



Name: _____

Teacher: _____

Class: _____

FORT STREET HIGH SCHOOL

2012
HIGHER SCHOOL CERTIFICATE COURSE
ASSESSMENT TASK 3: TRIAL HSC

Mathematics

Time allowed: 3 hours
 (plus 5 minutes reading time)

Outcomes Assessed	Questions
Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1-10
Manipulates algebraic expressions to solve problems from topic areas such as geometry, co-ordinate geometry, quadratics, trigonometry, probability and logarithms	11,13
Demonstrates skills in the processes of differential and integral calculus and applies appropriate techniques to solve problems	12, 14
Synthesises mathematical solutions to harder problems and communicates them in appropriate form	15, 16

Total Marks 70

Section I 10 marks

Multiple Choice, attempt all questions,
 Allow about 15 minutes for this section

Section II 90 Marks

Attempt Questions 11-16,
 Allow about 2 hours 45 minutes for this section

General Instructions:

- Questions 11-16 are to be started in a new booklet
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used

Section I	Total 10	Marks
Q1-Q10		
Section II	Total 90	Marks
Q11	/15	
Q12	/15	
Q13	/15	
Q14	/15	
Q15	/15	
Q16	/15	
	Percent	

SECTION I Multiple Choice (10 marks)

CIRCLE CORRECT RESPONSE

1. What is the value of $\frac{(1.49)^2 - 1.98}{\sqrt{11.62 + 8.34 \times 2.72}}$ correct to three significant figures?

A. 0.040

B. 0.0409

C. 0.041

D. 0.0410

2. Find the values that satisfy $x^2 - x < 6$

A. $x < -2, x > 3$

B. $x < -3, x > 2$

C. $-3 < x < 2$

D. $-2 < x < 3$

3. If $\log_a 5 = 1.03$ and $\log_a 2 = 0.64$ then the value of $\log_a 10$ is:

A. 2.06

B. 1.67

C. 0.6592

D. 3.2

4. $\sum_{n=5}^{14} 17 - 2n =$

A. -11

B. -20

C. -18

D. -28

5. Expressed in radian measure, 235° is:

A. $\frac{\pi}{235}$

B. $\frac{235}{\pi}$

C. $\frac{47\pi}{36}$

D. $\frac{36\pi}{47}$

6. The third and seventh terms of a geometric series are 1.25 and 20 respectively. What is the first term?

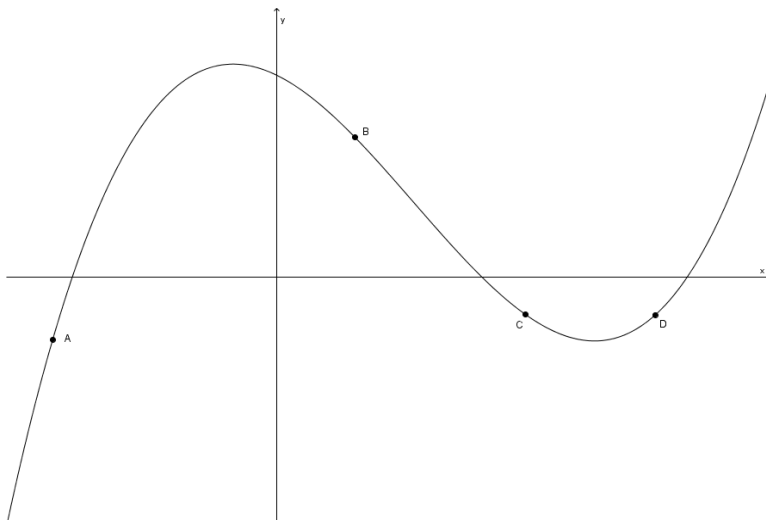
A. ± 2

B. $\frac{5}{16}$

C. ± 4

D. $\frac{5}{1024}$

7. State which point on the sketch fits the description $y < 0, \frac{dy}{dx} > 0, \frac{d^2y}{dx^2} < 0$



A. A

B. B

C. C

D. D

8. What is the greatest value of the function $y = 4 - 2 \cos x$?

- A. 2
- B. 4
- C. 6
- D. 8

9. A game is played in which two coloured dice are thrown once. The six faces of the red die are numbered 3, 5, 7, 8, 9 and 11. The six faces of the white die are numbered 1, 2, 4, 6, 10 and 12. The player wins if the number on the white die is larger than the number on the red die. What is the probability that the player wins once in two successive games?

- A. $\frac{7}{18}$
- B. $\frac{11}{18}$
- C. $\frac{77}{162}$
- D. $\frac{77}{324}$

10. What is the centre and radius of the circle with the equation $x^2 + y^2 + 6x - 8y - 11 = 0$?

- A. Centre $(-3, -4)$ and radius 36
- B. Centre $(-3, 4)$ and radius 36
- C. Centre $(-3, -4)$ and radius 6
- D. Centre $(-3, 4)$ and radius 6

SECTION II All necessary working must be shown

Question 11 (15 marks) **Begin a NEW booklet**

Marks

(a) Evaluate $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x - 3}$ 2

(b) If $\sqrt{50} - 3\sqrt{75} + \sqrt{18} = a\sqrt{2} - b\sqrt{3}$, find the value for a and b . 2

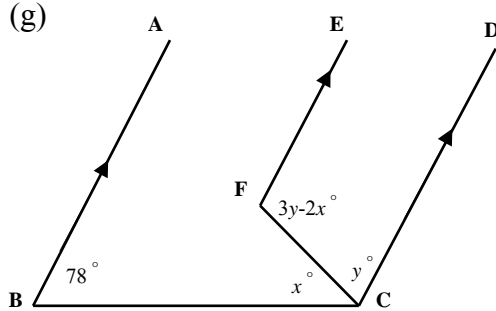
(c) State the domain and range of the function: $f(x) = 2\sqrt{x-1} + 3$ 2

(d) Evaluate a , in simplest exact form, if $\int_1^3 \frac{a}{1+x} dx = \log_e 16$. 2

(e) Solve for x : $|2x - 1| = 3x - 4$ 2

(f) For what values of k will $x^2 - (k - 3)x + k = 0$ have equal roots? 2

(g) 3



$AB \parallel EF \parallel DC$. Find the value of x and y , giving reasons.

