Question One (12 marks)

a) Solve $|2x+1| = 7$ [2]

b) Solve $3 - 2x \leq 5$ [2]

c) Find integers x and y such that $\left(\sqrt{7}-2\right)^2 = x + y\sqrt{7}$ [2]

d) For the arithmetic sequence $3, 7, 11, 15, \ldots\ldots\ldots\ldots T_n \ldots\ldots$

i) Find a rule to describe the nth term [1]

ii) Find T_{99} [1]

iii) Find the sum of the first 99 terms [1]

e) Differentiate $x^2 \log_e x$ with respect to x [2]

f) Given $f(x) = \dfrac{1}{\sqrt{x-2}}$, what is the domain of $f(x)$? [1]

Question Two (12 marks)

a) Solve $x^2 = 9x$ [2]

b) Differentiate each of the following with respect to x;

i) $\left(6x^2 - 2\right)^5$ [2]

ii) $\dfrac{3\tan x}{x^2}$ [2]

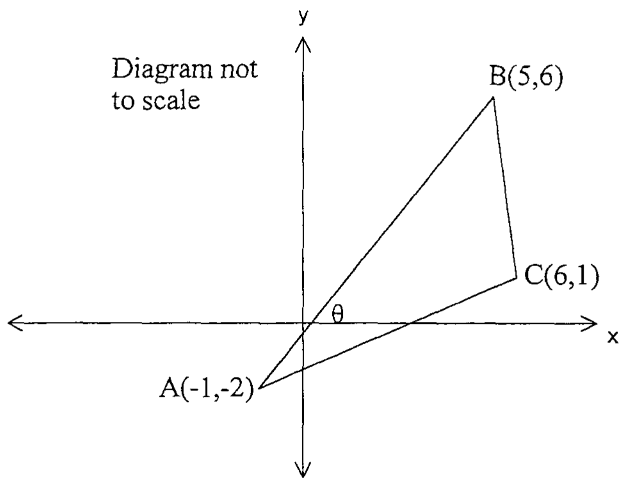
c) Solve $2x^2 + 5x - 3 \geq 0$ [2]

d) Find $\displaystyle\int \frac{dx}{(6x+1)^2}$ [2]

e) Find $\displaystyle\int \frac{x}{6-x^2}\,dx$ [2]

Question Three (12 marks)

a) The diagram shows the points $A(-1,-2)$, $B(5,6)$ and $C(6,0)$. The acute angle between the x axis and AB is θ



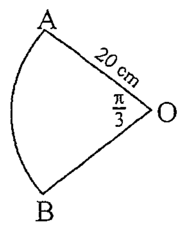
i) Find the gradient of AB [1]

ii) Find the size of θ to the nearest degree [1]

iii) Show that the equation of AB is $4x-3y-2=0$ [2]

iv) Find the perpendicular distance of C from AB [2]

b) In the diagram AB is an arc of a circle with radius 20cm. The angle AOB is $\frac{\pi}{3}$ radians.



i) Find the exact length of arc AB [2]

ii) Find the exact area of sector AOB [2]

iii) If angle AOB is increased by x°, find the new area of sector AOB in terms of x [2]

Question Four (12 marks)

a) The following table lists the values of a function for 5 values of x [2]

x	0	1	2	3	4
$f(x)$	15	24	20	10	5

Use the table above and Simpson's rule to estimate $\int_0^4 f(x)dx$

b) Find $\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$ [2]

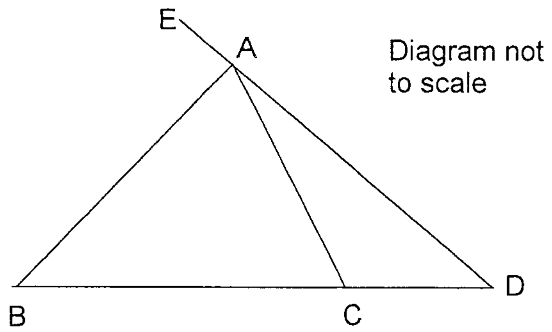
c) Sketch the curve of $y = 3\sin 2x$ for $0 \le x \le 2\pi$ [2]

d) The first term of a geometric sequence is 32 and the fourth term is $-\frac{1}{2}$.

