

Student's name

Student's number

Teacher's name



PLC PRESBYTERIAN
LADIES' COLLEGE
SYDNEY
1888

2010
TRIAL
HIGHER SCHOOL CERTIFICATE
EXAMINATION

Mathematics

General Instructions

- Reading time - 5 minutes
- Working time - 3 hours
- Write using blue or black pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total Marks - 120

- Attempt questions 1-10
- All questions are of equal value

1	2	3	4	5	6	7	8	9	10	Total	Total
										/120	%

Question 1 (12 marks)		Start a new sheet of writing paper.	Marks
a)	Evaluate $\frac{e^5}{(-2)^2}$ correct to 3 significant figures.		2
b)	Simplify $\frac{x-1}{1-x}$		1
c)	Solve $ 2x-1 \geq 6$ and graph your solution on a number line.		3
d)	Given $\frac{2}{\sqrt{3}-2} = a\sqrt{3} + b$, find the values of a and b .		2
e)	The velocity of a particle is given by the equation $v = \frac{\log_e(t-1)}{2}$, metres per second. Find the acceleration at $t = 2$ seconds.		2
f)	Factorise fully $x^6 - 64$.		2

End of Question 1

Question 2 (12 marks) **Start a new sheet of writing paper.**

Marks

a) Differentiate, with respect to x :

i) $\frac{\sin x}{x}$ **2**

ii) $\log_e (5x-4)^3$ **2**

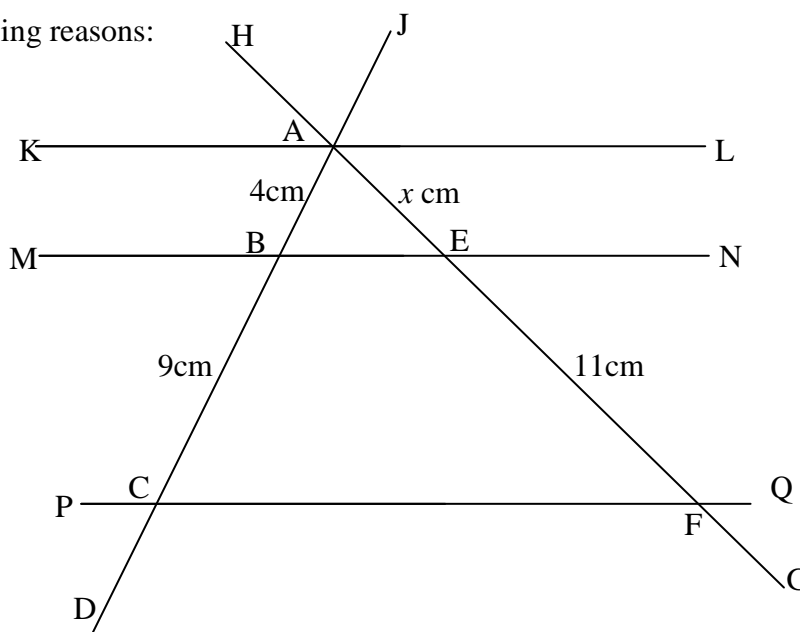
b) Find $\int 2e^{3x-5} dx$. **1**

c) Evaluate $\int_1^2 \frac{3x}{4x^2-2} dx$ leaving your answer in exact form. **3**

d) Find the exact value of $\sin 495^\circ \tan 240^\circ$ **2**

e) If KL, MN and PQ are all parallel lines, **2**

find x giving reasons:



End of Question 2

Question 3 (12 marks) **Start a new sheet of writing paper.**

Marks

$A(2, 6)$ and $B(-3, 1)$ are points on the number plane.

- a) On your answer sheet, plot the points on a number plane (at least one-third of a page). **1**
- b) Find the coordinates of C , the midpoint of AB . **1**
- c) Find the gradient of the line AB . **1**
- d) Show that the equation of AB is $y = x + 4$. **1**
- e) Show that the equation of the perpendicular bisector to AB is $x + y - 3 = 0$. **2**
- f) Show that $D(6, -3)$ lies on the perpendicular bisector. **1**
- g) Find the coordinates of E such that $ADBE$ is a rhombus. **1**
- h) Show that the perpendicular distance from D to AB is $\frac{13\sqrt{2}}{2}$ units. **2**
- i) Find the distance AB in surd form. **1**
- j) Hence, or otherwise, find the area of $ADBE$. **1**

End of Question 3

Question 4 (12 marks) **Start a new sheet of writing paper.**

Marks

a) Solve $2 \sin \theta + \sqrt{3} = 0$ for $0 \leq \theta \leq 2\pi$ **2**

b) Find the value(s) of k in $x^2 - kx + 3k - 8 = 0$ if:

i) 2 is a root of the quadratic. **1**

ii) The roots are equal in magnitude but opposite in sign. **1**

iii) The roots are real. **3**

iv) The roots are reciprocals of one another. **1**

c)