Question12 (15 marks) Begin a NEW booklet**Marks**(a) Differentiate with respect to x :

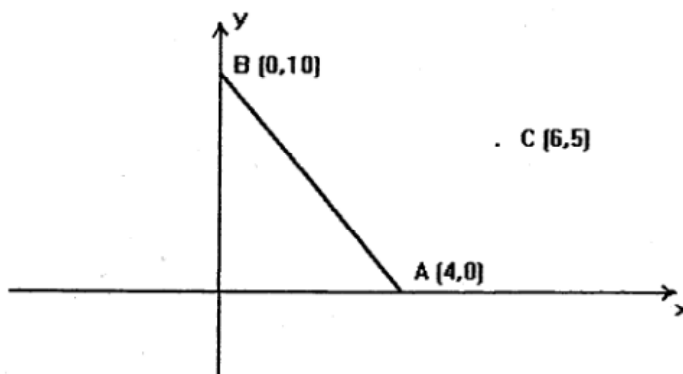
i) $y = \sqrt{1 - x^2}$ 1

ii) $y = \frac{\cos x}{x}$ 1

(b) Find:

i) $\int \frac{1}{e^{3x}} dx$ 1

ii) $\int (1 + 3\sec^2 \frac{x}{2}) dx$ 1

(c) The gradient of a curve is given by $f'(x) = 6x^2 - 6x + 5$. If the curve passes through the point $(2, 13)$, find its equation. 2(d) Find the equation of the normal to the curve $y = \ln\left(\frac{2x-1}{x+1}\right)$, at the point where $x = 2$. 3(e) 6

The points A, B and C have coordinates $(4, 0)$, $(0, 10)$ and $(6, 5)$ respectively as shown on the diagram. Copy the diagram into your booklets.

(i) Find the gradient of the line AB.

(ii) Show that the equation of the line through point C perpendicular to AB is given by

$$2x - 5y + 13 = 0. \text{ Label this line } q.$$

(iii) Find the perpendicular distance of B from the line CD, giving your answer in surd form with rational denominator.

(iv) If the line q meets the x -axis at D, calculate the size of angle ADC , correct to the nearest degree.

Question 13 (15 marks) **Begin a NEW booklet**

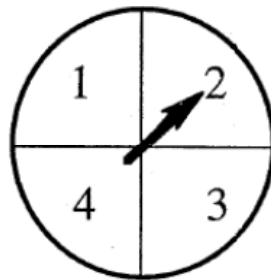
marks

(a) A party of walkers leave their base camp A at noon and walk in the direction $150^\circ T$ until they reach a point B on the bank of the river. Unable to cross the river, they follow it on a bearing of $015^\circ T$ and retire at 2 pm when they reach point C. The bearing and distance of C from A are $120^\circ T$ and 2.5 km respectively.

- i) In your booklet, draw a diagram and mark on it the given information, in the correct positions.
- ii) Find the distances AB and BC.
- iii) Hence, calculate the average speed, correct to one decimal place, of these walkers from A to C.

(b)

3

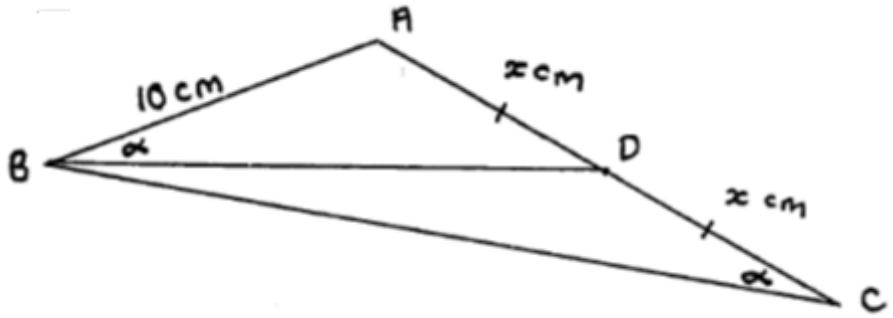


Dani and Chris used the spinner shown above to play a game. Dani spun the spinner twice and added the results of the two spins to get his score. Chris then took his turn. The player with the highest score won the game.

- i) Use a tree diagram or a sample space to show all the possible scores Dani could have achieved when he played the game.
- ii) What is the probability that Dani scored 6 in the game?
- iii) Dani's score was 6. What is the probability that Chris won the game?

Question 13 is continued over

(c)



In the diagram above, $AB = 10 \text{ cm}$, $AD = DC = x \text{ cm}$ and $\angle ABD = \angle BCD$.

4

- i) Prove $\triangle ABD \sim \triangle ACB$
- ii) Hence find the exact value of x .

iv) If α and β are the roots of the quadratic equation $5x^2 - 3x - 2 = 0$, find the value of: 4

- i) $\alpha + \beta$
- ii) $\alpha\beta$
- iii) $\alpha^2 + \beta^2$
- iv) $(3 - \alpha)(3 - \beta)$

Question14 (15 marks) **Begin a NEW booklet**

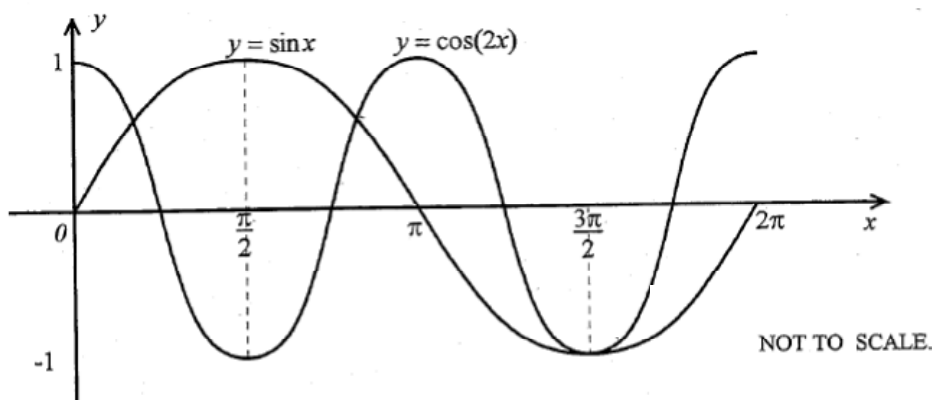
marks

- (a) Rapunzel has discovered something interesting about the way her hair grows. **6**
 When she was imprisoned in her tower (on the 1st of January) her hair was
 9 metres long. She measured her hair every year on the 1st of January, and made a note of its length. She
 recorded the following lengths on her cell wall for the first 4 years of her imprisonment:

