



BAULKHAM HILLS HIGH SCHOOL

**TRIAL 2013
YEAR 12 TASK 4**

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 180 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 11-16
- Marks may be deducted for careless or badly arranged work
- All the diagrams are not to scale

Total marks – 100

Exam consists of 10 pages.

This paper consists of TWO sections.

Section 1 – Page 2 and 3 (10 marks)

Questions 1-10

- Attempt Question 1-10
- Allow about 15 minutes for this section

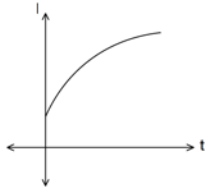
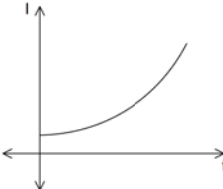
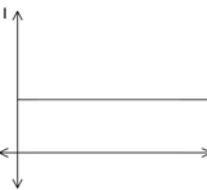
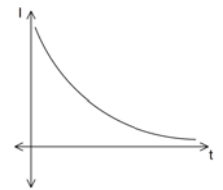
Section II – Pages 4-9 (90 marks)

- Attempt questions 11-16
- Allow about 2 hours and 45 minutes for this section

Table of Standard Integrals is on page 10

Section I - 10 marks**Use the multiple choice answer sheet for question 1-10****Allow about 15 minutes for this section.**

1. Evaluate to three significant figures $\frac{4.67 \times \sin 28^\circ}{\sqrt{4.6 \times 10^6}}$
- (A) 1.02
(B) 0.06
(C) 2.89×10^7
(D) 1.02×10^{-3}
2. The value of the limit $\lim_{x \rightarrow 10} \frac{x^2 - 100}{x - 10}$
- (A) undefined
(B) 0
(C) 8
(D) 20
3. Solve the equation $2x - 5 = \frac{x+3}{2}$
- (A) $x = \frac{-7}{3}$
(B) $x = \frac{-2}{3}$
(C) $x = \frac{8}{3}$
(D) $x = \frac{13}{3}$
4. The first term of an arithmetic progression is 3, and eleventh term is 23. The n^{th} term is
- (A) $T_n = 3 + 23(n - 1)$
(B) $T_n = 23 + 10(n - 1)$
(C) $T_n = 3 + 2(n - 1)$
(D) $T_n = 3 + 10(n - 1)$
5. Given that $\cos x = 0.5$ and $0^\circ < x < 90^\circ$, which of the following has the greatest value?
- (A) $\cos^2 x$
(B) $\sin x$
(C) $\tan x$
(D) 0.75

6. Given that $f(x) = x^2 + x$, find the values of b for which $f''(b) = f(b)$
- (A) $b = 2$ and 1
 (B) $b = -1$ and 2
 (C) $b = -2$ and -1
 (D) $b = -2$ and 1
7. If $\log_{10} 7 = a$ then $\log_{10} \left(\frac{1}{70}\right)$ is equal to
- (A) $-(1 + a)$
 (B) $(1 + a)^{-1}$
 (C) $\frac{a}{10}$
 (D) $\frac{1}{10a}$
8. The number of animals P on an island, at time t is given by $P = 7000e^{-kt}$, where k is a positive constant. Over time the number of animals on the island is
- (A) increasing exponentially.
 (B) decreasing exponentially.
 (C) increasing at a constant rate
 (D) decreasing at a constant rate.
9. If $z = 1 - y^3$ and $y = 1 - x$, then $z =$
- (A) $x^3 + 3x^2 + 3x$
 (B) $x^3 - 3x^2 + 3x$
 (C) $x^3 - 3x^2 - 3x$
 (D) $x^3 + 3x^2 - 3x$
10. Interest rates are increasing at an decreasing rate . Which of the following graphs represents the above statement.
- (A) 
- (B) 
- (C) 
- (D) 

End of Section I

Section II – Extended Response

Attempt questions 11-16.

Allow about 2 hours and 45 minutes for this section.

Answer each question on a SEPARATE PAGE. Clearly indicate question number.

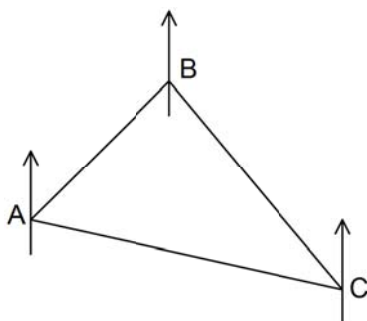
Each piece of paper must show your BOS#.