i) Find the common ratio [1]

ii) Find the limiting sum [1]

e) In the diagram below AB=AC=CD and $\angle ADC = x^\circ$



i) Prove that $\angle ABD = 2x^\circ$ [2]

ii) Prove that $\angle EAB = 3x^\circ$ [2]

Question Five (12 marks)

a) Find the equation of the tangent to the curve $y = e^{\frac{x}{2}}$ at the point where it crosses the Y axis. [3]

b) Lola is an archer. She fires three arrows at a target. The probability of any single arrow hitting the target is $\frac{4}{5}$.

Find the probability that;

i) The first arrow hits and the next two miss. [1]
ii) She hits the target exactly once. [1]
iii) She hits the target at least once. [1]

c) The roots of the quadratic equation $2x^2 + 5x - 8 = 0$ are α and β.
Find;

i) $\alpha + \beta$ [1]

ii) $\alpha\beta$ [1]

iii) $\frac{2}{\alpha} + \frac{2}{\beta}$ [1]

iv) $\alpha^2 + \beta^2$ [1]

v) $\alpha^3 + \beta^3$ [2]

Question Six (12 marks)

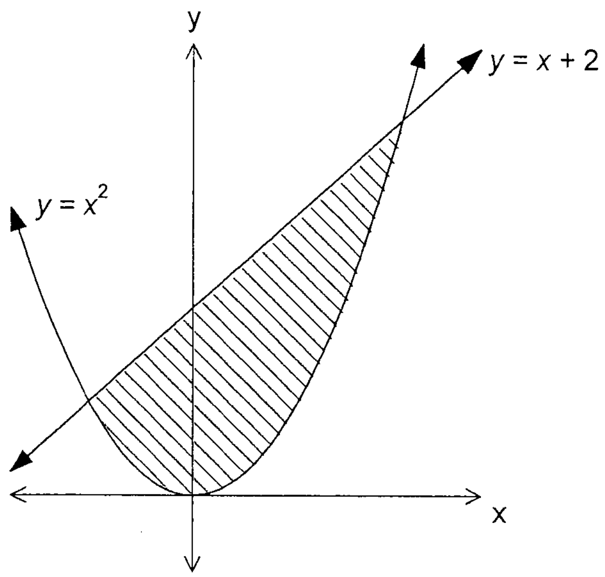
a) Given $f(x) = 2x^3 - 9x^2 + 12x - 1$.

i) Find any stationary points on the graph of $f(x)$ and determine their nature. [4]

ii) Show that a point of inflexion exists on $f(x)$ at $x = 1.5$ [2]

iii) Sketch the graph of $f(x)$ for $0 \le x \le 3$ [2]

b) The diagram below shows the graphs of $y = x^2$ and $y = x + 2$



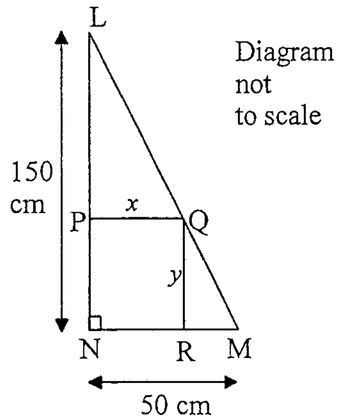
i) Show that the graphs intersect at $x = -1$ and $x = 2$ [1]

ii) Calculate the exact volume of the solid formed by rotating the area enclosed by the curves around the x-axis. [3]

Question Seven (12 marks)

a) Solve $\sin 2\theta = 0.5$ for $0 \le \theta \le 2\pi$ [3]

b) A right triangle LMN has base 50 cm and height 150cm. A rectangle PQRN is inscribed inside the triangle with dimensions x cm by y cm as shown.