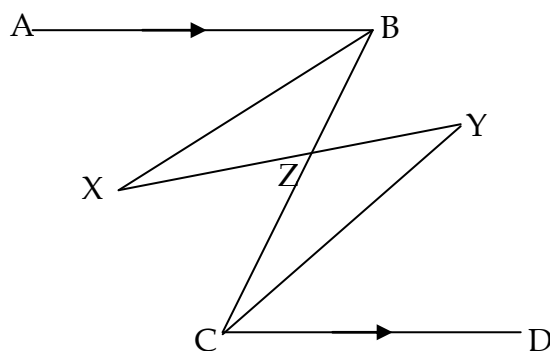


DIAGRAM NOT DRAWN TO SCALE

In the diagram above, AB is parallel to CD . XB bisects $\angle ABC$ and YC bisects $\angle BCD$. $BX = CY$,

i) Copy the diagram into your answer booklet, showing all information. **1**

ii) Prove that Z is the midpoint of BC . **3**

End of Question 4

Question 5 (12 marks) **Start a new sheet of writing paper.** **Marks**

- a) If the sum of n terms of the series $15+13+11+\dots$ is 55, find the number of terms possible in the series. **2**
- b) i) Find the sum of the series: $4+8+16+\dots+1024$ **2**
- ii) Hence, or otherwise, simplify $2^4 \times 2^8 \times 2^{16} \times \dots \times 2^{1024}$, leaving your answer in index form. **1**
- c) i) Find the vertex of the parabola with focus at $(3,2)$ and directrix at $x=5$ **1**
- ii) Hence, or otherwise, state the equation of the parabola. **1**
- iii) Show that the points of intersection of the parabola and the line $2x+y-6=0$ are $(3,0)$ and $(0,6)$. **2**
- iv) Find the area between the parabola and the line in the first quadrant. **3**

End of Question 5

Question 6 (12 marks) **Start a new sheet of writing paper.**

Marks

$$\text{If } f(x) = 6x^3 + 9x^2 - 3,$$

- a) i) Show that $6x^3 + 9x^2 - 3 = 3(x+1)^2(2x-1)$ **1**
- ii) Hence find the x intercepts. **2**
- b) Determine the y intercept. **1**
- c) Find the stationary point(s) and determine their nature. **4**
- d) Find the point(s) of inflexion. **2**
- e) On a number plane (at least one-third of a page) sketch the curve **2**
- $$f(x) = 6x^3 + 9x^2 - 3$$
- showing all of the above features.

End of Question 6

Question 7 (12 marks) **Start a new sheet of writing paper.**

Marks

a) Find the equation of the tangent to the curve $y = x^2 + \frac{1}{x}$ at the point where $x = 1$ (answer in gradient-intercept form) **3**

b) The velocity, v m/s, of a particle moving along a straight line is given by $v = 2 + t - 3t^2$ where t is in seconds. If the particle is initially at the origin,

i) Find when the particle is at rest **2**

ii) Sketch the velocity function from $t = 0$ to $t = 4$ seconds. **2**

iii) Find the total distance travelled by the particle in the first 4 seconds. **3**

c) The graph of $y = f'(x)$ is given below:

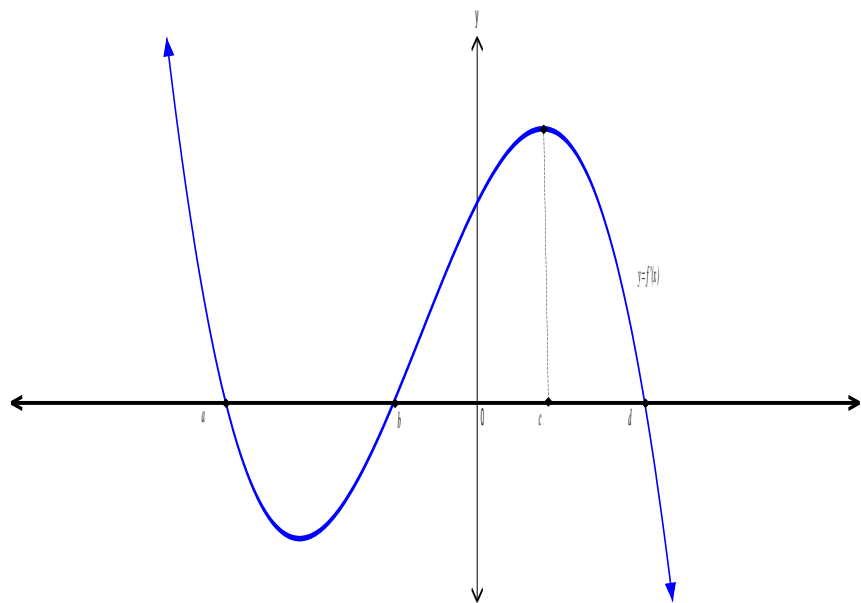


DIAGRAM NOT DRAWN TO SCALE

Copy or trace the curve into your answer booklet and carefully sketch and label the graph of $y = f(x)$ on the same set of axes. You may need to highlight your answer. **2**

End of Question 7

Question 8 (12 marks) **Start a new sheet of writing paper.** **Marks**

- a) i) Sketch $y = 1 - \sin 2x$ for $-\pi \leq x \leq \pi$ **2**
- ii) Show that the equation of the tangent to the curve $y = 1 - \sin 2x$ at $x = 0$ is $2x + y = 1$. **2**
- iii) For what range of values of m does the equation $1 - \sin 2x = 1 - mx$ have exactly 3 solutions in the domain $-\pi \leq x \leq \pi$. **2**
- b) Find the volume generated when the area between the curve $y = \tan x$ and $x = 0$ and $x = \frac{\pi}{4}$ is rotated about the x -axis. **3**
- c) Constance is doing yo-yo tricks. She is doing “around the world”. That is, she is keeping the yo-yo at its maximum length for a complete revolution. The length of the string on the yo-yo is 0.4 metres.
- i) Find the distance the yo-yo travels from a point A to another point B if the angle subtended at the centre is $\frac{3\pi}{5}$ radians. **1**
- ii) Find the area of the sector that the yo-yo sweeps around from A to B. **2**

End of Question 8

Question 9 (12 marks)

Start a new sheet of writing paper.

