^2 + 15n - 900$$

$$= n^2 + 3n - 180$$

$$0 = (n+15)(n-12)$$

$$n = -15, 12$$

As  $n > 0$ , there are 12 strips needed.

d)  $4e^{2x} - e^x = 0$ , let  $u = e^x$ , then

$$0 = 4u^2 - u$$

$$= u(4u - 1)$$

$$u = 0, \frac{1}{4}, \text{ hence}$$

$$e^x = 0, e^x = \frac{1}{4}$$

$e^x = 0$  has no solution.

$$e^x = \frac{1}{4}$$

$$x = \ln\left(\frac{1}{4}\right)$$

$$= -2 \ln 2$$

$$(= -1.386294361)$$

Students needed to recognise the extra 5cm centre and deduct it; for full marks, justification for the positive solution was needed.

① forms quadratic

① answer justified

① resolves quad

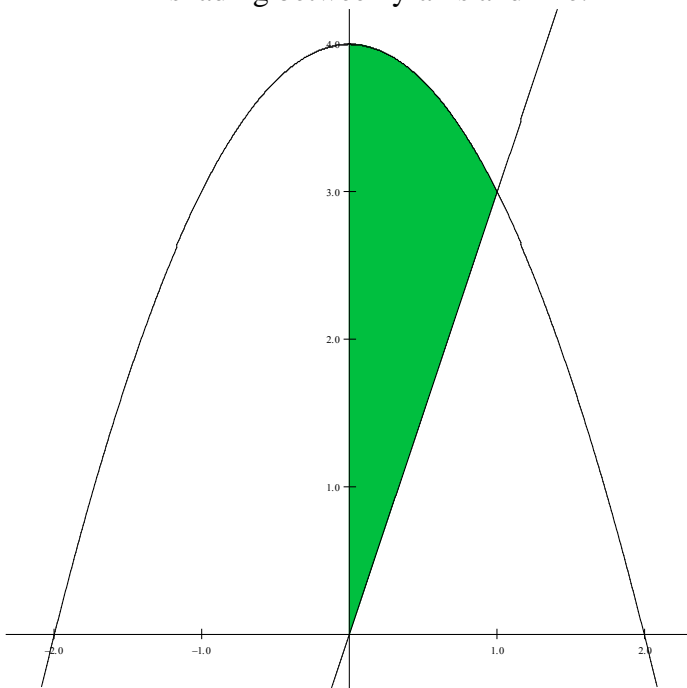
For full marks, students needed to give the two expressions for  $e^x$  and state that  $e^x = 0$  has no solution.

① answer justified

**Question Nine:**

a)  $y \leq 4 - x^2$

- i. ① shading below parabola,  
 ② shading between y-axis and line.



- ii. Solving  $y = 3x$  and  $y = 4 - x^2$  simultaneously:

$$3x = 4 - x^2$$

$$0 = x^2 + 3x - 4$$

$$= (x + 4)(x - 1)$$

$$x = -4, 1$$

Hence intersection at (1, 3)

So volume of solid to y-axis is  $V = \pi \int_a^b x^2 dy$

$$y = 4 - x^2 \text{ becomes } x^2 = 4 - y$$

$$y = 3x \text{ becomes } x = \frac{y}{3} \text{ or } x^2 = \frac{y^2}{9}$$

$$V = \pi \int_0^3 \frac{y^2}{9} dy + \pi \int_3^4 4 - y dy$$

**Marking**

**Comments**

① values

① both  $x^2$  eqns

① int & limits

Poorly done. Most students failed to establish the equation in terms of y, and hence did not find the correct y values for the integration. Many also did not recognise the need to split the integral into two parts.