In Question 11-16, your responses should include relevant mathematical reasoning and/or calculation

Question 11 (15 marks)		Marks
a)	Solve for x $\frac{x + 6}{2} \leq x$	2
b)	Express $\frac{3+\sqrt{2}}{6+\sqrt{2}}$ with rational denominator.	2
c)	Factorise $3x^2 - 48y^2$	2
d)	Solve for x $ 2x - 5 = 3$	2
e)	Find the value of k for which $(k - 2)x^2 - 2kx - 1 = 0$ has real and distinct roots.	3
f)	If α and β are the roots of $2x^2 - 3x + 6 = 0$ then write the value of : (i) $\alpha + \beta$ (ii) $\alpha\beta$ (iii) $(2 + \alpha)(2 + \beta)$	1 1 2
End of Question 11		

Question 12 (15 marks)

- a) A bush walker walks from Point A on a bearing of 030° for 2.4km to Point B. He changes direction at B to a bearing of 145° to avoid a swamp and follows this course for 3.6km to Point C.



- (i) Copy the diagram in your booklet and mark on it all the given information.
 (ii) Calculate the distance from Point A to Point C (to one decimal place).
 (iii) What is the bearing of Point C from Point A?

2

2

- b) Differentiate the following functions with respect to x

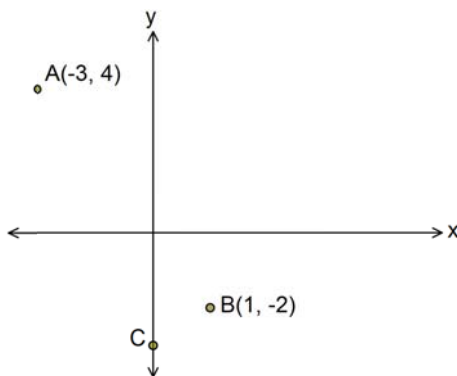
(i) $\cos^2 3x$

2

(ii) $\ln\left(\frac{1-x}{1+x}\right)$

2

- c)



- (i) Show that the equation through $A(-3, 4)$ and $B(1, -2)$ is $3x + 2y + 1 = 0$.
 (ii) Find the perpendicular distance from $C(0, -3)$ to the line AB.
 (iii) Hence find the area of $\triangle ABC$.

1

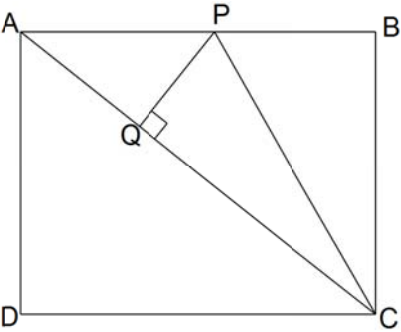
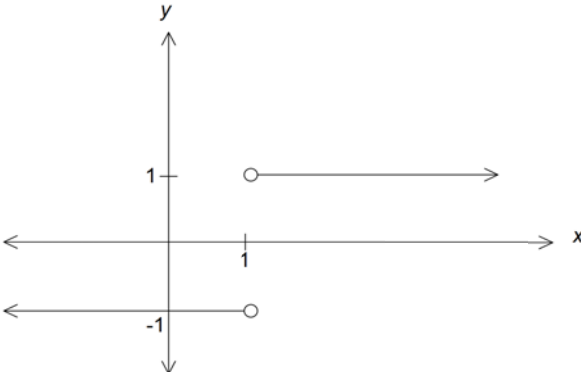
2