Marks

- a) Simplify: **1**
- $$\tan \theta \sqrt{1 - \sin^2 \theta}$$
- b) Evaluate $\sum_{n=0}^4 \sin^2 \left(\frac{n\pi}{3} \right)$. **3**
- c) Using Simpson's Rule with 5 function values, find the area bounded by the curve $y = 2^x$, the x -axis, $x = -1$ and $x = 1$. Leave your answer in surd form. **3**
- d) A radioactive element used in a hospital decays according to $y = Ae^{-kt}$, where k is a constant and t is time in years. If the element has a mass, y , of 200g in 2009, and 182g in 2010:
- i) Show that $k = -\log_e \left(\frac{91}{100} \right)$ **2**
- ii) Find its mass in 2039. Answer to 1 decimal place. **1**
- iii) Find its half-life (that is, the length of time elapsed when it has lost half of its original mass). **2**

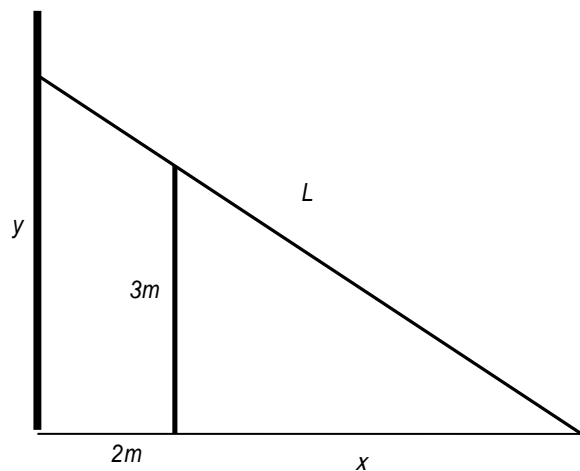
End of Question 9

Question 10 (12 marks)

Start a new sheet of writing paper.

Marks

- a) Solve: **3**
 $\log_e(6x+9) - \log_e(x-1) = \log_e(3x-1)$
- b) The general term of a geometric series is defined by $T_n = (x-2)^n$.
find:
- i) The value(s) of x for which the series has a limiting sum **1**
- ii) This limit in terms of x **2**
- c) A 3m vertical fence stands 2 metres from a high vertical wall. A ladder is placed from the horizontal ground to the wall, resting on the fence. The base of the ladder is x metres from the fence.



- i) Show that the square of the length of the ladder is given by **2**
$$L^2 = (x+2)^2 \left(1 + \frac{9}{x^2}\right)$$
- ii) How long is the shortest ladder that can reach from the ground outside the fence to the wall, correct to 2 decimal places? Show all working. **4**

End of Examination

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STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

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Course	2U Maths	Name of task/exam	TRIAL EXAM

Question 1:

$$a) \frac{e^5}{(-2)^2} = 37.103\dots$$

$$= 37.1 \quad (3 \text{ sig figs})$$

$$b) \frac{x-1}{1-x} = \frac{x-1}{-(-1+x)}$$

$$= \frac{\cancel{(x-1)}}{-\cancel{(x-1)}}$$

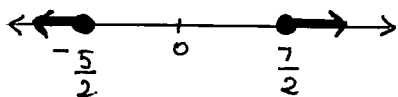
$$= -1$$

$$c) |2x-1| \geq 6$$

$$2x-1 \leq -6, \quad 2x-1 \geq 6$$

$$2x \leq -5, \quad 2x \geq 7$$

$$x \leq -\frac{5}{2}, \quad x \geq \frac{7}{2}$$



$$d) \frac{2}{\sqrt{3}-2} = a\sqrt{3} + b$$

$$\text{LHS} = \frac{2}{\sqrt{3}-2} \times \frac{\sqrt{3}+2}{\sqrt{3}+2}$$

$$= \frac{2\sqrt{3}+4}{3-4}$$

$$= -2\sqrt{3}-4$$

$$\therefore -2\sqrt{3}-4 = a\sqrt{3} + b$$

$$\therefore a = -2, \quad b = -4$$

$$e) v = \frac{\log_e(t-1)}{2}$$

$$a = \frac{1}{2} \left(\frac{1}{t-1} \right)$$

$$a = \frac{1}{2(t-1)}$$

when $t=2$

$$a = \frac{1}{2(1)}$$

$$a = \frac{1}{2} \text{ m/s}^2$$

$$f) x^6 - 64 = (x^3)^2 - 8^2$$

$$= (x^3 - 8)(x^3 + 8)$$

$$= (x-2)(x^2+2x+4)(x+2)(x^2-2x+4)$$

OR

$$x^6 - 64 = (x^2)^3 - 4^3$$

$$= (x^2 - 4)(x^4 + 4x^2 + 16)$$

$$= (x-2)(x+2)(x^4 + 4x^2 + 16)$$

Question 2:

$$a) i) \frac{d}{dx} \left(\frac{\sin x}{x} \right)$$

$$= \frac{vu' - uv'}{v^2}$$

$$= \frac{x \cos x - \sin x (1)}{x^2}$$

$$u = \sin x$$

$$u' = \cos x$$

$$v = x$$

$$v' = 1$$

$$= \frac{x \cos x - \sin x}{x^2}$$

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a) ii) $\frac{d}{dx} \log_e (5x-4)^3$

$$= \frac{3(5x-4)^2(5)}{(5x-4)^3}$$

$$= \frac{15}{(5x-4)}$$

OR $\frac{d}{dx} \log_e (5x-4)^3$

$$= \frac{d}{dx} 3 \log_e (5x-4)$$

$$= 3 \times \frac{5}{(5x-4)}$$

$$= \frac{15}{(5x-4)}$$

b) $\int 2e^{3x-5} dx = \frac{2e^{3x-5}}{3} + c$

c) $\int_1^2 \frac{3x}{4x^2-2} dx = \frac{3}{8} \int_1^2 \frac{8x}{4x^2-2} dx$

$$= \frac{3}{8} \left[\log_e (4x^2-2) \right]_1^2$$

$$= \frac{3}{8} \left[\log_e 14 - \log_e 2 \right]$$

$$= \frac{3}{8} \log_e \left(\frac{14}{2} \right)$$

$$= \frac{3}{8} \log_e 7$$

d) $\sin 495^\circ \tan 240^\circ = \frac{1}{\sqrt{2}} \times \sqrt{3}$

$$= \frac{\sqrt{3}}{\sqrt{2}}$$

$$\text{or } \frac{\sqrt{6}}{2}$$

e) $\frac{AB}{BC} = \frac{AE}{EF}$

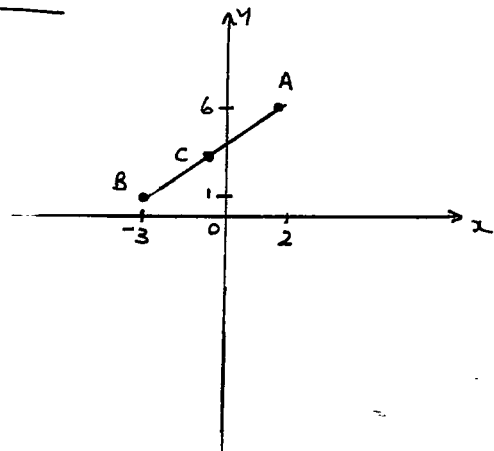
(When 3 or more lines are cut by two transversals, the ratio of intercepts are equal.)

$$\frac{4}{9} = \frac{x}{11}$$

$$x = \frac{44}{9}$$

Question 3:

a)



b) $C \left(-\frac{1}{2}, \frac{7}{2} \right)$

c) $m_{AB} = \frac{\text{rise}}{\text{run}}$

$$m_{AB} = \frac{5}{5}$$

$$\therefore m_{AB} = 1$$

d) $y - 6 = 1(x - 2)$

$$y - 6 = x - 2$$

$$y = x + 4$$

e) $m_{\perp} = -1$ (since $m_1 m_2 = -1$)

& midpt $\left(-\frac{1}{2}, \frac{7}{2} \right)$

\therefore equation: $y - \frac{7}{2} = -1 \left(x + \frac{1}{2} \right)$

$$2y - 7 = -2x - 1$$

$$2x + 2y - 6 = 0$$

$$x + y - 3 = 0$$

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f) $D(6, -3) \quad x+y-3=0$

LHS = $6 - 3 - 3$
 = 0
 = RHS

$\therefore (6, -3)$ lies on perpendicular bisector.

g) $(-7, 10)$

b) $d_{\perp} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

AB: $x - y + 4 \quad D(6, -3)$

$\therefore d_{\perp} = \frac{|6 - (-3) + 4|}{\sqrt{1^2 + (-1)^2}}$
 = $\frac{13}{\sqrt{2}}$
 = $\frac{13}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
 = $\frac{13\sqrt{2}}{2}$ units

i) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
 = $\sqrt{(-3 - 2)^2 + (1 - 6)^2}$
 = $\sqrt{25 + 25}$
 = $5\sqrt{2}$ units

j) $A = \left(\frac{1}{2} \times \frac{13\sqrt{2}}{2} \times 5\sqrt{2}\right) \times 2$
 $A = 65$ units²

Question 4:

a) $2 \sin \theta + \sqrt{3} = 0$

$\sin \theta = -\frac{\sqrt{3}}{2} \quad \swarrow \downarrow \searrow$

$\theta = \pi + \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$

$\theta = \frac{4\pi}{3}, \frac{5\pi}{3}$

b) $x^2 - kx + 3k - 8 = 0$

i) 2 roots $\Rightarrow x = 2$ satisfies

$2^2 - k(2) + 3k - 8 = 0$

$4 - 2k + 3k - 8 = 0$

$k - 4 = 0$

$k = 4$

ii) roots equal in magnitude, opposite in sign.