9 m, 10 m, 11.05 m, 12.15 m, 13.3 m

- i) How much did Rapunzel's hair grow in the second year?
 ii) Write a series showing the amount her hair had grown in each of the first 4 years.
 iii) Using the series in part (ii), find how much Rapunzel's hair grew in the eighteenth year.
 iv) Prince Charming arrived to rescue Rapunzel after she had been in the tower for 21 years. If
 the tower is 40 metres high, and assuming that her hair continues to grow according to the series
 in (ii), will Rapunzel's hair be long enough to reach the ground ? Justify your answer by finding
 the length of Rapunzel's hair at this time.

(b)



The diagram shows parts of the curves $y = \sin x$ and $y = \cos 2x$.

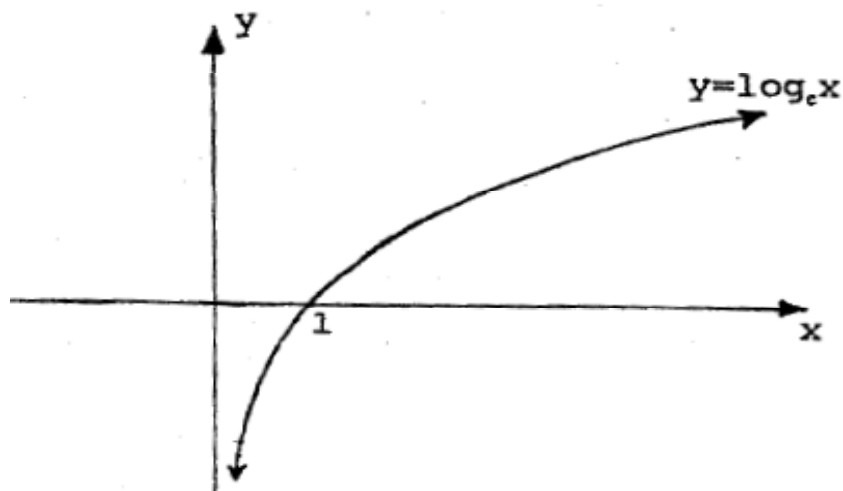
6

- i) Show that the curves intersect at $x = \frac{5\pi}{6}$ and $x = \frac{3\pi}{2}$.
 ii) Calculate the area between the curves from $x = \frac{5\pi}{6}$ to $x = \frac{3\pi}{2}$. Leave the
 answer in exact form.
- c) The region enclosed by the y axis, the line $y = x - 1$ and the line $y = 3$ is rotated about the
 y axis.
- i) Sketch the region **1**
 ii) Show that the volume of the resulting solid of revolution is given by **1**

$$V = \pi \int_{-1}^3 (y^2 + 2y + 1) dy$$

 iii) Hence, calculate this volume correct to two decimal places. **1**

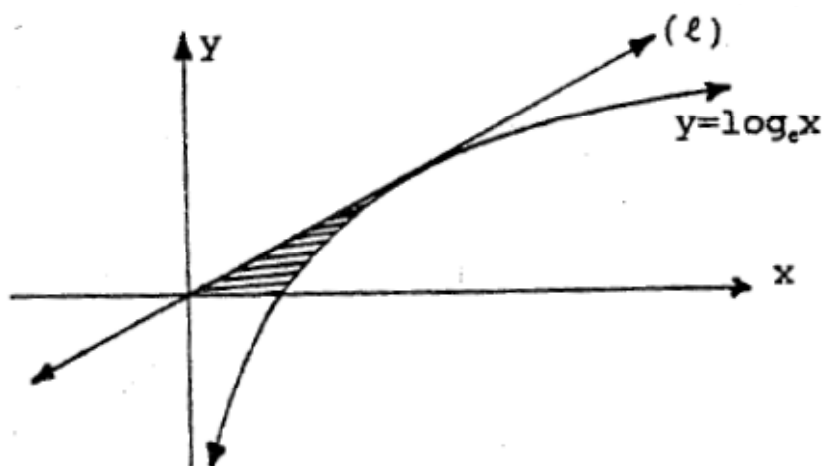
(a)



6

The diagram shows the graph of the curve $y = \log_e x$, not drawn to scale.

- (i) Find the equation of the tangent (l) at $x = e$ to the curve and show that it passes through the origin.
- (ii) Show that the area bounded by the curve, the y -axis and the lines $y = 0$ and $y = 1$ is $(e - 1)$ square units.
- (iii) Using the results of part (ii) or otherwise, find the area of the shaded region between $y = \log_e x$, the tangent (l) and the x -axis as shown in the diagram below.



Question 15 is continued over

- b) A river 60m wide is measured for depth every 10m across its width. 2

The measurements from bank to bank are given in the table. By using Simpson's Rule, estimate the cross-sectional area of the river at this point.

Measurement number	1	2	3	4	5	6	7
Depth in metres	0	5.5	11.0	13.2	8.5	4.5	0

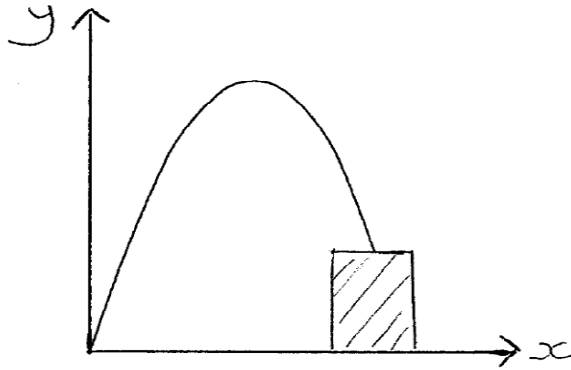
- (c) (i) Expand $e^{-x}(1 - e^{-x})$ 1
- (ii) For the curve $y = e^{-x} - e^{-2x}$: 6
- (α) find where it cuts the axes.
- (β) find the co-ordinates of the stationary points and determine their nature.
- (γ) determine the values of x for which the curve is monotonic decreasing and hence, or otherwise, discuss the behaviour of the curve for large values of x .
- (δ) sketch the curve.