$$\begin{aligned}
&= \pi \int_0^3 \frac{y^2}{9} dy + \int_3^4 4 - y dy \\
&= \pi \left( \left[ \frac{y^3}{27} \right]_0^3 + \left[ 4y - \frac{y^2}{2} \right]_3^4 \right) \\
&= \pi \left( \left( \frac{27}{27} - 0 \right) + \left( \left( 16 - \frac{16}{2} \right) - \left( 12 - \frac{9}{2} \right) \right) \right) \\
&= \pi \left( 1 + 8 - \frac{15}{2} \right) \\
&= \frac{3\pi}{2} \text{ cu. units}
\end{aligned}$$

① answer

b)  $AB = x$ , hence  $CD, CY$  and  $XY$  are all also  $x$ .

i. Total length of fencing is given by:

$$700 = AB + CD + CY + XY + BC$$

$$BC = 700 - 4x$$

For rhombus  $CDXY$ :

$$A = 2 \times \frac{1}{2} CD \cdot DY \cdot \sin 30^\circ$$

$$= \frac{x^2}{2}$$

For rectangle  $ABCD$ :

$$A = AB \cdot BC$$

$$= x(700 - 4x)$$

$$= 700x - 4x^2$$

Total area is therefore:

$$A = 700x - 4x^2 + \frac{1}{2}x^2$$

$$= 700x - \frac{7x^2}{2} \quad \text{as reqd.}$$

① Perim link

Poorly done. Most students did not use the sine version of the area of a triangle.

① areas & alg

ii. For a possible maximum,  $\frac{dA}{dx} = 0$ :

$$\therefore \frac{dA}{dx} = 700 - 7x; \quad \frac{dA}{dx} = 0 \text{ gives}$$

$$0 = 700 - 7x$$

$$x = 100$$

$$\frac{d^2A}{dx^2} = -7 \Rightarrow \text{ccd, or a max tp.}$$

Hence max area is

$$A = 700 \times 100 - \frac{7 \times 100^2}{2}$$

$$= 35000 \text{ sq.m}$$

① x-value

① max shown

① answer

iii. Paddock is therefore  $100\text{m}$  by  $300\text{m}$ .

① answer



**Question Ten:**

a) \$P invested at 9% p.a.

i. First Year:  $A_1 = P(1 + 0.09)$

or  $A_1 = 1.09P$

ii. Second Year:  $A_2 = (A_1 + P)(1 + 0.09)$

or  $A_2 = (1.09P + P)(1.09)$

$$= 1.09^2 P + 1.09P$$

$$= P(1.09^2 + 1.09)$$

iii. After  $n$  years:  $A_n = (A_{n-1} + P)(1 + 0.09)$

Using the above pattern, this becomes:

$$A_n = P(1.09^n + 1.09^{n-1} + \dots + 1.09)$$

Now, this is a GP with

 $a = 1.09, r = 1.09, n = n$ , thus

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{1.09(1.09^n - 1)}{1.09 - 1}$$

$$= \frac{1.09(1.09^n - 1)}{0.09}$$

$$= \frac{1.09}{0.09}(1.09^n - 1)$$

$$= \frac{109(1.09^n - 1)}{9}$$

Hence  $A_n = \frac{109P}{9}(1.09^n - 1)$ , as reqd.iv. For  $A_n = \$1,000,000$  and  $n=30$ :