\therefore Let roots be $\alpha, -\alpha$

Sum of roots: $\alpha - \alpha = -\frac{b}{a}$

$0 = -\frac{(-k)}{1}$

$k = 0$

iii) real roots $\Rightarrow \Delta \geq 0$

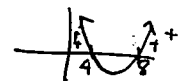
$b^2 - 4ac \geq 0$

$(-k)^2 - 4(1)(3k - 8) \geq 0$

$k^2 - 12k + 32 \geq 0$

$(k - 8)(k - 4) \geq 0$

$k \leq 4, k \geq 8$



iv) reciprocal roots

Let roots be $\alpha, \frac{1}{\alpha}$

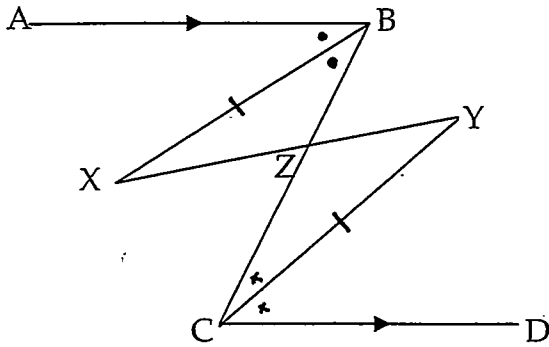
product of roots $\alpha \left(\frac{1}{\alpha}\right) = \frac{c}{a}$

$1 = 3k - 8$

$k = 3$

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c) i)



ii) $\angle ABC = \angle BCD$ (alternate angles equal $AB \parallel CD$)
 $\angle ABX = \angle XBZ$ (XB bisects $\angle ABC$)
 $\angle ZCY = \angle YCD$ (CY bisects $\angle BCD$)
 $\therefore \angle ABX = \angle XBZ = \angle ZCY = \angle YCD$

In $\triangle BXZ$ and $\triangle CYZ$,
 $BX = CY$ (given)
 $\angle XBZ = \angle ZCY$ (proved above)
 $\angle BZX = \angle CZY$ (vertically opposite)
 $\therefore \triangle BXZ \cong \triangle CYZ$ (AAS)
 $\therefore BZ = CZ$ (corresponding sides in congruent triangles are equal).
 $\therefore Z$ is midpoint of BC .

Question 5:

a) $15 + 13 + 11 + \dots + \dots$ $S_n = 55$
 $a = 15$
 $d = -2$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

$$55 = \frac{n}{2} [30 + (n-1)(-2)]$$

$$110 = n [30 - 2n + 2]$$

$$110 = n [32 - 2n]$$

$$110 = 32n - 2n^2$$

$$2n^2 - 32n + 110 = 0$$

$$n^2 - 16n + 55 = 0$$

$$(n-11)(n-5) = 0$$

$$\therefore n = 5, 11.$$

b) i) $4 + 8 + 16 + \dots + 1024$

$$T_n = ar^{n-1}$$

$$a = 4$$

$$r = 2$$

$$1024 = 4(2)^{n-1}$$

$$T_n = 1024$$

$$256 = 2^{n-1}$$

$$2^8 = 2^{n-1}$$

$$\therefore n = 9.$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

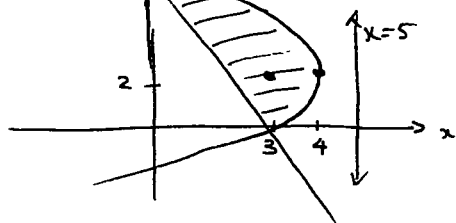
$$= \frac{4(2^9 - 1)}{1}$$

$$= 2044$$

ii) 2^{2044}

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c) i)



$$V(4, 2)$$

ii) eqn

$$(y-2)^2 = -4(1)(x-4)$$

$$(y-2)^2 = -4(x-4)$$

iii) $2x + y - 6 = 0$

$$y = -2x + 6$$

subst. into parabola

$$(y-2)^2 = -4(x-4)$$

$$\therefore (-2x+6-2)^2 = -4x+16$$

$$(-2x+4)^2 = -4x+16$$

$$4x^2 - 16x + 16 = -4x + 16$$

$$4x^2 - 12x = 0$$

$$4x(x-3) = 0$$

$$\therefore \left. \begin{array}{l} x=0 \\ y=6 \end{array} \right\} \left. \begin{array}{l} x=3 \\ y=0 \end{array} \right\}$$

iv) $A = \int_0^6 \text{parabola} - A_{\text{triangle}}$

$$= \int_0^6 \text{parabola} - \frac{1}{2} \times 3 \times 6$$

$$= \int_0^6 \text{parabola} - 9$$

$$(y-2)^2 = -4(x-4)$$

$$-\frac{1}{4}(y-2)^2 = x-4$$

$$x = 4 - \frac{1}{4}(y-2)^2$$

$$\therefore \int_0^6 \left(4 - \frac{1}{4}(y-2)^2\right) dy - 9$$

$$= \left[4y - \frac{1}{4} \frac{(y-2)^3}{1 \times 3}\right]_0^6 - 9$$

$$= \left(24 - \frac{1}{4} \left(\frac{64}{3}\right) - \left(0 - \frac{1}{4} \left(-\frac{8}{3}\right)\right)\right) - 9$$

