SAINT IGNATIUS' COLLEGE

Trial Higher School Certificate

2010

MATHEMATICS

Directions to Students

• Reading Time : 5 minutes	• Total Marks **120**
• Working Time : 3 hours	
• Write using blue or black pen. (sketches in pencil).	• Attempt Question 1 – 10
• Board approved calculators may be used	• All questions are of equal value
• A table of standard integrals is provided at the back of this paper.	
• All necessary working should be shown in every question.	
• **Answer each question in the booklets provided and clearly label your name and teacher's name.**	

Question 1 (Start a new Booklet) Marks

(a) Calculate $e^{3 \cdot 1}$ correct to 2 significant figures 2

(b) Solve for x: $x^2 + 5x = 24$ 2

(c) Find the primitive of $(2x + 5)^4$ 2

(d) Calculate the exact value of $\cos \frac{\pi}{6}$ 2

(e) Expand and simplify $(3 + 2\sqrt{2})^2$ 2

(f) Find the sum of the first 40 terms of the series $4 + 10 + 16 + \ldots$ 2

Question 2 (Start a new Booklet) Marks

(a) Find the derivative of

(i) $x \sin 3x$ 2

(ii) $\frac{e^x}{x}$ 2

(b) Integrate the following

(i) $\frac{3x}{x^2 + 4}$ 2

(ii) $3 \sec^2 \frac{x}{2}$ 1

(c) Find the equation of the tangent to the curve $y = x^3 + 1$ at the point where $x = 1$. 3

(d) Evaluate $\sum_{n=1}^{3} n^3 + 3$ 2

Question 3 (Start a new Booklet) Marks

(a) $A(1, -4)$ is a point on the line J: $3x + 2y + 5 = 0$

(i) Show that the point $B(-3,2)$ lies on the line. 1

(ii) Find the equation of the line perpendicular to J passing through the point $C(3,1)$. 2

(iii) Calculate the distance AB. 2

(iv) Find the perpendicular distance from C to the line J. 2

(v) Calculate the area of ΔABC. 1

(b) The Pacific Star cruise ship travels 215 km on a bearing of 085° from Sydney. It then travels 112km on a bearing of 135°.

(i) How far is the ship from Sydney? 2

(ii) What is the final bearing of the ship from Sydney? 2
(give answer correct to the nearest degree)

Question 4 (Start a new Booklet) Marks

(a) For the parabola $x^2 + 4x - 12y + 40 = 0$:

(i) Use completing the square method to write the equation in the form $(x-h)^2 = 4a(y-k)$ 1

(ii) Find the focal length. 1

(iii) Write down the coordinates of the vertex. 1

(iii) Find the focus. 1

(b) Solve $\log x + \log (x+4) = \log 12$ 2

(c) William drops a ball out of a window that is 25 m above the ground. On the first rebound, it rises to a height of 20 m. On subsequent rebounds, it rises to a height equal to $\frac{4}{5}$ of its previous height. If there is no interference with the ball, calculate the total distance through which the ball moves before coming to rest. 2

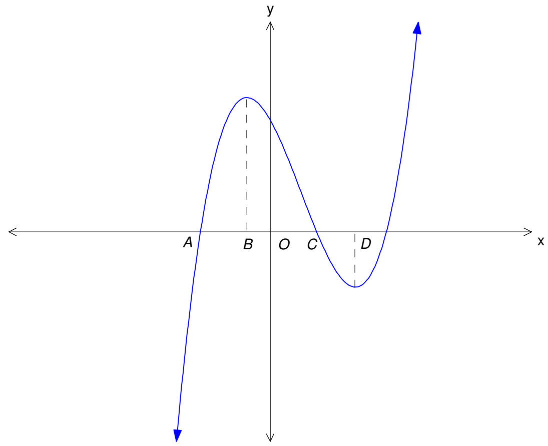
(d) At a Primary School Sports Carnival the combined year relay race has one participant from each year group from Kinder to Year 6. The Kinder child runs 15 m to a point and returns to the start, then the year 1 student runs 20 m and returns to the start. Each child runs in turn with each year group running 5 m further than the previous year group.

(i) How far does the year 6 child have to run? 2

(ii) How far is run by the students in one complete race? 2

Question 5 (Start a new Booklet) Marks

(a) Copy the following diagram into your answer booklet. 2



The curve represents the function $y = f(x)$. On the same set of axes draw the derivative function $y = f'(x)$.