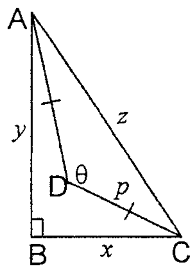


i) Show that triangle LMN is similar to triangle QMR. [2]

ii) Hence or otherwise show that $y = 150 - 3x$. [2]

iii) Find the maximum area of rectangle PQRN [3]

c) In the diagram below: In right triangle ABC, AB $= y$, BC $= x$, AC$= z$.
In isosceles triangle ADC, AD = DC$=p$, $\angle ADC = \theta$ [2]



Show that $x^2 + y^2 = \dfrac{p^2 \sin^2 \theta}{\cos^2 \dfrac{\theta}{2}}$

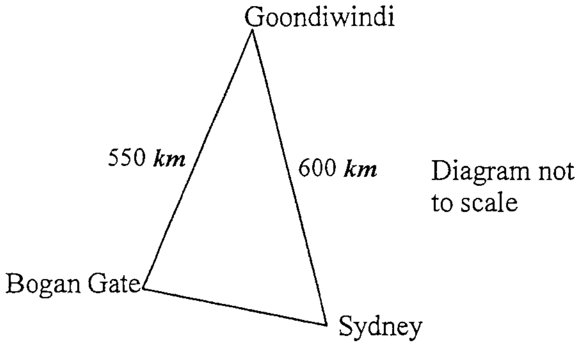
Question Eight (12 marks)

a) Given the equation of the parabola $x^2 - 4x - 2y = 0$:

i) Find the coordinates of the vertex. [2]

ii) Find the coordinates of the focus. [2]

b)



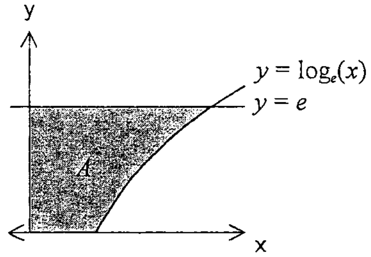
Three towns in Southeast Australia form a triangle as shown above. The bearing of Sydney form Goondiwindi is 172^0. The bearing of Bogan Gate from Goondiwindi is 200^0.

i) Find the area enclosed by the three towns. [2]

ii) Find the distance from Bogan Gate to Sydney [1]

c) Show that $\dfrac{\cos\theta}{1+\sin\theta} + \tan\theta = \sec\theta$ [2]

d) Find the shaded area bounded by the curves $y = \log_e x$, $y = e$ and the co ordinate axes [3]



Question Nine (12 marks)

a) In a class of 23 students, on Sunday night 13 watched 'The Prime Ministers Carbon Tax Speech', 12 watched 'MasterChef' while 7 watched both.

i) Find the probability that a student chosen at random watched neither. [2]

ii) Find the probability that a student chosen at random watched MasterChef only [1]

b) Simplify $\log_a 1 + \log_a 2 + \log_a 4 + \log_a 8 + \log_a 16$. [2]

c) Melanie's Aunty Umina has agreed to give her a lump sum of money to help her out over the five years that it will take her to complete her double degree at university. The money is deposited into an account which will pay 0.5% interest per month on the balance of the account. Melanie intends to withdraw \$400 each month after the interest has been added. Let the amount of the deposit be $\$x$.

i) Show that after Melanie has made her second withdrawal, the balance (B_n) of her account will be:

$$B_2 = \$\left[x \times 1.005^2 - 400(1 + 1.005)\right]$$ [2]

ii) How much should Aunty Umina deposit so that Melanie has enough money to make withdrawals for five years? [3]

d) Solve the following for x: [2]

$$\log_{10} x^{\frac{3}{2}} - \log_{10} \sqrt{x} = 5$$

Question Ten (12 marks)