$$= \left(24 - \frac{16}{3} - \frac{2}{3}\right) - 9$$

$$= 9 \text{ units}^2$$

Question 6 :

a) i) show $6x^3 + 9x^2 - 3 = 3(x+1)^2(2x-1)$

$$\text{RHS} = 3(x+1)^2(2x-1)$$

$$= 3(x^2 + 2x + 1)(2x-1)$$

$$= (3x^2 + 6x + 3)(2x-1)$$

$$= 6x^3 + 12x^2 + 6x - 3x^2 - 6x - 3$$

$$= 6x^3 + 9x^2 - 3$$

$$= \text{LHS}$$

\therefore shown

ii) x -intercepts \Rightarrow set $y=0$

$$\therefore 6x^3 + 9x^2 - 3 = 0$$

$$3(x+1)^2(2x-1) = 0$$

$$\therefore x = -1, \frac{1}{2}$$

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b) y-intercept \Rightarrow set $x=0$.

$$\therefore y = 6(0)^3 + 9(0)^2 - 3$$

$$y = -3$$

c) $f(x) = 6x^3 + 9x^2 - 3$

$$f'(x) = 18x^2 + 18x$$

for stat. pts $f'(x) = 0$

$$\therefore 18x^2 + 18x = 0$$

$$18x(x+1) = 0$$

$$\therefore x = 0 \quad x = -1.$$

$$y = -3 \quad y = 0$$

$$\therefore (0, -3) \quad \& \quad (-1, 0)$$

are stationary points

$$f''(x) = 36x + 18$$

when $x = 0$

$$f''(0) = 18$$

\therefore concave up

$\therefore (0, -3)$ min stat. pt.

when $x = -1$

$$f''(-1) = -18$$

\therefore concave down

$\therefore (-1, 0)$ max stat. pt.

d) for points of inflexion

$$y'' = 0$$

$$\therefore 36x + 18 = 0$$

$$36x = -18$$

$$x = \frac{-18}{36}$$

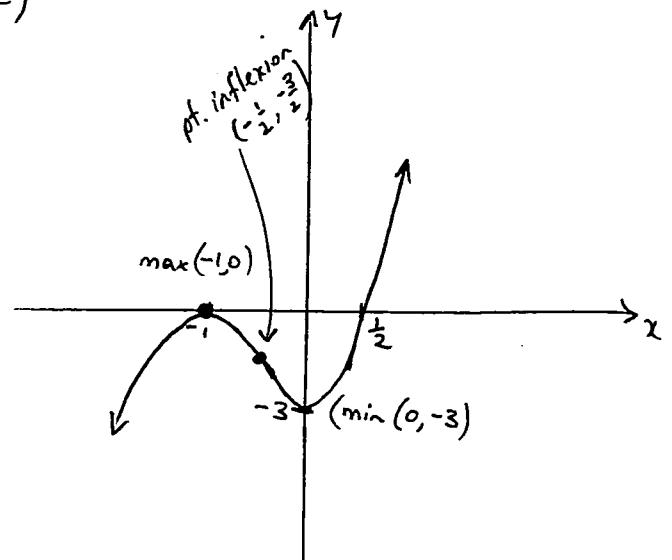
$$x = -\frac{1}{2}$$

x	$-\frac{3}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$
y''	-9	0	9

\therefore concavity changes

$\therefore (-\frac{1}{2}, -\frac{3}{2})$ is point of inflexion.

e)



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Question 7 :

a) $y = x^2 + \frac{1}{x}$

$y = x^2 + x^{-1}$

$\frac{dy}{dx} = 2x - x^{-2}$

at $x=1$

$m_{\text{tang}} = 2 - 1^{-2}$

$= 2 - 1$

$\therefore m = 1$

\therefore eqn of tangent :

$y - y_1 = m(x - x_1)$

$y - 2 = 1(x - 1)$

$y - 2 = x - 1$

$y = x + 1$

b) $v = 2 + t - 3t^2$

$t=0 \quad x=0$

i) at rest $v=0$

$0 = 2 + t - 3t^2$

$3t^2 - t - 2 = 0$

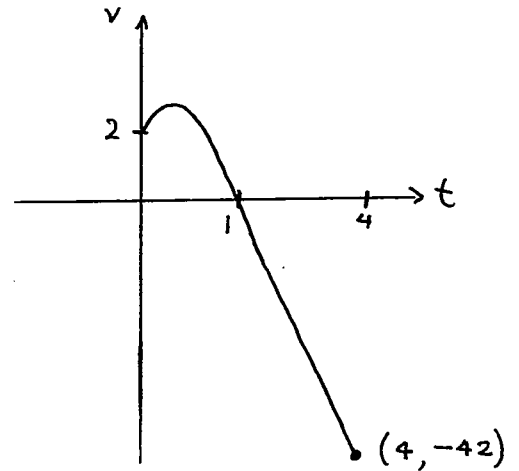
$(3t+2)(t-1) = 0$

$t = -\frac{2}{3}, \quad t = 1$

since $t \geq 0$

$\therefore t = 1 \text{ sec.}$

ii)



iii)

