Caringbah High School

2013

Trial HSC Examination

Mathematics

General Instructions
- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11–16, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section I Pages 2–3 **10 marks**
- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II Pages 4–11 **90 marks**
- Attempt Questions 11–16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 The equation of the directrix of the parabola $x^2 = 16y$

A) $x = -4$ B) $y = -4$

C) $x = 4$ D) $y = 4$

2 If $y = 3\cos 2x$ then $\frac{dy}{dx} =$

A) $6\sin 2x$ B) $6\cos 2x$

C) $-6\cos 2x$ D) $-6\sin 2x$

3 If α , β are the roots of $2x^2 - 4x + 1 = 0$ then $\alpha^2+\beta^2$ is equal to

A) $\frac{7}{2}$ B) $\frac{15}{4}$

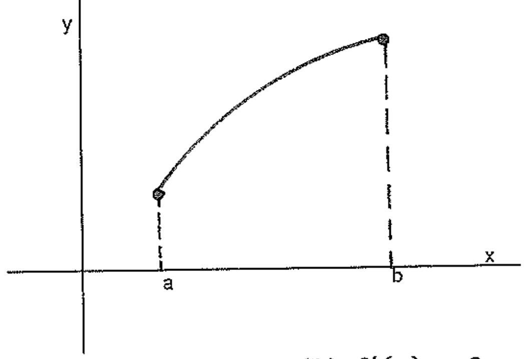
C) $\frac{17}{4}$ D) 3

4 The value of $t^3 - 3t$ if $t = 2\sqrt{2}$ is

A) $16 - 6\sqrt{2}$ B) $26\sqrt{2}$

C) $10\sqrt{2}$ D) *none of these*

5 For the function $y = f(x), a \leq x \leq b$ graphed below which of the following are true



(A) $f'(x) > 0$ and $f''(x) < 0$ (B) $f'(x) < 0$ and $f''(x) < 0$

(C) $f'(x) > 0$ and $f''(x) > 0$ (D) $f'(x) < 0$ and $f''(x) > 0$

6 What is the value of

$$\int_0^1 (x+1)^{-1}\,dx$$

A) $ln2 - 1$ B) $ln2$

C) $ln2 + 1$ D) 0

7 The displacement at any time t (seconds) of a particle is given by $x = 2\sin(\frac{\pi}{3}t)$. Its velocity at $t = 4$ seconds in metres per second is

A) $\frac{-\pi}{3}$ B) +2

C) -2 D) $\frac{\pi}{3}$

8

$$\sum_{n=3}^{10}(3n+1)$$

A) 143.5 B) $5(3^8 - 1)$

C) $5(3^7 - 1)$ D) 164

9 The interior angle size of a 12 sided regular figure is

A) 150^0 B) 144^0

C) 210^0 D) 180^0

10 The domain of $y = \sqrt{4x - x^2}$ is

A) $x \leq 0$ B) $x \leq 4$

C) $0 \leq x \leq 4$ D) $x \geq 0$

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

a) Find correct to 2 decimal places the value of (2)

$$\frac{8.64^2 + 9.86}{7.5 \times 3.21}$$

b) Factorise $3a^2 + a - 2$ (2)

c) Solve the equation $5x - 3(x - 4) = 2$ (2)

d) Differentiate

(i) $y = 4x^2 + \dfrac{1}{x^2} + 3 - \sqrt{x}$ (2)

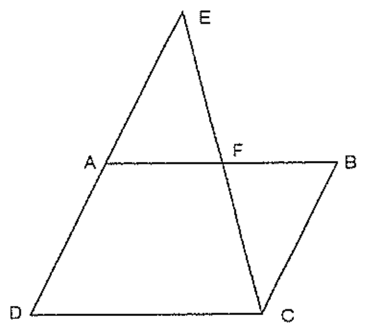
(ii) $y = x\tan 2x$ (2)

(iii) $y = \dfrac{e^{2x}}{x+1}$ (2)

e) Find the equation of the tangent to the curve $y = (2x - 1)^4$ at the point where $x = 1$. (3)

Question 12 (15 marks) Use a SEPARATE writing booklet.

a)



In the diagram ABCD is a parallelogram DA is produced to E, so that AE=AD. AB intersects EC in F. Prove that AF=FB (2)