$$1,000,000 = \frac{109P}{9}(1.09^{30} - 1)$$

$$\frac{9000000}{109} = P(1.09^{30} - 1)$$

$$P = \frac{9000000}{109(1.09^{30} - 1)}$$

$$= 6730.597606$$

$$= \$6731 \quad (\text{nearest } \$)$$

**Marking****Comments**

1 answer

1 answer

1 GP &amp; values

1 GP resolved

1 answer

Need to state this is a GP and write the formula

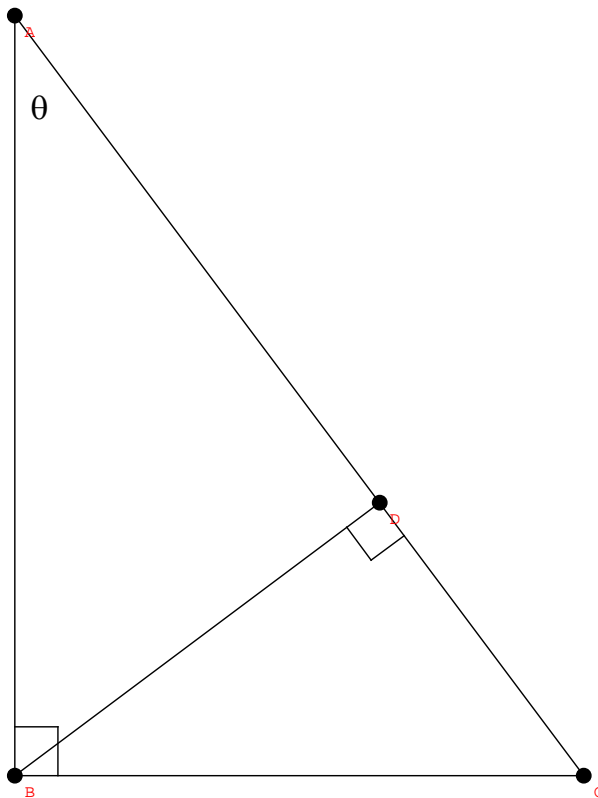
This step 'fudged' by many.

Check degree of accuracy required (no marks lost if gave to nearest  $\phi$ ).

b)

i.

1 diagram



ii. Given  $6AD + BC = 5AC$ , expressions for  $AD, BC$  and  $AC$  in terms of  $\theta$ :

$$\cos \theta = \frac{AD}{AB} \quad (\text{from } \triangle ADB)$$

$$AD = AB \cos \theta$$

$$\tan \theta = \frac{BC}{AB} \quad \text{and} \quad \cos \theta = \frac{AB}{AC} \quad (\text{from}$$

$$BC = AB \tan \theta$$

$$AC = \frac{AB}{\cos \theta}$$

$\triangle ABC$ )

Substituting these into the expression above:

$$6AB \cos \theta + AB \tan \theta = 5 \frac{AB}{\cos \theta}$$

Hence,  $6 \cos \theta + \tan \theta = 5 \sec \theta$  as reqd.

iii.  $6 \cos \theta + \tan \theta = 5 \sec \theta$ , becomes

$$6 \cos \theta + \frac{\sin \theta}{\cos \theta} = \frac{5}{\cos \theta}$$

$$6 \cos^2 \theta + \sin \theta = 5$$

$$6(1 - \sin^2 \theta) + \sin \theta = 5$$

$$6 - 6 \sin^2 \theta + \sin \theta - 5 = 0$$

Hence  $6 \sin^2 \theta - \sin \theta - 1 = 0$  as reqd.

If students saw the common connection of  $AB$ , they generally did well with the question.

1 exp for each

1 correct subst & alg

Many poor with Trig identities!

1 sin/cos resolved

1 subst & alg

iv. Let  $u = \sin \theta$

$$0 = 6u^2 - u - 1$$

$$= 6u^2 - 3u + 2u - 1$$

$$= 3u(2u - 1) + 1(2u - 1)$$

$$= (2u - 1)(3u + 1)$$

Hence

$$0 = 2\sin \theta - 1 \quad 0 = 3\sin \theta + 1$$

$$\frac{1}{2} = \sin \theta \quad \frac{-1}{3} = \sin \theta$$

$$\theta = 30^\circ \quad \text{or} \quad \theta \approx 199^\circ 28'$$

Reject  $199^\circ 28'$ , as  $\theta$  is in a right triangle,  
so  $\theta = 30^\circ$

① quad resolved

①  $\theta$  correct

Need to solve 2 eqns (a negative angle was not acceptable – need the reflex angle).

Need to give both solutions and reject the invalid one with correct reasoning given.