$v = 2 + t - 3t^2$

$x = \int (2 + t - 3t^2) dt$

$x = 2t + \frac{t^2}{2} - \frac{3t^3}{3} + C$

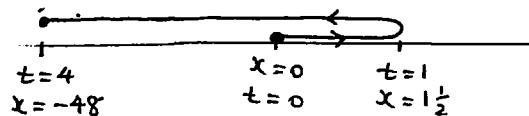
when $t=0 \quad x=0 \quad \therefore C=0$

$\therefore x = 2t + \frac{1}{2}t^2 - t^3$

at $t=0 \quad x=0$

at $t=1 \quad x = 2 + \frac{1}{2} - 1 = 1\frac{1}{2}$

at $t=4 \quad x = 2(4) + \frac{1}{2}(4)^2 - (4)^3 = -48$



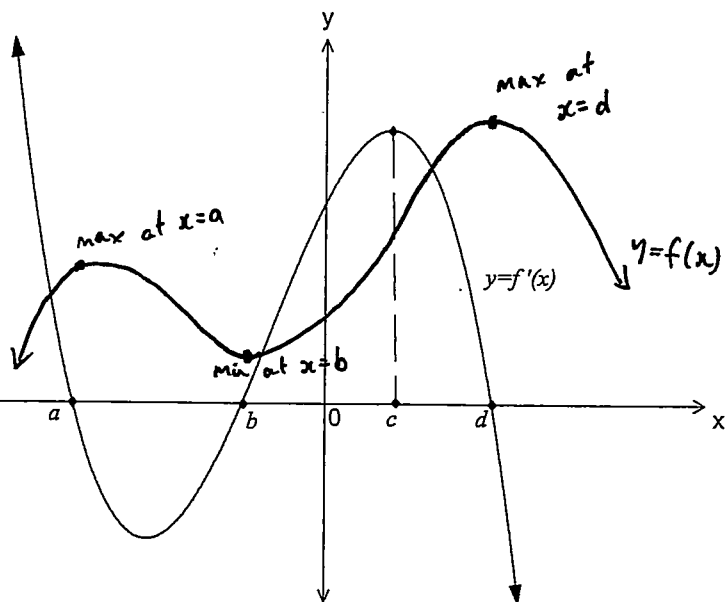
\therefore total distance travelled

$= 1\frac{1}{2} + 1\frac{1}{2} + 48$

$= 51 \text{ m}$

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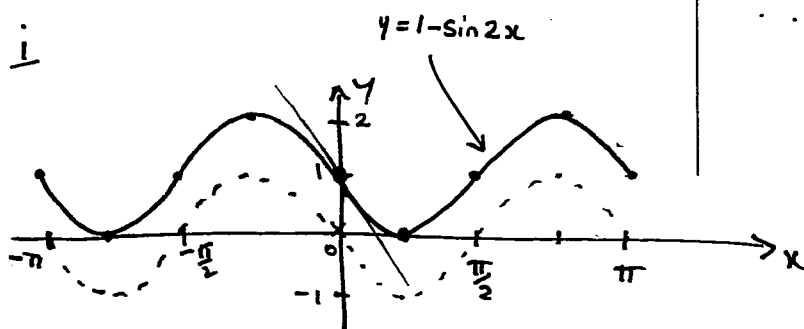
c)



Question 8 :

a) i) $y = 1 - \sin 2x \quad -\pi \leq x \leq \pi$

a) i



ii) $y' = -2 \cos 2x$
at $x = 0$

$$m_{\text{tang}} = -2 \cos 0 = -2$$

$$\therefore y - 1 = -2(x - 0)$$

$$y - 1 = -2x$$

$$2x + y = 1$$

iii) $0 < m < 2$

b) $V = \pi \int_a^b y^2 dx$

$$= \pi \int_0^{\pi/4} \tan^2 x dx$$

we know $\tan^2 x + 1 = \sec^2 x$

$$\therefore \tan^2 x = \sec^2 x - 1$$

$$\therefore V = \pi \int_0^{\pi/4} (\sec^2 x - 1) dx$$

$$= \pi \left[\tan x - x \right]_0^{\pi/4}$$

$$= \pi \left[\left(\tan \frac{\pi}{4} - \frac{\pi}{4} \right) - (0 - 0) \right]$$

$$= \pi \left[1 - \frac{\pi}{4} \right]$$

$$= \frac{\pi}{4} (4 - \pi) \text{ units}^3$$

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c) $r = 0.4 \text{ m}$

i) $l = r\theta$

$$= 0.4 \times \frac{3\pi}{5}$$

$$= 0.24\pi \text{ m or } \frac{6\pi}{25} \text{ m}$$

ii) $A = \frac{1}{2} r^2 \theta$

$$= \frac{1}{2} (0.4)^2 \left(\frac{3\pi}{5}\right)$$

$$= 0.048\pi \text{ m}^2$$

$$\text{OR } \frac{6\pi}{125} \text{ m}^2$$

Question 9:

a) $\tan \theta = \sqrt{1 - \sin^2 \theta}$

$$= \frac{\sin \theta}{\cos \theta} \sqrt{\cos^2 \theta}$$

$$= \frac{\sin \theta}{\cancel{\cos \theta}} \times \cancel{\cos \theta}$$

$$= \sin \theta$$

c) $A = \frac{h}{3} [y_0 + y_n + 4(y_1 + y_3) + 2(y_2)]$

$$= \frac{1}{3} [2^{-1} + 2^1 + 4(2^{-\frac{1}{2}} + 2^{\frac{1}{2}}) + 2(2^0)]$$