2

- d) Find the value of $\tan x$ when $\tan^2 x + \sec^2 x = 9$

2

End of Question 12

Question 14 (15 marks)	Marks
a) Tangents of the parabola $y = x^2 + ax - 3$ at $x = 1$ and $x = 0$ are known to be perpendicular. Find the value of a .	2
b) In the diagram ABCD is a square. PC bisects $\angle ACB$ and Q is the foot of the perpendicular from P to AC. <div style="text-align: center; margin: 10px 0;">  </div> (i) Copy the diagram into your booklet (ii) Prove that $\triangle PBC \equiv \triangle PQC$. (iii) Hence if $BC = x$ cm and $PB = \frac{1}{3}AB$, prove that the area of $\triangle APQ$ is $\frac{x^2}{6}$ cm ² .	3 3
c) A train travels from Oxford to Cambridge. Its velocity \dot{x} after leaving Oxford is given by $\dot{x} = \frac{1}{5}t(4 - t)$ where x is measured in kilometres and time in minutes. Find (i) The time taken to travel to Cambridge. (i) The maximum velocity attained. (ii) The distance between Oxford and Cambridge.	1 2 2
d) Sketch a possible function which could have the gradient function as graphed below. <div style="text-align: center; margin: 10px 0;">  </div>	2
End of Question 14	

Question 13 (15 marks)

a) Find

(i) $\int \frac{2}{3x} - e^{-3x} dx$

2

(ii) $\int_0^{\frac{\pi}{6}} \sec^2 2x dx$

2

b) Find the equation of the tangent to the curve

$y = 3 \sin \frac{x}{2}$ at $x = \frac{3\pi}{2}$

3c) For the function $y = x^3 - 6x^2 + 9x + 1$ find the

(i) Stationary points and determine their nature.

3

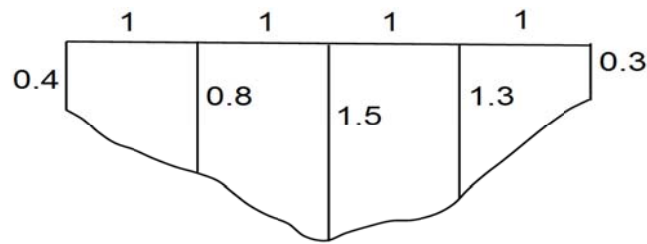
(ii) Coordinates of any point of inflexion.

2(iii) Hence sketch the curve for $-1 \leq x \leq 5$ **3****End of Question 13**

Question 15 (15 marks)

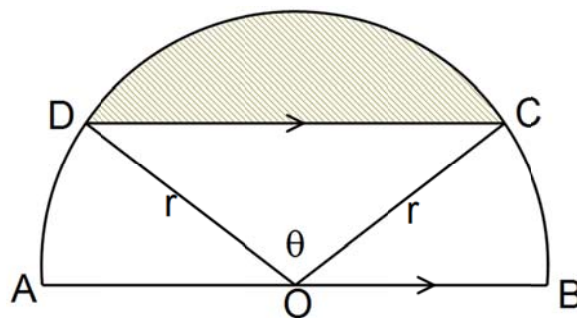
Marks

- a) The depth of water in the cross section of a creek 4m wide was measured and recorded in the table below.



- (i) By applying Simpson's Rule with the five depth measurements, find the cross sectional area of the creek. **2**
- (ii) If the water is flowing at a rate 0.25ms^{-1} , calculate the volume of water which passes this point in one day. **1**

- b) The shaded segment has an area one half of the semicircle with centre O and radius r . $CD \parallel AB$.



Prove that $\frac{\pi}{2} = \theta - \sin\theta$.

3

- c) The population of a country given by $P = P_0 e^{kt}$, is increasing at a rate proportional to its size. The population was 9 million in 1954 and 20 million in 2004.

- (i) Find the value of P_0 and k **3**
- (ii) At what rate is the population increasing after 10 years?? **2**
- (iii) In what year will the population reach 50 million? **2**

- d) If $y^2 = (2m - y)(2n - y)$ show that $\frac{1}{m}, \frac{1}{y}, \frac{1}{n}$ is an arithmetic sequence. **2**