(b) Consider the function $y = 2x^3 - 9x^2 + 12x$.

(i) Show that the only x-intercept exists at the origin. 1

(ii) Find the stationary points and determine their nature. 3

(iii) Show that a point of inflexion exists at the point $(1.5, 4.5)$. 1

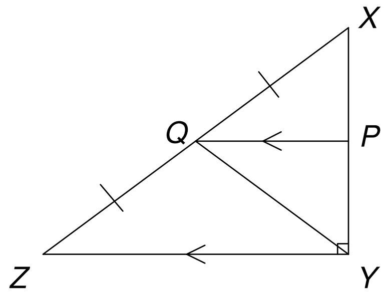
(iv) For what values of x is the curve monotonically decreasing? 1

(v) Sketch the curve of the function $y = 2x^3 - 9x^2 + 12x$ in the domain $0 \le x \le 3$. 2

(vi) Find the values of k for which $2x^3 - 9x^2 + 12x = k$ has only one solution. 2

Question 6 (Start a new Booklet) Marks

(a)



In the diagram above ΔXYZ is right angled. PQ is parallel to YZ and Q is the midpoint of XZ.

(i) Copy the diagram into your answer booklet.

(ii) Give a reason why $\angle XPQ = 90^\circ$. 1

(iii) Prove that $\Delta XPQ \equiv \Delta YPQ$. 2

(iv) Prove $QZ = QY$. 1

(b) A cylindrical tank is filled with water. The volume of water in the tank is determined by the function $V = 7t^3 + 15t^2 - 3t$, where t is time in seconds and the volume in litres. What is the rate of change of the volume of the tank after 12 seconds have elapsed? 2

(c) Find the **exact** length of the radius of a circle in which an arc length of 10 cm subtends an angle of 50° at the centre of the circle. 2

(d) Given that α and β are the roots of the quadratic equation $2x^2 - 7x + 5 = 0$, find ;

(i) $\alpha + \beta$ 1

(ii) $\alpha\beta$ 1

(iii) $\alpha^2 + \beta^2$ 2

Question 7 (Start a new Booklet) Marks

(a) Prove that $\dfrac{\sin\theta}{1+\cos\theta} = \dfrac{1-\cos\theta}{\sin\theta}$ 3

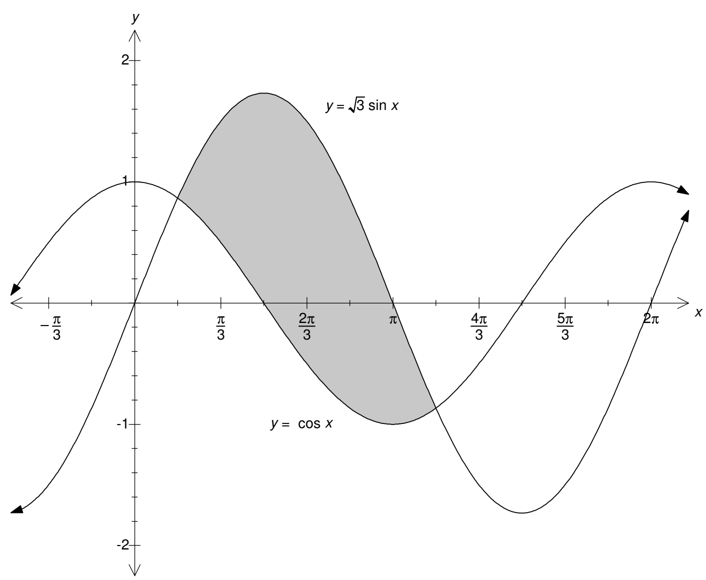
(b) The area bound by the curve $y = \sqrt{\sin x}$, $x = 0$, $x = \dfrac{\pi}{3}$ and the x-axis is rotated about the x-axis. Find the volume of the solid formed. 3

(c) A function $f(x)$ has a table of values 2

x	0	0.5	1	1.5	2
f(x)	3.24	4.16	2.25	1.15	0

Use Simpson's Rule to calculate $\displaystyle\int_0^2 f(x)\,dx$ (correct to 2 decimal places)

(d)