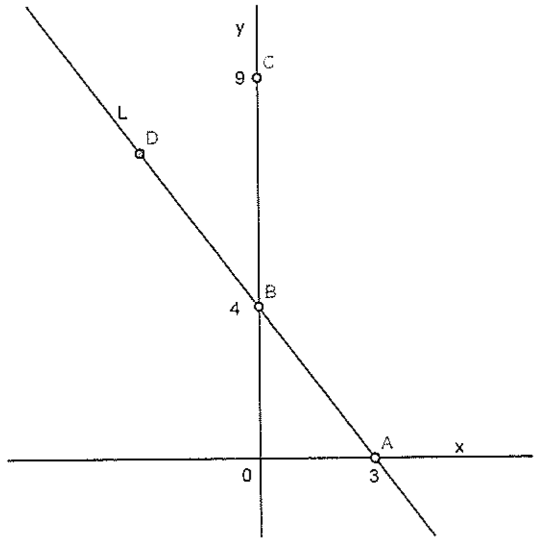
b) The parabola P has equation $x^2 = 16(y + 1)$

(i) Draw a neat $\frac{1}{3}$ page sketch of P and clearly indicate on it

(α) equation of the directrix (1)
(β) the coordinates of its focus (1)
(γ) the coordinates of its vertex (1)

(ii) Find the coordinates of the points where P cuts the x axis (1)

(iii) Calculate the area bounded by P and the x axis (2) c)



The line L cuts the x axis at A (3,0) and the y axis B(0,4). C is the point (0,9). D is a point on the line L

Copy this diagram onto your page ($\frac{1}{3}$ page size)

(i) Find the equation of the line L (2)

(ii) If B is the midpoint of AD find the coordinates of D (1)

(iii) Show that ∠ACD is a right angle (2)

(iv) Find the area of triangle ACD (2)

Question 13 (15 marks) Use a SEPARATE writing booklet.

a) Given the curve $f(x) = 7 + 4x^3 - 3x^4$

i) Find any stationary points and determine their nature (2)

ii) Find any inflexion points (1)

iii) Sketch the curve and label with the above points (2)

b) The velocity v m/s of a particle after t seconds is given by
$v = 7 - 6t - t^2, \quad t \geq 0$

(i) Find the acceleration of the particle when $t = 2$ (2)

(ii) When is the particle at rest (2)

(iii) If the particle is at the origin at $t = 0$ find the distance travelled in the first 4 seconds (2)

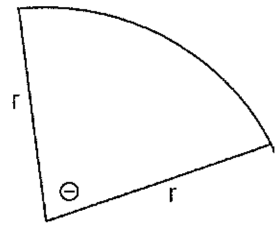
c) Find the volume generated by rotating about the line $y = 0$, the region in the first quadrant bounded by the curve $y^2 = \frac{1}{x}$, the x axis and the lines $x = 1$ and $x = e$ (2)

d) Solve for θ , $0^0 \leq \theta \leq 360^0$ the equation (2)

$\sqrt{3}\,\tan x - 1 = 0$

Question 14 (15 marks) Use a SEPARATE writing booklet.

a) A flower bed is to be made in the shape of a minor sector with angle θ in radians and radius r in metres



(i) If the area of the bed is $16m^2$ show that $\theta = \frac{32}{r^2}$ (1)

(ii) Show that the perimeter P metres of the flower bed is given by $P = \frac{32}{r} + 2r$ (1)

(iii) Find the minimum value of the perimeter P and the value of θ for which it occurs (2)

b) Food prepared for takeaway is likely to get contaminated by bacteria. In order to kill the bacteria the food is heated to 135^0C for a period of time. The number of live bacteria in the food after t minutes is given by $N(t) = 4000e^{-0.4t}$ $(t \geq 0)$

(i)Find the initial number of bacteria present in the food (1)

(ii)Find the number of bacteria still alive after 5 minutes (1)

(iii)Find the time taken in minutes (1 d.p) to kill 50% of the bacteria (1)

(iv)Find the rate of decrease in the number of bacteria when 50% have been killed (2)

c) (i) On the same number plane draw a neat sketch of the graphs (2)

$y = 9 - x^2$ and $y = x^2 - 9$

(ii) Find the points of intersection of the graphs (1)

(iii) Find the area enclosed between the graphs in (i) (3)

Question 15 (15 marks) Use a SEPARATE writing booklet.

a) Given $ln10 = x$ and $ln7 = y$ find an expression in terms of x and y for $ln0.07$ (2)

b) Find the exact value of (leave in terms of e)

(i) $\int_0^1 e^{2x}\,dx$ (1)