End of Question 15

Question 16 (15 marks)	Marks
<p>a) Kelvin borrows \$200 000 from his bank. Interest is compounded monthly at 0.425% per month. A_n is the amount owed after n payments, \$$M$ is the amount of the monthly instalments and the loan is repaid after n months.</p> <p>(i) Show that $A_2 = 200\,000(1.00425)^2 - M(1.00425) - M$</p> <p>(ii) Show that the monthly repayment, \$$M$ is given by</p> $M = \frac{200\,000(1.00425)^n(0.00425)}{1.00425^n - 1}$ <p>(iii) Find the amount of the monthly instalments if Kelvin agrees to repay the loan over 30 years.</p> <p>(iv) How much will Kelvin pay in total after 30 years?</p> <p>(v) If Kelvin instead decided to pay monthly instalment of \$1331 from the beginning of the loan, how long will he take to repay the loan?</p>	<p>1</p> <p>3</p> <p>1</p> <p>1</p> <p>2</p>
<p>b) If $\sin x \neq \pm 1$ show that</p> $1 + \sin^2 x + \sin^4 x + \sin^6 x + \dots = \sec^2 x$	<p>2</p>
<p>c) (i) If r is the radius of the cone show that $r^2 = 2hR - h^2$</p> <p>(ii) Show that volume of the cone that can be inscribed in a sphere of radius R is given by $V = \frac{1}{3}\pi(2h^2R - h^3)$ where h is the height of the inscribed cone.</p> <div data-bbox="528 1400 1082 1892" data-label="Diagram"> </div> <p>(ii) Show that the volume of the largest cone is $\frac{8}{27}$ of the volume of the sphere.</p>	<p>1</p> <p>1</p> <p>3</p>

End of Exam

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \quad \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2})$$

NOTE: $\ln x = \log_e x, \quad x > 0$

M.C

Q1 D Q2 D Q3 D Q4 C Q5 C

Q6 D Q7 A Q8 B Q9 B Q10 A

311 a) $\frac{x+6}{2} \leq x$

$x+6 \leq 2x$

$6 \leq x$ — (2) with working.

$x > 6$

b) $\frac{3+\sqrt{2}}{6+\sqrt{2}} \times \frac{6-\sqrt{2}}{6-\sqrt{2}}$ — (1)

$\frac{18-3\sqrt{2}+6\sqrt{2}-2}{36-2}$

$\frac{16+3\sqrt{2}}{34}$ — (1)

c) $3(x^2-16y^2)$ — (1)

$3(x-4y)(x+4y)$ — (1)

d) $2x-5=3$ | $2x+5=-3$

$2x=8$

$x=4$

$2x=2$

$x=1$ — (1)

e) for real & distinct roots

$\Delta > 0$

$4k^2-4k-1 \times (k-2) > 0$ — (1)

$4k^2+4k-8 > 0$

$k^2+k-2 > 0$

$(k+2)(k-1) > 0$ — (1)

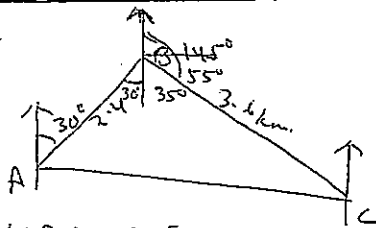
$k > 1$ or $k < -2$ — (1)

f) (i) $\alpha + \beta = \frac{-b}{a} = \frac{3}{1}$ — (1)

(ii) $\alpha\beta = \frac{c}{a} = \frac{6}{2} = 3$ — (1)

$4 + 2(\alpha + \beta) + \alpha\beta = 4 + 2 \times \frac{3}{1} + 3 = 10$ — (2)

Q12a



(i) $\angle CBA = 65^\circ$ — (1)

$AC^2 = (2.4)^2 + (3.6)^2 - 2 \times 2.4 \times 3.6 \cos 65^\circ$ — (1)

$AC = 3.4 \text{ km}$

(ii) $\frac{\sin A}{3.6} = \frac{\sin 65^\circ}{3.4}$ — (1)

$\sin A = \frac{3.6 \times \sin 65^\circ}{3.4}$

$A = 73^\circ 39'$ or 74°

\therefore Bearing of C from A = $30 + 73^\circ 39' = 103^\circ 39'$ — (1)
or 107°

b) (i) $y = \cos^2 3x$

$y' = 2 \cos 3x \times -\sin 3x \times 3$

$= -6 \sin 3x \cos 3x$ — (2)
-1 for any error.