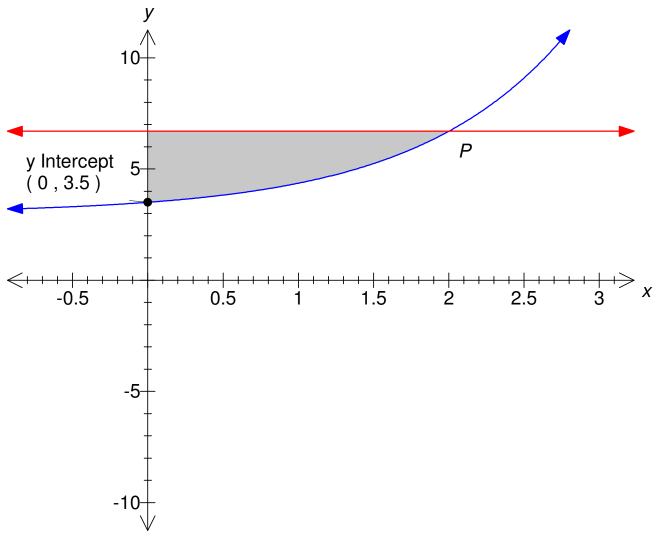
The graph above shows the sketch of the curves $y = \sqrt{3}\,\sin x$ and $y = \cos x$.

(i) Solve the curves simultaneously to show that the x-values of the points of intersection are $\dfrac{\pi}{6}$ and $\dfrac{7\pi}{6}$. 1

(ii) Find the shaded area in the diagram. 3

Question 8 (Start a new Booklet) Marks

(a)



(i) The curve $f(x)$ displayed above is in the form $f(x) = ae^{x} + 3$. Given that the curve passes through the point $(0, 3.5)$, show that $a = 0.5$. **1**

(ii) The x-value of the point P on the curve is 2. What is the y-value of the point P? **1**

(iii) Find the area of the shaded region. **2**

(b) A particle moves such that its position, x metres, from a fixed point O is given by the function $x = t^3 - 7\tfrac{1}{2}t^2 + 18t + 2$, where t is measured in seconds.

(i) Find the particle's initial position and velocity. **3**

(ii) When is the particle at rest? **1**

(iii) What is the acceleration of the particle when it is first at rest? **2**

(iv) Find the distance travelled by the particle in the first three seconds. **2**

Question 9 (Start a new Booklet) Marks

(a) Jenni invests \$30000 into an account on the 1^{st} of March. She receives 9% p.a. interest compounded monthly. On the first day of each month after that she withdraws \$250 immediately after the interest is paid.

(i) How much money did she have in the account immediately after making the first withdrawal? 1

(ii) Show that after making the n *th* withdrawal the balance of the account is given by $\left(33\,333\frac{1}{3} - 3\,333\frac{1}{3} \times 1.0075^n\right)$ 2

(iii) Find the number of withdrawals that Jenni can make before there is no money left in the account. 2

(b) An ant colony has a population that is described by the function $P = P_0\, e^{kt}$. If the ant conlony initially had 300 ants and after 100 days the population had increased to 550 ants, find:

(i) the value of P_0 and k. 3

(ii) the time taken for the population of the colony to reach 1000 ants (write your answer correct to the nearest day). 2

(c) It was found that on the 1^{st} of June, 30 students in a school had the flu. Over the next month the number of cases of students being sick with the flu increased at decreasing rate. Draw a graph that would describe this situation. 2

Question 10 (Start a new Booklet) Marks

(a) Find the derivative of the function $y = x \log_e x$ and hence find $\int \log_e x \, dx$. 2

(b) A sector of a circle has an area of $\frac{3\pi}{4} cm^2$, while its arc length is $\frac{\pi}{4} cm$. Find the radius and angle of the sector. 4

(c) A cylindrical can is to hold $20 \pi m^3$. The material for the top and bottom costs $\$10/m^2$ and material for the side costs $\$8/m^2$.

(i) Show that the total cost of the material for the can be expressed by the formula $C = 20\pi r^2 + 16\pi r h$ 1

(ii) Show that $h = \frac{20}{r^2}$ 1

(iii) Find an expression for the cost in terms of r and hence find the values of r and h such that the cost of the materials is a minimum. 4

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \ x \neq 0, \ \text{if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \ a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \ a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \ a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$