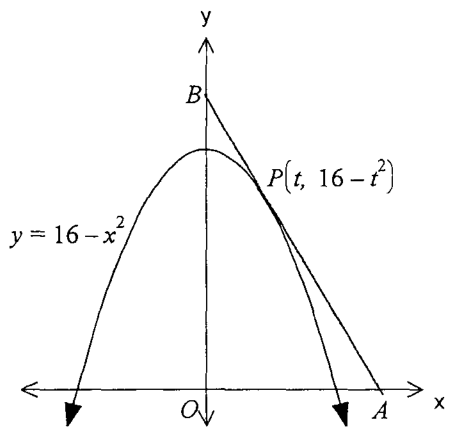
a) The total prize money for a surfing carnival is \$320 000. The prize money for the winner is \$50 000. After first place the prize money decreases by a constant amount until the tenth place is awarded:

i) Find the prize for finishing tenth. [2]

ii) Find the prize for finishing fifth. [2]

b) The diagram below shows the graph of the parabola $y = 16 - x^2$.

$P(t,\ 16-t^2)$ is a variable point that lies on the parabola in the first quadrant.



i) Find the equation of the tangent at P [3]

ii) The tangent at P cuts the x and y axes at A and B respectively. [5]
Find the value of t that will make the area of right triangle AOB a minimum.

End of paper

MATHEMATICS TRIAL 2011 - SOLUTIONS

Q1

a) $2x+1 = \pm 7$

$2x+1 = 7$

$2x = 6$

$\boxed{x = 3}$

$2x+1 = -7$

$2x = -8$

$\boxed{x = -4}$

b) $-2x \leq 2$

$\boxed{x \geq -1}$

c) $(\sqrt{7}-2)^2 = 7 - 4\sqrt{7} + 4$

$= 11 - 4\sqrt{7}$

$\therefore \boxed{x = 11}$ $\boxed{y = -4}$

d) $T_n = 3 + (n-1)4$

$= 3 + 4n - 4$

i) $\boxed{T_n = 4n - 1}$

ii) $T_{99} = 4 \times 99 - 1$

$\boxed{T_{99} = 395}$

iii) $S_{99} = \frac{99}{2}(6 + 98 \times 4)$

$\boxed{S_{99} = 19701}$

e) $y' = 2x \log_e x + x^2 \times \frac{1}{x}$

$= 2x \log_e x + x$

$\boxed{y' = x(2\log_e x + 1)}$

f) $x - 2 > 0$

$x > 2$

$\boxed{\text{Domain: } x > 2}$

Q2

a) $x^2\ 9x = 0$

$x(x-9) = 0$

$\boxed{x = 0}$ $\boxed{x = 9}$

b) i) $y' = 5(6x^2-2)^4(12x)$

$\boxed{y' = 60x(6x^2-2)^4}$

ii) $y' = \frac{3\sec^2 x \times x^2 - 2x \times 3\tan x}{x^4}$

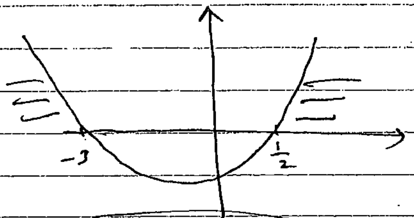
$= \frac{x(3x\sec^2 x - 6\tan x)}{x^4}$

$\boxed{y' = \frac{3x\sec^2 x - 6\tan x}{x^3}}$

c) $2x^2 + 6x - x - 3 \geq 0$

$2x(x+3) - 1(x+3) \geq 0$

$(x+3)(2x-1) \geq 0$



$\boxed{x \geq \frac{1}{2} \quad x < -3}$

d) $\int (6x+1)^{-2}\,dx$

$\frac{(6x+1)^{-1}}{6x-1} + C$

$\boxed{= \frac{-1}{6(6x+1)} + C}$

e) $-\frac{1}{2}\int \frac{-2x}{6-x^2}\,dx$

$\boxed{= -\frac{1}{2}\log_e(6-x^2) + C}$

Q3.