(ii) $y = \ln(1-x) - \ln(1+x)$ — (1)

$y' = \frac{1}{1-x} \times -1 - \frac{1}{1+x}$

$= \frac{-1}{1-x} - \frac{1}{1+x}$ — (1) = $\frac{-2}{1-x^2}$

g) (i) $m_{AB} = \frac{-2-4}{1+3} = \frac{-3}{2}$ — (1)

$y+2 = \frac{-3}{2}(x-1)$

$2y+4 = -3x+3$

$3x+2y+1=0$

(ii) $D = \frac{|3 \times 0 + 2 \times -3 + 1|}{\sqrt{9+3}}$ — (1)

$= \frac{5}{\sqrt{3}}$

(iii) $d_{AB} = \sqrt{4^2+36} = \sqrt{52}$ — (1)

$Ar = \frac{1}{2} \times \frac{5}{\sqrt{3}} \times \sqrt{52}$

$= \frac{1}{2} \times \frac{5}{\sqrt{3}} \times \sqrt{13} \times 2$ — (1)
 $= 5 \text{ u}^2$

d) $\tan^2 x + 1 + \tan^2 x = 9$ — (1)

$2 \tan^2 x = 8$

$\tan x = \pm 2$ — (1)

Q13a) $\int \frac{2}{3x} - e^{-3x} dx$

$= \frac{2}{3} \ln x + \frac{e^{-3x}}{3} + C$

— (1) — (1)

(-1) if no C

(ii) $\int_0^{\pi/6} \sec^2 2x dx = \frac{1}{2} \tan 2x \Big|_0^{\pi/6}$ — (1)

$= \frac{1}{2} \tan \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ — (1)

$$y = 3 \cos \frac{2x}{2} \times \frac{1}{2}$$

$$m_T = \frac{3}{2} \times \cos \frac{3\pi}{4} = -\frac{3}{2\sqrt{2}} \quad \text{--- (1)}$$

$$y = 3 \times \sin \frac{3\pi}{4} = \frac{3}{\sqrt{2}} \quad \text{--- (1)}$$

eq. of tangent is

$$y - \frac{3}{\sqrt{2}} = -\frac{3}{2\sqrt{2}} \left(x - \frac{3\pi}{2} \right) \quad \text{--- (1)}$$

$$2\sqrt{2}y - 6 = -3x + \frac{9\pi}{2}$$

$$3x + 2\sqrt{2}y - 6 - \frac{9\pi}{2} = 0.$$

$$y = x^3 - 6x^2 + 9x + 1$$

$$y' = 3x^2 - 12x + 9$$

For stat pts $y' = 0$

$$3(x^2 - 4x + 3) = 0$$

$$(x-3)(x-1) = 0$$

$$\left. \begin{array}{l} x = 3, 1 \\ y = 1, 5 \end{array} \right\} \quad \text{--- (1)}$$

coordinates of stat pts are (3, 1), (1, 5)

$$y'' = 6x - 12$$

at $x = 1$

$$y'' = 6 - 12 = -6 < 0 \quad \text{--- (1)}$$

\therefore max. at (1, 5)

at $x = 3$

$$y'' = 18 - 12 = 6 > 0 \quad \text{--- (1)}$$

\therefore min. at (3, 1)