Question 16 (15 marks) Begin a NEW booklet

marks

- (a) A water spout reaches a maximum height of 40m at a point 20m away from its source on the ground. It lands on the roof of a building 25 m away. How high is the building? (A water spout traces out a parabola.)

3



- (b) Fabio and Su Lin worked out that they would save \$50000 in 5 years by depositing all their combined monthly salary of $\$S$ at the beginning of each month into a savings account and withdrawing \$1600 at the end of each month for living expenses. The savings account paid interest at the rate of 3% p.a. compounding monthly.

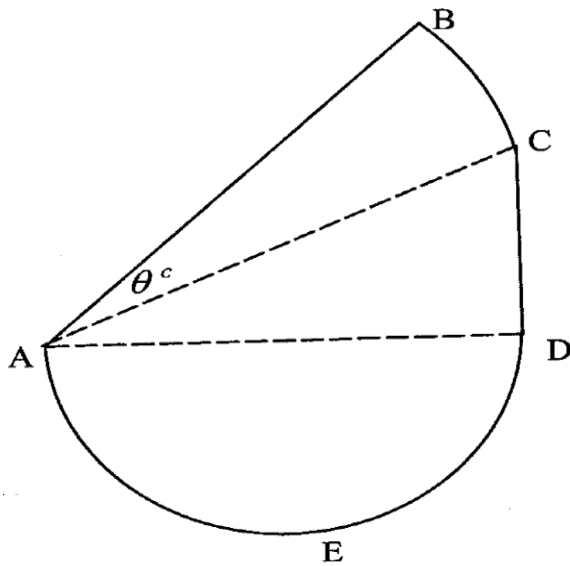
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- (i) Show that at the end of the second month, the balance in their savings account, immediately after making their withdrawal of \$1600, would be given by $\$[(1.0025^2 + 1.0025)S - 1600(1.0025 + 1)]$.
- (ii) Hence write down an expression for the balance in their account at the end of the 60th month.
- (iii) Calculate their combined monthly salary.

Question 16 continued

(c)

7



ABC is a sector of centre A and radius AB.

ACD is a right-angled triangle with $\angle ADC = 90^\circ$.

AED is a semi-circle of radius $2r$. $CD = 3r$. The perimeter of ABCDE is 24m.

(i) Show that $\theta = \frac{24 - 2\pi r - 8r}{5r}$

(ii) Show that the total Area = $60r - 14r^2 - 3\pi r^2$

(iii) Show that the total area is a maximum when $r = \frac{30}{3\pi + 14}$

(iv) Find the maximum area to the nearest m^2 .

END OF EXAMINATION

- Q1. D Q2. D Q3. B Q4. B
- Q5. C Q6. B Q7. A Q8. C
- Q9. C Q10. D

(10 marks)

11) $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x - 3}$
 $= \lim_{x \rightarrow 3} \frac{(x-3)(x^2 + 3x + 9)}{x-3}$
 $= 9 + 9 + 9$
 $= 27$

12) $5\sqrt{2} - 15\sqrt{3} + 3\sqrt{2} = a\sqrt{2} - b\sqrt{3}$
 $8\sqrt{2} - 15\sqrt{3} = a\sqrt{2} - b\sqrt{3}$
 $a = 8$ and $b = 15$

13) $f(x) = 2\sqrt{x-1} + 3$
 D: $x \geq 1$
 R: $y \geq 3$

14) $\int_1^3 \frac{a}{1+x} dx = \log_e 16$
 $a [\ln(1+x)]_1^3 = \log_e 16$
 $a (\ln 4 - \ln 2) = \ln 16$
 $a \ln\left(\frac{4}{2}\right) = \ln 16$
 $a = \frac{\ln 16}{\ln 2}$
 $= \frac{4 \ln 2}{\ln 2}$
 $a = 4$

students factorised this incorrectly

many simple maths errors

well done

students used $>$ instead of \geq

students took $\frac{1}{a}$ out of the integral instead of a

students left the answer as $\frac{\ln 16}{\ln 2}$

$2x - 1 = 3x - 4$
 $3 = x$
 Check: LHS = 5
 RHS = 5

$-2x + 1 = 3x - 4$
 $5 = 5x$
 $x = 1$
 Check: LHS = 1
 RHS = -1
 \therefore not a soln

\therefore the only soln is $x = 3$

15) $x^2 - (k-3)x + k = 0$
 for equal roots $\Delta = 0$
 i.e. $b^2 - 4ac = 0$
 $[-(k-3)]^2 - 4 \times 1 \times k = 0$
 $k^2 - 6k + 9 - 4k = 0$
 $k^2 - 10k + 9 = 0$
 $(k-9)(k-1) = 0$
 $\therefore k = 1$ or 9

16) $3y - 2x + y = 180$ (co-int. L's, EF || DC)
 $4y - 2x = 180$
 $2y - x = 90$ (1)

$78 + x + y = 180$ (co-int. L's, AB || CD)
 $x + y = 102$ (2)

D + (2): $3y = 192$
 $y = 64$
 Sub in (1): $x + 64 = 102$
 $x = 38$

mostly well done

students did not check their answers.

must check both solns

mostly well done

1 for setting up eqns

$$1) y = \sqrt{1-x^2}$$

$$y = (1-x^2)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} \cdot -2x$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}}$$