i) $m_{AB} = \frac{6+2}{5+1}$

$= \frac{8}{6}$

$\boxed{m_{AB} = \frac{4}{3}}$

ii) $m = \tan\theta$

$\frac{4}{3} = \tan\theta$

$\boxed{\theta = 53^\circ}$

iii) $y - 6 = \frac{4}{3}(x-5)$

$3y - 18 = 4x - 20$

$\boxed{4x - 3y - 2 = 0}$

iv) $d = \frac{|4(6) - 3(1) - 2|}{\sqrt{16+9}}$

$= \frac{|24-3-2|}{5}$

$\boxed{d = \frac{19}{5}}$

Question Five

a) $y = e^{x/2}$

$\frac{dy}{dx} = \frac{1}{2}e^{x/2}$ ✓

at $(0,1)$, $m = \frac{1}{2}$ ✓

Equation:

$y - 1 = \frac{1}{2}(x - 0)$

$y - 1 = \frac{x}{2}$

$y = \frac{x}{2} + 1$ ✓

OR $x - 2y + 2 = 0$.

b)

i) $P(HMM)$

$= \frac{4}{5} \times \frac{1}{5} \times \frac{1}{5}$

$= \frac{4}{125}$ ✓

ii) $P(HMM) + P(MHM) + P(MMH)$

$= 3 \times \frac{4}{125}$

$= \frac{12}{125}$ ✓

iii) $P(\text{at least once})$

$= 1 - P(MMM)$

$= 1 - \frac{1}{125}$

$= \frac{124}{125}$ ✓

c) $2x^2 + 5x - 8 = 0$

i) $\alpha + \beta = \frac{-b}{a} = \frac{-5}{2}$ ✓

ii) $\alpha\beta = \frac{c}{a} = \frac{-8}{2} = -4$ ✓

iii) $\frac{2}{\alpha} + \frac{2}{\beta} = \frac{2\beta + 2\alpha}{\alpha\beta}$

$= \frac{2(\alpha+\beta)}{\alpha\beta}$

$= \frac{2\left(-\frac{5}{2}\right)}{-4}$

$= \frac{5}{4}\,(1.25)$ ✓

iv) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{-5}{2}\right)^2 - 2(-4)$

$= \frac{25}{4} + 8$

$= 14\frac{1}{4}\left(= \frac{57}{4}\right)$ ✓

iv) $\alpha^3 + \beta^3$

$= (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$ ✓

$= (\alpha+\beta)\left[-\alpha\beta + (\alpha+\beta)^2 - 2\alpha\beta\right]$

$= (\alpha+\beta)\left[(\alpha+\beta)^2 - 3\alpha\beta\right]$

$= \left(\frac{-5}{2}\right)\left[\left(\frac{-5}{2}\right)^2 - 3(-4)\right]$

$= \left(\frac{-5}{2}\right)\left(\frac{73}{4}\right)$

$= -\frac{365}{8}\left(-45\frac{5}{8}\right)$ ✓

OR $(\alpha+\beta)\left[\alpha^2 + \beta^2 - \alpha\beta\right]$

$= -\frac{5}{2}\left[\frac{57}{4} + 4\right]$

$= -\frac{365}{8}$.

Question Six

a) i) $f(x) = 2x^3 - 9x^2 + 12x - 1$

$f'(x) = 6x^2 - 18x + 12$

$f''(x) = 12x - 18$ ✓

For stationary points $f'(x) = 0$:

$6x^2 - 18x + 12 = 0$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 1, 2$ ✓

When $x = 1$, $y = 4$ and

$f''(1) = -6 < 0$ (concave down)

$\therefore (1,4)$ is a maximum t.pt. ✓

When $x = 2$, $y = 3$ and

$f''(2) = 6 > 0$ (concave up)