$$y'' = \frac{d^2y}{dx^2} = 0$$

$$6x - 12 = 0$$

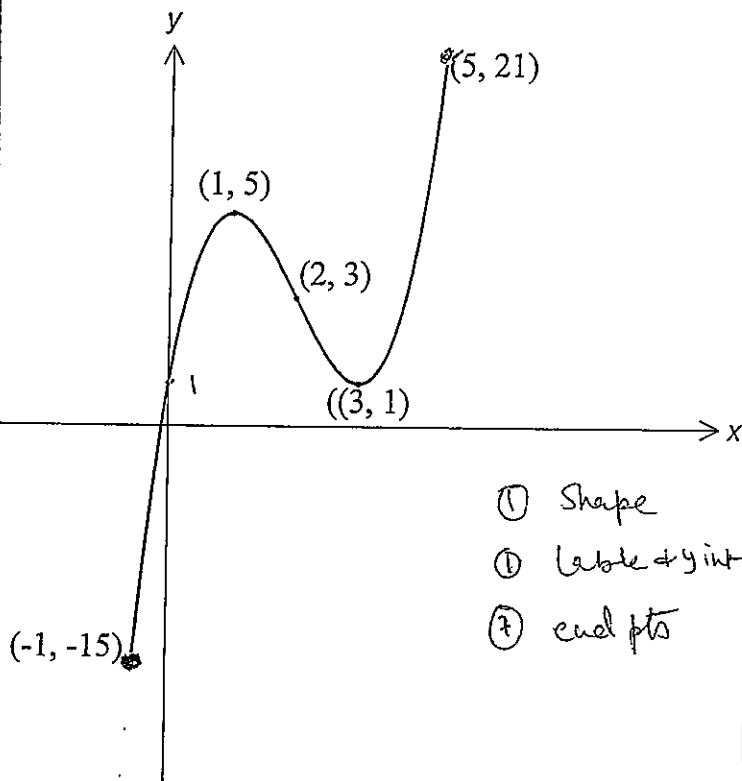
$$x = 2 \quad \text{--- (1)}$$

x	1	2	3
y''	-6	0	6

\therefore concavity changes. Hence (2, 3) is a P.O.I

(ii) For $-1 \leq x \leq 5$
when $x = -1, y = -15$
 $x = 5, y = 21$

\therefore end pts are (-1, -15) (5, 21)



(1) Shape

(1) Label + y int

(7) end pts

$$y' = 2x + a$$

at $x = 1$

$$m_1 = 2 + a$$

at $x = 0$

$$m_2 = a$$

Since tangents are \perp lar. (1)

$\therefore m_1 m_2 = -1$

$$(2+a) \times a = -1$$

$$a^2 + 2a + 1 = 0$$

$$(a+1)^2 = 0 \quad \text{--- (1)}$$

$$a = -1$$

b) (ii) In ΔPBC & ΔPQC (1)

$$\angle PBC = \angle PQC = 90^\circ \text{ (given, } \angle \text{ in a sq.)}$$

$$\angle BCP = \angle QCP \text{ (PC bisects } \angle ACB)$$

AC is common

$$\therefore \Delta PBC \equiv \Delta PQC \text{ (AAS)} \quad \text{--- (1)}$$

(ii) $AB = BC = x$ (sides of sq.)

$$\therefore PB = \frac{x}{3}$$

$$\text{Ar. } \Delta PBC = \frac{1}{2} x \times \frac{1}{3} x$$

$$= \frac{x^2}{6} \quad \text{--- (1)}$$

$$\therefore \text{Ar } \Delta PQC = \frac{x^2}{6} \text{ (}\because \Delta PBC \equiv \Delta PQC)$$

$$\text{Ar } \Delta ABC = \frac{1}{2} x \times x = \frac{1}{2} x^2 \quad \text{--- (1)}$$

$$\therefore \text{Ar } \Delta APQ = \frac{x^2}{6} - 2 \times \frac{x^2}{6}$$

$$= \frac{x^2}{6} \text{ cm}^2 \quad \text{--- (1)}$$

c) (i) When $\dot{x} = 0$

$$\frac{1}{5}t + (4-t) = 0$$

$$t = 0 \text{ or } 4$$

$$\text{time} = 4 \text{ min.} \quad \text{--- (1)}$$

$$\text{ii) } v = \dot{x} = \frac{4}{5}t - \frac{1}{5}t^2$$

$$\frac{dv}{dt} = \dot{\dot{x}} = \frac{4}{5} - \frac{2}{5}t$$

$$\frac{d^2v}{dt^2} = -\frac{2}{5}$$

max. vel. when $\frac{dv}{dt} = 0$

$$\frac{4}{5} - \frac{2}{5}t = 0$$

$$t = 2$$

Since $\frac{d^2v}{dt^2} = -\frac{2}{5} < 0$ for all t , $t = 2$

gives max. vel. } (1)

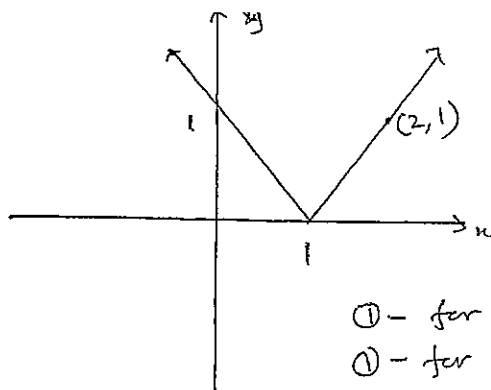
$$\text{max. vel} = \frac{2}{5}(4-2) = \frac{4}{5} \text{ km/min.}$$

$$\text{iii) } x = \int_0^4 \left(\frac{4}{5}t - \frac{1}{5}t^2 \right) dt \quad \text{--- (1)}$$

$$= \left[\frac{2}{5}t^2 - \frac{1}{15}t^3 \right]_0^4$$

$$= \frac{32}{5} - \frac{64}{15} = 2\frac{2}{15} \text{ km.} \quad \text{--- (1)}$$

d)