✓
accept
 $\frac{dy}{dx} = -x(1-x^2)^{-1/2}$

Watch
Chain
rule

$$ii) y = \frac{\cos x}{x}$$

$$\frac{dy}{dx} = \frac{v \frac{dv}{dx} - u \cdot \frac{du}{dx}}{v^2}$$

$$= \frac{x \cdot -\sin x - \cos x \cdot 1}{x^2}$$

$$\boxed{\frac{dy}{dx} = \frac{-x \sin x - \cos x}{x^2}}$$

Well
done

$$2) i) \int \frac{1}{e^{3x}} dx = \int e^{-3x} dx$$

$$\boxed{= -\frac{1}{3} e^{-3x} + c}$$

Some
confusion
 $\neq -3e^{-3x}$

$$ii) \int \left(1 + 3 \sec^2 \frac{x}{2}\right) dx$$

$$= x + 3 \tan \frac{x}{2} + c$$

$$\boxed{= x + 6 \tan \frac{x}{2} + c}$$

Well
done

$$3) f'(x) = 6x^2 - 6x + 5$$

$$f(x) = \frac{6x^3}{3} - \frac{6x^2}{2} + 5x + c$$

$$f(x) = 2x^3 - 3x^2 + 5x + c$$

$$\text{at } (2, 13) \quad 13 = 16 - 12 + 10 + c$$

$$c = -1$$

$$\therefore f(x) = 2x^3 - 3x^2 + 5x - 1$$

Wide range
of constants.

$$1) y = \ln \left(\frac{2x-1}{x+1} \right)$$

$$y = \ln(2x-1) - \ln(x+1)$$

$$y' = \frac{2}{2x-1} - \frac{1}{x+1}$$

$$\text{at } x=2: y' = \frac{2}{3} - \frac{1}{3}$$

$$= \frac{1}{3} \Rightarrow m = \frac{1}{3}$$

$$\therefore \perp m = -3$$

$$\text{then } x=2: y = \ln 1 = 0 \Rightarrow (2, 0)$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -3(x - 2)$$

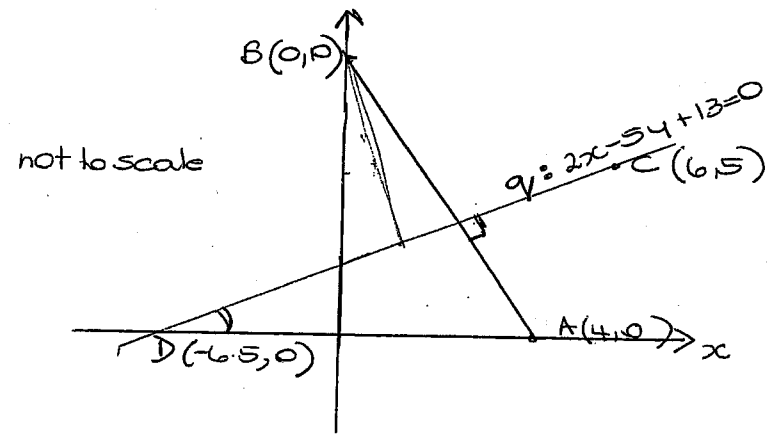
$$\boxed{y = -3x + 6}$$

or

$$\boxed{3x + y - 6 = 0}$$

Some
log differ
problems

\Rightarrow



$$i) m_{AB} = \frac{10-0}{0-4}$$

$$m_{AB} = \frac{-10}{-4}$$

$$m_{AB} = \frac{5}{2}$$

$$ii) \perp m = \frac{2}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{2}{5}(x - 6)$$

$$5y - 25 = 2x - 12$$

$$2x - 5y + 13 = 0$$

(as req'd)

Mostly
well
done

for
working

$$a=2 \quad b=-5 \quad c=13$$

$$\begin{aligned} \text{sp. dist.} &= \left| \frac{ax+by+c}{\sqrt{a^2+b^2}} \right| \\ &= \left| \frac{0-50+13}{\sqrt{4+25}} \right| \\ &= \left| \frac{-37}{\sqrt{29}} \right| \\ &= \frac{37}{\sqrt{29}} \times \frac{\sqrt{29}}{\sqrt{29}} \\ &= \frac{37\sqrt{29}}{29} \text{ units.} \end{aligned}$$

Some people didn't rationalise as asked

$$\begin{aligned} \checkmark) \quad CD \text{ (q)} \text{ cuts } x \text{ axis} &\Rightarrow y=0 \\ 2x &=-13 \\ x &=-6.5 \Rightarrow D(-6.5, 0) \end{aligned}$$

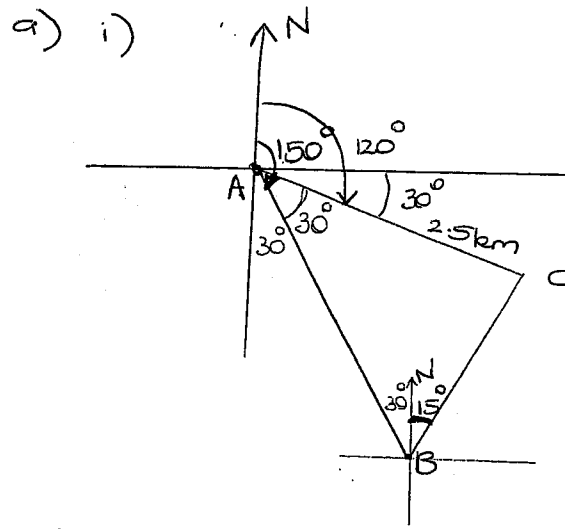
not needed.

now $m = \frac{2}{5}$ from ii)

$$\begin{aligned} \therefore \tan \theta &= m \\ \tan \angle ADC &= \frac{2}{5} \end{aligned}$$

$$\angle ADC = 22^\circ \text{ (nearest deg.)}$$

$$\frac{21.8^\circ}{\text{OK}}$$



$$\text{ii) } \frac{AB}{\sin 105^\circ} = \frac{2.5}{\sin 45^\circ}$$

$$AB = \frac{2.5 \sin 105^\circ}{\sin 45^\circ}$$

$$= 3.415063509 \checkmark$$

$$AB = 3.42 \text{ km (2dp)}$$

$$\frac{BC}{\sin 30^\circ} = \frac{2.5}{\sin 45^\circ}$$

$$BC = \frac{2.5 \times \frac{1}{2}}{\frac{1}{\sqrt{2}}}$$

$$= 1.767766953 \checkmark$$

$$BC = 1.77 \text{ km (2dp)}$$

$$\begin{aligned} \text{iii) } \text{av. speed} &= \frac{D}{T} \\ &= \frac{AB+BC}{2} \end{aligned}$$

$$= \frac{2.591415231}{2}$$

$$= 2.6 \text{ km/h}$$

(correct bldp)

* OR 2.6 km/h if using full display on calculator

* Diagrams were very poorly done
They must be $\frac{1}{3}$ A4 page in pencil using a ruler!