$\therefore (2,3)$ is a minimum t.pt. ✓

ii) $f''(x) = 0$ for inflexion point

$12x - 18 = 0$

$x = \frac{18}{12} = 1.5$ ✓

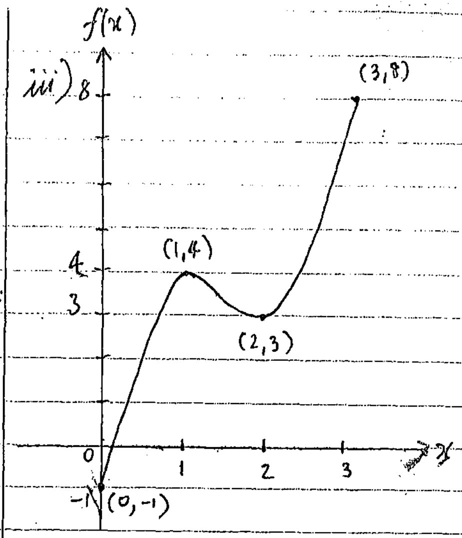
x	1.4	1.5	1.6
$f''(x)$	-1.2 < 0	0	1.2 > 0

✓

$\therefore$ change in concavity

$\therefore (1.5, 3.5)$ is a point of inflexion.

iii)



b) i) $y = x^2 \ldots ①$

$y = x + 2 \ldots ②$

$① = ②$: $x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$ ✓

$\therefore x = -1$ and $x = 2$.

ii) $V = \pi \int_a^b y^2\,dx$

$V = \pi \int_{-1}^{2} (x+2)^2 - (x^2)^2\,dx$ ✓

$= \pi\left[\frac{(x+2)^3}{3} - \frac{x^5}{5}\right]_{-1}^{2}$ ✓

$= \pi\left[\left(\frac{4^3}{3} - \frac{2^5}{5}\right) - \left(\frac{1}{3} + \frac{1}{5}\right)\right]$

$= \pi\left[\frac{224}{15} - \frac{8}{15}\right]$

$= \frac{72\pi}{5}$ units^3 ✓

a) $\sin 2\theta = \frac{1}{2}$

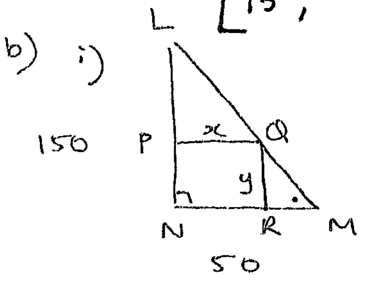
$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}, 2\pi + \frac{\pi}{6}, 2\pi + \frac{5\pi}{6}$

$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ (3)

$\theta = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

$[15°, 75°, 195°, 255°]$ (2)

b) i)



In $\triangle LNM$ and $\triangle QRM$

$\angle N = \angle QRM = 90°$

(angles of rectangle)

$\angle M$ is common

$\therefore \angle NLM = \angle RQM$ (2)

(angle sum of $\triangle$'s)

$\therefore \triangle LNM \parallel\!\parallel \triangle QRM$ (equiangular)

ii) $\frac{LN}{QR} = \frac{NM}{RM}$

$\frac{150}{y} = \frac{50}{50-x}$ (2)

$150(50-x) = 50y$

$3(50-x) = y$

$\therefore y = 150 - 3x$

iii) Area $= xy$

$= x(150-3x)$

$A = 150x - 3x^2$

$\frac{dA}{dx} = 150 - 6x$

at $\frac{dA}{dx} = 0$

$150 - 6x = 0$

$150 = 6x$

$\therefore x = 25$

$\frac{d^2A}{dx^2} = -6 < 0$

$\therefore$ Max Value.