(1) - for shape

(1) - for $y=1$, any pt on the other branch is marked

Q15 (i) $A = \frac{1}{3} [0 \cdot 4 + 4 \times 0.8 + 2 \times 1.5 + 4 \times 1.3 + 0.3] \quad \text{--- (1)}$

$$= 4.03 \quad \text{--- (1)}$$

$$\text{(ii) Vol.} = 4.03 \times 0.25 \times 60 \times 60 \times 24$$

$$= 87120 \text{ m}^3 \quad \text{--- (1)}$$

$$\text{b) Ar of seg CDE} = \frac{1}{4} \pi r^2 \quad \text{--- (1)}$$

$$\text{Ar of } \Delta DOC = \frac{1}{2} r^2 \sin \theta \quad \text{--- (1)}$$

$$\text{Ar of sector DOC} = \frac{1}{2} r^2 \theta$$

$$\therefore \text{Ar of seg CDE}$$

$$= \text{Ar of sector} - \text{Ar of } \Delta$$

$$\frac{1}{4} \pi r^2 = \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin \theta \quad \text{--- (1)}$$

$$\frac{1}{4} \pi = \theta - \sin \theta$$

c) (i) Initially $t=0$, $P = 9000000$

$$\therefore 9000000 = P_0 e^0 = P_0 \quad \text{--- (1)}$$

(ii) When $t = 2004 - 1954 = 50$

$$P = 2 \times 10^7$$

$$\therefore 2 \times 10^7 = 9 \times 10^6 e^{50k}$$

$$\frac{20}{9} = e^{50k}$$

$$\ln\left(\frac{20}{9}\right) = 50k \quad \text{--- (1)}$$

$$k = \frac{1}{50} \ln\left(\frac{20}{9}\right) = 0.015970$$

(ii)

$$\frac{dP}{dt} = k \cdot P_0 e^{kt}$$

$$= kP$$

$$= \frac{1}{50} \ln\left(\frac{20}{9}\right) \times e^{\frac{10}{50} \times \ln\left(\frac{20}{9}\right)}$$

$$= 168620 \text{ per year} \quad \text{--- (1)}$$

(iii) $P = 9 \times e^{\frac{1}{50} \ln\left(\frac{20}{9}\right) \times 60} \quad \text{--- (1)}$

$$= 23.4632 \text{ millions}$$

(iv) $50 = 9 \times e^{kt} \quad \text{--- (1)}$

$$\frac{50}{9} = e^{kt}$$

$$\ln\left(\frac{50}{9}\right) = kt$$

$$t = \frac{1}{k} \times \ln\left(\frac{50}{9}\right)$$

$$\text{(1) } = \frac{\ln\left(\frac{50}{9}\right)}{\frac{1}{50} \ln\left(\frac{20}{9}\right)} = 107.375 \text{ million in year } 2061 \quad \text{--- (1)}$$

15d

$$\text{If } y^2 = (2m-y)(2n-y)$$

$$\text{then } y^2 = 4mn - 2my - 2ny + y^2$$

$$4mn - 2my - 2ny = 0$$

$$2mn = my + ny$$

\div mny

$$\frac{2}{y} = \frac{1}{n} + \frac{1}{m} \quad \text{--- (1)}$$

$$\frac{1}{y} + \frac{1}{y} = \frac{1}{n} + \frac{1}{m}$$

$$\frac{1}{y} - \frac{1}{m} = \frac{1}{n} - \frac{1}{y} \quad \text{--- (1)}$$

$\therefore \frac{1}{m}, \frac{1}{y}, \frac{1}{n}$ are an arithmetic sequence.