* many students did not mark on the given info as asked. They marked 'deduced' info instead.

for placing given info in correct position

Errors in ii) were mainly due to incorrect angle sizes.

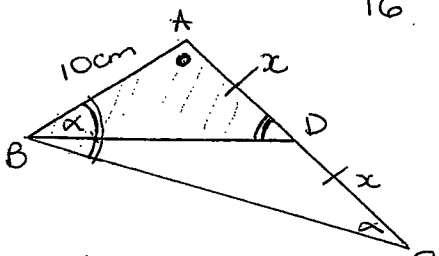
* some tried to use right Δ ratios ??

* many students included AC!

1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

or can use a tree diagram
 * many students recorded the outcomes, not the scores as asked in the question.

1) $P(\text{Dani}, 6) = \frac{3}{16}$
 2) $P(\text{Chris wins}) = P(\text{score} > 6)$
 $= P(\text{score } 7) + P(\text{score } 8)$
 $= \frac{3}{16}$



1) In Δ 's ABD & ACB
 $\angle A$ is common
 $\angle ABD = \angle ACB (= \alpha)$ given
 $\therefore \Delta ABD \parallel \Delta ABC$ equiangular

$\therefore \frac{AD}{AB} = \frac{AB}{AC}$ (corres. sides of sim. Δ 's are in same ratio)
 $\frac{x}{10} = \frac{10}{2x}$
 $2x^2 = 100$
 $x^2 = 50$
 $x = 5\sqrt{2}$ ($x > 0$)

5) $5x^2 - 3x - 2 = 0$
 $a=5 \quad b=-3 \quad c=-2$
 1) $x+b = \frac{-b}{a}$
 $x+b = \frac{3}{5}$
 2) $\frac{ax+b}{a} = \frac{-b}{a}$
 $\frac{5x+b}{5} = \frac{-3}{5}$
 3) $x^2 + b^2 = (x+b)^2 - 2xb$
 $= \frac{9}{25} + \frac{4}{5}$
 $= \frac{29}{25}$
 4) $(3-x)(3+x) = 9 - 3x - 3x + x^2$
 $= 9 - 3(x+b) + x^2$
 $= 9 - 3(\frac{3}{5}) + x^2$
 $= \frac{34}{5}$

generally well done!

* MOST STUDENTS NEED TO REVISE SETTING OUT SIMILARITY PROOFS and the CORRECT TESTS!!!

(Many are confusing congruence tests with similarity tests)

reason should be stated for setting up the proportion statement

* generally very well done but some students must learn the main expressions $x+b$, x^2+b^2 , $(x+b)^2$

9) $9m, 10m, 11.05m, 12.15m, 13.3m$
 i) $1.05m$ ✓
 ii) $1 + 1.05 + 1.10 + 1.15$ ✓
 iii) $T_n = a + (n-1)d$ AP: $a=1, d=0.05$
 $T_{18} = 1 + 17 \times 0.05$
 $= 1.85$
 \therefore grew $1.85m$ in the 18th year

iv) $S_n = \frac{n}{2} [2a + (n-1)d]$
 $S_{21} = \frac{21}{2} [2 + 20 \times 0.05]$ ✓
 $S_{21} = 31.5$
 Length of hair after 21 yrs
 $= 9 + S_{21}$
 $= 40.5m$ ✓

Since the tower is 40m high, her hair will be long enough. ✓

b) $y = \sin x$
 $x = \frac{5\pi}{6} : y = \sin \frac{5\pi}{6} = \frac{1}{2}$
 $y = \cos 2x = \cos 2(\frac{5\pi}{6}) = \cos \frac{5\pi}{3} = \frac{1}{2}$
 \therefore curves intersect at $x = \frac{5\pi}{6}$ ✓
 $x = \frac{3\pi}{2} : y = \sin \frac{3\pi}{2} = -1$
 $y = \cos 2(\frac{3\pi}{2}) = \cos 3\pi = -1$
 \therefore curves intersect at $x = \frac{3\pi}{2}$ ✓

ii) $A = \int_{\frac{5\pi}{6}}^{\frac{3\pi}{2}} (\cos 2x - \sin x) dx$
 $= \left[\frac{1}{2} \sin 2x + \cos x \right]_{\frac{5\pi}{6}}^{\frac{3\pi}{2}}$ ✓
 $= \left(\frac{1}{2} \sin 3\pi + \cos \frac{3\pi}{2} \right) - \left(\frac{1}{2} \sin \frac{5\pi}{3} + \cos \frac{5\pi}{6} \right)$
 $= (0 + 0) - \left(-\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{2} \right) + \frac{\sqrt{3}}{6}$ ✓
 $= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}$ ✓

* Most students are not ready the requirements of the question
 ii) (i) 2nd year, not 1st!
 (ii) "Write a series..."
 (iii) "... in the 18th year..."
 • many did not know the definition of a series.

• Several students found T_{21} instead of S_{21} .
 • many forgot to add the 9m growth to the initial 9m length, thus forever trapping Rapunzel unjustly in her tower forever.