$\therefore$ Maximum

Area $= xy$

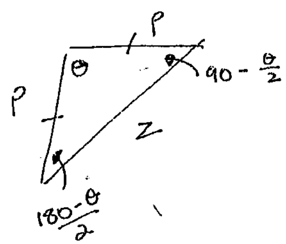
$A = 150x - 3x^2$

Area $= 150(25) - 3(25)^2$

Area $= 3750 - 1875$

Area $= 1875\ cm^2$ (3)

c) $x^2 + y^2 = z^2$



$\frac{z}{\sin\theta} = \frac{p}{\sin(90-\frac{\theta}{2})}$

$z = \frac{p\sin\theta}{\sin(90-\frac{\theta}{2})}$

$z = \frac{p\sin\theta}{\cos\frac{\theta}{2}}$

$\therefore z^2 = \frac{p^2\sin^2\theta}{\cos^2\frac{\theta}{2}}$ (2)

$\therefore x^2 + y^2 = \frac{p^2\sin^2\theta}{\cos^2\frac{\theta}{2}}$

Question 8

$x^2 - 4x - 2y = 0$

i) $x^2 - 4x = 2y$

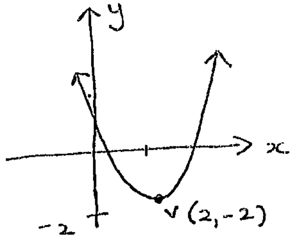
$x^2 - 4x + (-2)^2 = 2y + 4$

$(x-2)^2 = 2(y+2)$

$(x-h)^2 = 4a(y-k)$

$\therefore$ Vertex (h, k)

$V(2, -2)$



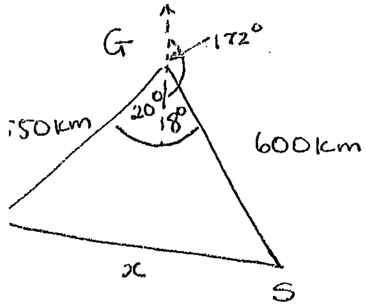
(2)

ii) $4a = 2$

$a = \frac{1}{2}$

(2)

$\therefore$ Focus $(2, -1\frac{1}{2})$



i) Area $= \frac{1}{2} ab \sin C$

$= \frac{1}{2}(600)(550) \sin 28^\circ$

$A = 77462.8 \text{ km}^2$

(2)

ii) $x^2 = (600)^2 + (550)^2 - 2(600)(550)\cos 28^\circ$

$x^2 = 79754.59$

$x = 282.4 \text{ km}$

(1)

Question 8

c) $\dfrac{\cos\theta}{1+\sin\theta} + \tan\theta = \sec\theta$

L.H.S $= \dfrac{\cos\theta}{1+\sin\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{\cos^2\theta + \sin\theta(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$

$= \dfrac{\cos^2\theta + \sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)}$ (with $\cos^2\theta + \sin^2\theta$ marked as 1)

(2)

$= \dfrac{(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$

$= \dfrac{1}{\cos\theta}$

$= \sec\theta$

$=$ R.H.S.

d) Area $= \int_1^e \ln x \, dx$

$y = \ln x$

$e^y = x$

$= \int_0^e e^y \, dy$

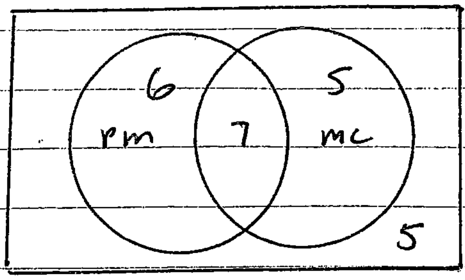
$= \left. e^y \right]_0^e$

$= e^e - 1$ units^2

(3)