$$= \frac{1}{6} \left[\frac{9}{2} + 6\sqrt{2} \right]$$

$$= \frac{3}{4} + \sqrt{2}$$

b) $\sum_{n=0}^4 \sin^2 \left(\frac{n\pi}{3} \right)$

$$= \sin^2 \left(0 \right) + \sin^2 \left(\frac{\pi}{3} \right) + \sin^2 \left(\frac{2\pi}{3} \right)$$

$$+ \sin^2 \left(\frac{3\pi}{3} \right) + \sin^2 \left(\frac{4\pi}{3} \right)$$

$$= 0 + \left(\frac{\sqrt{3}}{2} \right)^2 + \left(\frac{\sqrt{3}}{2} \right)^2 + 0 + \left(-\frac{\sqrt{3}}{2} \right)^2$$

$$= \frac{3}{4} + \frac{3}{4} + \frac{3}{4}$$

$$= \frac{9}{4}$$

d) i) $y = A e^{-kt}$

2009 $t=0$ $y=200$

$$200 = A e^0$$

$$A = 200$$

$$y = 200 e^{-kt}$$

2010 $t=1$ $y=182$

$$182 = 200 e^{-k}$$

$$\frac{182}{200} = e^{-k}$$

$$\ln \left(\frac{182}{200} \right) = -k$$

$$k = -\ln \left(\frac{91}{100} \right)$$

ii) when $t=30$ $y=?$

$$y = 200 e^{-(-\ln \frac{91}{100})30}$$

$$y = 11.89$$

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iii) $t = ?$ $y = 100$
 $100 = 200 e^{-(-\ln \frac{91}{100})t}$
 $\frac{1}{2} = e^{(\ln \frac{91}{100})t}$
 $\ln\left(\frac{1}{2}\right) = \ln\left(\frac{91}{100}\right)t$
 $\therefore t = 7.349... \text{ yrs}$
 $t = 7.3 \text{ years}$

Question 10:

a) $\ln(6x+9) - \ln(x-1) = \ln(3x-1)$

$\therefore \ln\left(\frac{6x+9}{x-1}\right) = \ln(3x-1)$

$\therefore \frac{6x+9}{x-1} = 3x-1$

$\therefore 6x+9 = (3x-1)(x-1)$

$6x+9 = 3x^2 - 3x - x + 1$

$\therefore 3x^2 - 10x - 8 = 0$

$(3x+2)(x-4) = 0$

$x = -\frac{2}{3}, x = 4$

but $x \neq -\frac{2}{3}$ as $(x-1) > 0$

$\therefore x = 4$ is only solⁿ.

b) i) $T_n = (x-2)^n$

$r = (x-2)$

$\therefore -1 < x-2 < 1$

$1 < x < 3$

ii) $S_\infty = \frac{a}{1-r}$

$= \frac{x-2}{1-(x-2)}$

$= \frac{x-2}{-x+3}$

$S_\infty = \frac{x-2}{3-x}$

c) i) $L^2 = y^2 + (x+2)^2$ (by pyth. thm)

$L^2 = (x+2)^2 + y^2$

we know $\frac{3}{x} = \frac{y}{x+2}$ (similar Δ s)

$\therefore y = \frac{3(x+2)}{x}$

$\therefore L^2 = (x+2)^2 + \left[\frac{3(x+2)}{x}\right]^2$

$L^2 = (x+2)^2 \left[1 + \frac{9}{x^2}\right]$

ii) $L = (x+2)\sqrt{1 + \frac{9}{x^2}}$

$= (x+2)\left(1 + \frac{9}{x^2}\right)^{\frac{1}{2}}$

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$$\frac{dL}{dx} = (x+2)^{\frac{1}{2}} \left(1 + \frac{9}{x^2}\right)^{-\frac{1}{2}} (-18x^{-3})$$

$$+ \left(1 + \frac{9}{x^2}\right)^{\frac{1}{2}} (1)$$

$$\frac{dL}{dx} = \frac{(x+2)(-18)}{2\sqrt{1+\frac{9}{x^2}}(x^3)} + \frac{\sqrt{1+\frac{9}{x^2}}}{1}$$

for min $\frac{dL}{dx} = 0$

$$\therefore 0 = (x+2)(-18) + 2x^3\left(1 + \frac{9}{x^2}\right)$$

$$18(x+2) = 2x^3\left(1 + \frac{9}{x^2}\right)$$

$$\cancel{18x} + 36 = 2x^3 + \cancel{18x}$$

$$x^3 = 18$$

$$x = \sqrt[3]{18}$$

x	$\sqrt[3]{18}^-$	$\sqrt[3]{18}$	$\sqrt[3]{18}^+$
$\frac{dL}{dx}$	-	0	+

- \quad \quad \quad +

\(\therefore\) min .

$$\therefore L = \left(\sqrt[3]{18} + 2\right) \sqrt{1 + \frac{9}{(\sqrt[3]{18})^2}}$$

$$L = 7.02 \text{ m (2 dp)}$$