• while many made and solved the equation $\cos 2x = \sin x$, too many used an equation to show. Solve or show, don't mix the methods!
 show substitutions

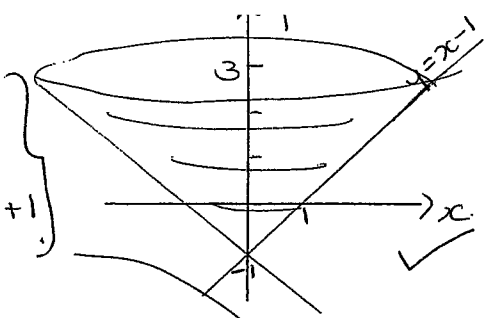
• despite being clearly asked to evaluate between $\frac{5\pi}{6}$ and $\frac{3\pi}{2}$, many tried to add together every possible area/integral (wasting time).
 • Many also seem unaware that $\frac{5\pi}{6} < \frac{3\pi}{2}$.
 • exact value - this doesn't mean in trig form when the angle gives exact trig values!!

$$= x-1$$

$$x = y+1$$

$$x^2 = (y+1)^2$$

$$x^2 = y^2 + 2y + 1$$



$$V = \pi \int x^2 dy$$

$$= \pi \int (y^2 + 2y + 1) dy$$

$$= \pi \left[\frac{y^3}{3} + \frac{2y^2}{2} + y \right]_{-1}^3$$

$$= \pi \left[(9 + 9 + 3) - \left(-\frac{1}{3} + 1 - 1 \right) \right]$$

$$= \pi \left[21\frac{1}{3} \right]$$

$$= \frac{64\pi}{3} \text{ u}^3$$

$$= 67.02 \text{ u}^3 \quad (204\text{p})$$

show expansion for mark

Sketch the region - e. you must indicate the region somehow: shading is the standard way to do this. Simply drawing all the lines does not fulfil the requirements of the question!

As the integral is already given, you must make the link between (i) as the starting point and this integral! Too many wrote $V = \pi \int_{-1}^3 x^2 dy$
 $= \pi \int_{-1}^3 (y^2 + 2y + 1) dy$
 (this scored no marks)

too many stopped at $V = \frac{64}{3}\pi$ (those that could at least get the $\frac{64}{3}$) and did not fulfil the requirements of the question.
 • π also had a habit of mysteriously disappearing - leaving $21\frac{1}{3}$ as a common answer!

Question 15 (15 marks)

comments

a) i) $y = \ln x$

$$y' = \frac{1}{x}$$

at $x=e$, $y' = \frac{1}{e}$, $y = \ln e = 1$

Eqn of tangent:

$$y-1 = \frac{1}{e}(x-e) \quad \leftarrow \text{1 mark for eqn. of tangent}$$

$$ey - e = x - e$$

$$x - ey = 0$$

sub (0,0) into

$$x - ey = 0$$

$$0 - e(0) = 0$$

$0 = 0$ \leftarrow 1 mark for showing origin is on tangent.
 LHS = RHS

tangent passes through the origin.

ii) $A = \int_0^1 e^y dy$

$y = \ln x$
 $x = e^y$

$$= [e^y]_0^1$$

$$= e^1 - e^0$$

$$= (e-1) \text{ units}^2$$

1 mark for integral

1 mark for working.

A common mistake was to try and $\int \ln x dx$

iii) $A = (e-1) - A \text{ of } \Delta$

$$= (e-1) - \frac{e \times 1}{2} \quad \leftarrow \text{1 mark}$$

$$= e - 1 - \frac{e}{2}$$

$$= \left(\frac{e}{2} - 1 \right) \text{ units}^2 \quad \leftarrow \text{1 mark}$$

More CARE needed.

b) $h=10$

0	10	20	30	40	50	60
0	5.5	11	13.2	8.5	4.5	0
y_1	y_2	y_3	y_4	y_5	y_6	y_7

Many students did not recognise $h=10$ from g .

$$A = \frac{h}{3} \{ y_1 + y_7 + 4(y_2 + y_4 + y_6) + 2(y_3 + y_5) \}$$

$$= \frac{10}{3} \{ 0 + 0 + 4(5.5 + 13.2 + 4.5) + 2(11 + 8.5) \} \quad \leftarrow \text{1 mark}$$

$$= 439.3 \text{ units}^2 \quad \leftarrow \text{1 mark}$$

15 continued.	comments
$\begin{aligned} \text{ii) } e^{-x}(1-e^{-x}) \\ = e^{-x} - e^{-2x} \end{aligned}$	<p>① for expanding correctly</p>
$i) y = e^{-x} - e^{-2x}$	
$\begin{aligned} x) \text{ sub } x=0: \quad \text{sub } y=0 \\ y = e^0 - e^0 \quad 0 = e^{-x}(1-e^{-x}) \\ = 1-1 \quad e^{-x} \neq 0, e^{-x} = 1 \\ y=0 \quad \text{when } x=0 \end{aligned}$	<p>some errors in not recognising $e^{-x} \neq 0$ and when $e^{-x} = 1$ $x=0$</p>
$\therefore \text{ cuts } x \text{ \& } y \text{ axes at origin.}$	<p>① for origin.</p>
$3) \frac{dy}{dx} = -e^{-x} + 2e^{-2x}$	
$\text{at stationary pts, } \frac{dy}{dx} = 0$	
$\begin{aligned} -e^{-x} + 2e^{-2x} = 0 \\ e^{-x}(2e^{-x} - 1) = 0 \\ e^{-x} \neq 0, e^{-x} = \frac{1}{2} \\ \ln\left(\frac{1}{2}\right) = -x \\ x = -\ln\left(\frac{1}{2}\right) \\ x = (\ln 1 - \ln 2) \\ x = \ln 2. \end{aligned}$	<p>many students had trouble solving $2e^{-x} - 1 = 0$</p>
$\begin{aligned} \text{If } x = \ln 2, y = e^{-\ln 2} - e^{-2\ln 2} \\ = \frac{1}{2} - \frac{1}{4} \\ = \frac{1}{4} \end{aligned}$	<p>$x = \ln 2$</p>
$\therefore \text{ stationary pt. is at } \left(\ln 2, \frac{1}{4}\right) \quad \left(\approx 0.69, \frac{1}{4}\right)$	<p>① for stat. pt.</p>
$\frac{d^2y}{dx^2} = e^{-x} - 4e^{-2x}$	
$\text{at } x = \ln 2, \frac{d^2y}{dx^2} < 0$	
$\therefore \left(\ln 2, \frac{1}{4}\right) \text{ is a maximum turning pt.}$	<p>① for nature of stat. pt.</p>