5(i) $P = \$ 200000$
 $r = 0.425$

$$A_1 = 200000(1.00425) - M$$

$$A_2 = A_1(1.00425) - M$$
$$= [200000(1.00425) - M] \times (1.00425) - M$$
$$= 200000(1.00425)^2 - M(1.00425) - M$$

$$A_3 = A_2(1.00425) - M$$
$$= [200000(1.00425)^2 - M(1.00425) - M] \times (1.00425) - M$$
$$= 200000(1.00425)^3 - M(1.00425)^2 - M(1.00425) - M$$

$$A_n = 200000(1.00425)^n - M(1 + 1.00425 + \dots + (1.00425)^{n-1}) \quad \text{--- (1)}$$

$$A_n = 200000(1.00425)^n - M \left(\frac{1.00425^n - 1}{0.00425} \right)$$

A.P with $a=1$
 $r=1.00425$

If the loan is paid after n months

$$A_n = 0 \quad \text{--- (1)}$$

$$M \left(\frac{1.00425^n - 1}{0.00425} \right) = 200000(1.00425)^n$$

$$M = \frac{200000(1.00425)^n(0.00425)}{1.00425^n - 1}$$

(iii) 30 years = 30 x 12 months = 360 instalments

$$M = \frac{200000(1.00425)^{360}(0.00425)}{1.00425^{360} - 1}$$
$$= \$ 1085.90 \quad \text{--- (1)}$$

(iv) total repayment = $560 \times 1085.90 = 390924$ --- (1)

(v) $1331 = \frac{200000(1.00425)^n \times 0.00425}{1.00425^n - 1}$

$$+ 331(1.00425)^n - 1331 = 850(1.00425)^n$$

$$(1331 - 850)(1.00425)^n = 1331 \quad \text{--- (1)}$$

$$(1.00425)^n = 2.767151 \dots$$

$$n \ln(1.00425) = \ln(2.76715 \dots)$$

$$n = \frac{\ln(2.76715 \dots)}{\ln(1.00425)} \quad \text{--- (1)}$$

$$= 239.99 \dots = 240 \text{ months}$$

(2 marks with correct ans & working)

(b) $1 + \sin^2 x + \sin^4 x + \sin^6 x + \dots$

$$r = \sin^2 x$$

$-1 \leq \sin x \leq 1$ for all values of x

$$\sin x \neq \pm 1$$

$$\therefore -1 < \sin x < 1$$

$$0 \leq \sin^2 x < 1$$

\therefore limiting sum exists.

$$S_{\infty} = \frac{a}{1-r} = \frac{1}{1-\sin^2 x} = \frac{1}{\cos^2 x} = \sec^2 x \quad \text{--- (1)}$$

3) (i) Given R is the radius of sphere & h the height. AD of inscribed cone.

$$OD = AD - OA = h - R$$

$$BO^2 = OB^2 - OD^2$$

$$r^2 = h^2 - (h - R)^2 = (2hR - h^2) \quad \text{--- ①}$$

$$\begin{aligned} \text{(ii)} \quad V_{\text{cone}} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \pi (2hR - h^2) \times h \quad \text{--- ①} \\ &= \frac{1}{3} \pi (2h^2 R - h^3) \end{aligned}$$

$$\text{(iii)} \quad \frac{dV}{dh} = \frac{\pi}{3} (4hR - 3h^2)$$

For max or min $\frac{dV}{dh} = 0$

$$\frac{1}{3} \pi (4hR - 3h^2) = 0$$

$$h(4R - 3h) = 0$$

$$h = 0 \text{ or } h = \frac{4R}{3} \quad \text{--- ①}$$

not possible

$$\frac{d^2V}{dh^2} = \frac{1}{3} \pi (4R - 6h)$$

$$\text{at } h = \frac{4R}{3}$$

$$\frac{d^2V}{dh^2} = \frac{1}{3} \pi (4R - 6 \times \frac{4R}{3})$$

$$= -\frac{4\pi R}{3} < 0$$

\therefore Vol. is max. at $h = \frac{4R}{3}$ } ①

\therefore Vol. of the largest cone

$$\frac{1}{3} \pi (2 \times \frac{16R^2}{9} \times R - \frac{64R^3}{27})$$

$$= \frac{1}{3} \pi (\frac{32R^3}{9} - \frac{64R^3}{27})$$

$$= \frac{32\pi R^3}{81}$$

① have
with workii

$$V_{\text{sphere}} = \frac{4}{3} \pi R^3$$

$$\therefore \frac{V_{\text{of largest cone}}}{V_{\text{of sphere}}} = \frac{\frac{32\pi R^3}{81}}{\frac{4}{3} \pi R^3}$$

$$= \frac{8}{27}$$

\therefore V. of the largest cone is $\frac{8}{27}$ V. of the sphere