Question Nine

a)



i) $P = \frac{5}{23}$ ② ①

ii) $P = \frac{5}{23}$ ①

b) $\log_a 1 + \log_a 2 + \log_a 4 + \log_a 8 + \log_a 16$

$= 0 + \log_a 2 + 2\log_a 2 + 3\log_a 2 + 4\log_a 2$

$= 10\log_a 2$ ②

c) i) $B_1 = x \times 1.005 - 400$

$B_2 = B_1 \times 1.005 - 400$

$= (x \times 1.005 - 400) \times 1.005 - 400$

$= x \times 1.005^2 - (400)(1.005) - 400$

$= \$[x \times 1.005^2 - 400(1 + 1.005)$ ②

ii) $B_n = \$[x \times 1.005^n - 400(1 + 1.005 + 1.005^2 + .. 1.005^{n-1})]$

$n = 12 \times 5 = 60$

$B_{60} = \$[x \times 1.005^{60} - 400(1 + 1.005 + 1.005^2 + .. 1.005^{59})$

when $n = 60$ $B_n = 0$

$0 = [x \times 1.005^{60} - 400(1 + 1.005 + 1.005^2 + .. 1.005^{59})$

$x = \frac{400(1 + 1.005 + 1.005^2 + .. + 1.005^{59})}{1.005^{60}}$

Sequence in brackets geometric $a = 1, r = 1.005, n = 60$

$S_n = \frac{a(r^n - 1)}{(r - 1)}$

$x = \frac{400}{1.005^{60}} \times \frac{1(1.005^{60} - 1)}{(1.005 - 1)}$

$= \$20690.22$ ③

d) $\log_{10} x^{\frac{3}{2}} - \log_{10} \sqrt{x} = 5$

$\frac{3}{2}\log_{10} x - \frac{1}{2}\log_{10} x = 5$

$\log_{10} x = 5$

ie $10^5 = x$ ②

Question 10

a) i) $S_n = \frac{n}{2}(a+l)$, $n=10$, $a=\$50000$, $l=?$

$320000 = \frac{10}{2}(50000+l)$

$64000 = 50000 + l$

$l = \$14,000$ ✓ ②

zero marks for geometric

ii) $a + 9d = l$

$50000 + 9d = 14000$

$9d = -36000$

$d = -4000$ ✓

$T_5 = a + 4d$

$= 50000 - 4(4000)$

$= \$34000$ ✓ ②

b) i) $y = 16 - x^2$ $P(t, 16-t^2)$

$\frac{dy}{dx} = -2x$

$m = -2t$ ✓

$y - y_1 = m(x - x_1)$

$y - (16 - t^2) = -2t(x - t)$ ✓

$y - 16 + t^2 = -2tx + 2t^2$

$y = -2tx + t^2 + 16$ ✓ correct simplification ③

ii) At A $y = 0$

$0 = -2tx + t^2 + 16$

$x = \frac{t^2+16}{2t}$ ✓

At B $x = 0$

$y = t^2 + 16$

Area AOB, $A = \frac{1}{2}\left(\frac{t^2+16}{2t}\right)\cdot(t^2+16)$ ✓

$= \frac{(t^2+16)^2}{4t}$

$$A = \frac{t^4 + 32t^2 + 256}{4t}$$

$$= \frac{1}{4}(t^3 + 32t + 256t^{-1})$$

$$\frac{dA}{dt} = \frac{1}{4}(3t^2 + 32 - 256t^{-2})$$ ✓

for max/min $\frac{dA}{dt} = 0$

$$\frac{1}{4}(3t^2 + 32 - 256t^{-2}) = 0$$

$$3t^2 + 32 - 256t^{-2} = 0$$

$$3t^4 + 32t^2 - 256 = 0$$

put $t^2 = m$

$$3m^2 - 32m - 256 = 0$$

⑤

$(3m - 16)(m + 16) = 0$ ✓ correct quadratic or equival

$m = \frac{16}{3}$ or $m = -16$ (no soln)

ie $t^2 = \frac{16}{3}$

$t = \pm\frac{4}{\sqrt{3}}$ but t in quad 1 $\therefore t > 0$

$t = \frac{4}{\sqrt{3}}$ ✓ correct answer

No CFPA here.

t	$\frac{4}{\sqrt{3}}^-$	$\frac{4}{\sqrt{3}}$	$\frac{4}{\sqrt{3}}^+$
$\frac{dA}{dt}$	$-$	0	$+$

$\therefore$ min