Question 15 continued.	comments
$\text{c) ii) } y) \text{ Since there is only one turning point and it is a maximum turning point, then curve is monotonically decreasing for } x > \ln 2.$	<p>① for $x > \ln 2$</p>
OR	<p>poorly answered</p>
$\frac{dy}{dx} = -e^{-x} + 2e^{-2x} < 0$	
$\text{i.e. } e^{-x}(2e^{-x} - 1) < 0$	<p>many students found their answer to be $x < \ln 2$ instead of $x > \ln 2$</p>
$\text{when } 2e^{-x} - 1 < 0 \text{ as } e^{-x} > 0$	
$\therefore e^{-x} < \frac{1}{2} \text{ for all real } x$	
$\text{i.e. } \ln e^{-x} < \ln \frac{1}{2}$	
$-x < \ln \frac{1}{2}$	
$x > -\ln \frac{1}{2}$	
$\text{i.e. } x > \ln 2.$	
$\text{as } x \rightarrow +\infty, y = e^{-x}(1-e^{-x}) \rightarrow 0 \text{ (because } e^{-x} \rightarrow 0)$	<p>① for $y \rightarrow 0$ well answered</p>
$8)$	
	<p>① for sketch poorly answered and often contradicted previous working.</p>

1) eqn of parabola is in form: $(x-h)^2 = -4a(y-k)$ ① mark.

e. $(x-20)^2 = -4a(y-40)$ * vertex $(h,k) = (20,40)$

Find 'a':

$$(0-20)^2 = -4a(0-40)$$

$$400 = 160a$$

$$a = 2.5$$

∴ eqn. of parabola:

$$(x-20)^2 = -10(y-40)$$

when $x = 25$:

$$(25-20)^2 = -10(y-40)$$

$$25 = -10y + 400$$

$$-375 = -10y$$

$$y = 37.5$$

∴ height of building is 37.5 metres

extension / students tried to use projectile motion formulae.

① for 'a' students used $y = ax^2 + bx + c$ which turns out to be much more difficult.

calculation error was common

2) $r = 3\% \text{ p.a. } a = 0.25\% / \text{month.}$

A_n = balance in savings account after n months, after withdrawal

$$A_1 = S(1.0025) - 1600 \quad \leftarrow \text{① mark}$$

$$A_2 = (A_1 + S)1.0025 - 1600$$

$$= A_1(1.0025) + S(1.0025) - 1600$$

$$= [S(1.0025 - 1600)]1.0025 + S(1.0025) - 1600$$

$$= S(1.0025)^2 - 1600(1.0025) + S(1.0025) - 1600$$

$$= S[1.0025 + 1.0025^2] - 1600[1 + 1.0025]$$

① mark for deriving A_2

i) $A_{60} = S[1.0025 + 1.0025^2 + \dots + 1.0025^{60}] - 1600[1 + 1.0025 + \dots + 1.0025^{59}]$

⇒ ① mark for A_{60}

b) ii) $A_{60} = 50000$

$$50000 = S(1.0025 + \dots + 1.0025^{60}) - 1600(1 + 1.0025 + \dots + 1.0025^{59})$$

$$50000 = S \left[\frac{1.0025(1.0025^{60} - 1)}{0.0025} \right] - 1600 \left[\frac{1(1.0025^{60} - 1)}{0.0025} \right] \quad \leftarrow \text{① mark}$$

$$50000 = S [401(1.0025^{60} - 1)] - 1600 [400(1.0025^{60} - 1)]$$

$$S = \frac{50000 + 640000 [1.0025^{60} - 1]}{401(1.0025^{60} - 1)}$$

$$S \doteq 2367.56 \quad \leftarrow \text{① mark}$$

students made the second geometric sum with 59 terms.

students made $S_{60} = 0$

Question 16 of 14

comments

c) i) In right $\triangle ADC$:

$AC = 5r$ (Pythagoras' Thm)

$\therefore AB = 5r$

$AB + BC + CD + DEA = 24$ (given)

$\therefore 5r + (5r)\theta + 3r + \frac{1}{2} \times \pi \times 4r = 24 \leftarrow \text{1 mark}$

$8r + 5r\theta + 2\pi r = 24$

$5r\theta = 24 - 2\pi r - 8r$

$\theta = \frac{24 - 2\pi r - 8r}{5r}$

1 mark for working

students did not show working many wrote a vague equation and then just wrote the answer.

ii) $A = A$ of sector $ABC + A$ of $\triangle ADC + A$ of semicircle

$= \frac{1}{2}(5r)^2\theta + \frac{4r \times 3r}{2} + \frac{1}{2} \times \pi \times (2r)^2 \leftarrow \text{1 mark}$

$= \frac{5r}{2} \left(\frac{24 - 2\pi r - 8r}{5r} \right) + 6r^2 + 2\pi r^2$

$= \frac{5r}{2} (24 - 2\pi r - 8r) + 6r^2 + 2\pi r^2$

$= 60r - 5\pi r^2 - 20r^2 + 6r^2 + 2\pi r^2$

$A = 60r - 14r^2 - 3\pi r^2$

1 mark for working

As above.

iii) A is maximum when $A' = 0$ and $A'' < 0$

$A' = 60 - 28r - 6\pi r$

$A' = 0$ when

$60 - 28r - 6\pi r = 0$

$60 = 6\pi r + 28r$

$60 = r(6\pi + 28)$

$r = \frac{60}{6\pi + 28} \div 2$

$r = \frac{30}{3\pi + 14}$

1 mark for finding r

mostly well done

students did not test their answers.

$A'' = -28 - 6\pi < 0$

$\therefore A$ is maximum when $r = \frac{30}{3\pi + 14}$

1 mark for testing for maximum

216 continued

Comments

iv) $A = 60(1.28) - 14(1.28)^2$

$= 3\pi \times (1.28)^2$

$= 38 \text{ m}^2$ (nearest m^2)

$r = \frac{30}{3\pi + 14} \approx 1.28$

Calculator error occurred regularly

1 mark for answer