(ii) $\int_1^{2e} \frac{x}{x^2+1}\,dx$ (2)

c) Given the series $ln2 + ln4 + ln8 + ln16 + \cdots$

Find the sum of the first 21 terms of the series leave answer in **exact** form (3)

d) Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} cos2x\,dx$ as an **exact** value. (3)

e) The curve $y = \sqrt{x^3}$ for $x \geq 0$ is rotated about the y axis between the values of $y = 0$ *and* $y = 1$

(i) Show that $x^2 = y^{\frac{4}{3}}$ (1)

(ii) Find the volume of the solid formed (2)

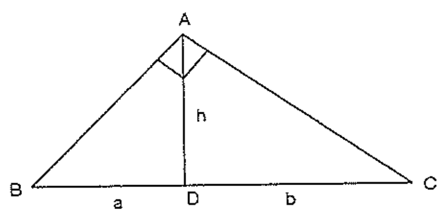
(leave answer in terms of π)

f) Find the value of y such that $\sqrt{80} + \sqrt{5} = \sqrt{y}$ (1)

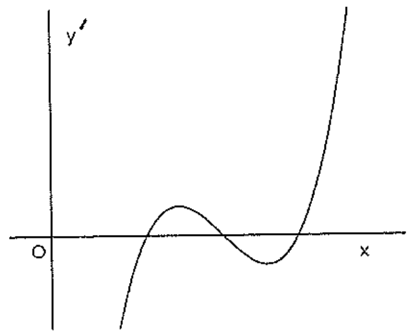
Question 16 (15 marks) Use a SEPARATE writing booklet.

a) In the diagram $\angle BAC = 90^0$. AD is perpendicular to BC (3)

Show giving reasons that $h = \sqrt{ab}$



b) The diagram below shows the sketch of the **derivative** of $y = f(x)$.
Draw a possible sketch of $y = f(x)$. (at least one third page size) (2)



c) Evaluate using the trapezoidal rule with 4 sub intervals (ans to 3d.p.) (3)

$$\int_1^5 \frac{1}{x^2+1}\,dx$$

d)

(i) Sketch the graph of $y = 2cos\frac{x}{2}$ for $0^0 \leq x \leq 2\pi$ (2)

(ii) How many solutions are there for $x = 2cos\frac{x}{2}$ You must justify your answer using the sketch above (1)

e) Given that $f'(x) = \frac{8x}{\pi^2} + cos2x$ and $f\left(\frac{\pi}{4}\right) = \frac{3}{4}$ find $f(x)$ (2)

f) Find the derivative of $y = \ln(\frac{x^2-4}{x+1})$ (2)

End of Exam

Name: ______________________________

Multiple Choice Answer Sheet

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

correct ↙

A ⊗ B ⊗ C ○ D ○

1)	A ○	B ○	C ○	D ○
2)	A ○	B ○	C ○	D ○
3)	A ○	B ○	C ○	D ○
4)	A ○	B ○	C ○	D ○
5)	A ○	B ○	C ○	D ○
6)	A ○	B ○	C ○	D ○
7)	A ○	B ○	C ○	D ○
8)	A ○	B ○	C ○	D ○
9)	A ○	B ○	C ○	D ○
10)	A ○	B ○	C ○	D ○

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax\,\tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

2013 2U Trial HSC Solutions

1) B
2) D
3) D
4) C
5) A
6) B
7) A
8) D
9) A
10) C

Question 11

a) 3.510263… (using calculator

= 3.51 (2 decimal places)

b)

$$3a^2 + a - 2 = \frac{(3a-2)(3a+3)}{3}$$

= $(3a - 2)(a + 1)$

c) $5x - 3(x - 4) = 2$

$5x - 3x + 4 = 2$

$2x = -10$

$x = -5$

d)i)

$$\frac{d}{dx}\left[4x^2 + x^{-2} + 3 - \sqrt{x}\right] = 8x - \frac{2}{x^3} - \frac{1}{2}x^{-\frac{1}{2}}$$

$$= 8x - \frac{2}{x^3} - \frac{1}{2\sqrt{x}}$$

ii) $y = x\tan(2x)$

$y' = \tan(2x)\,(1) + x \times 2\sec^2(2x)$

$= \tan(2x) + 2x\sec^2(2x)$

iii)

$$y = \frac{e^{2x}}{x+1}$$

$$y' = \frac{(x+1).2e^{2x} - e^{2x} \times 1}{(x+1)^2}$$

$$= \frac{2x.e^{2x} + 2e^{2x} - e^{2x}}{(x+1)^2}$$

$$= \frac{e^{2x}(2x+1)}{(x+1)^2}$$

e) $y = (2x - 1)^4$

$y' = 4(2x - 1)^3 \times 2$

$= 8(2x - 1)^3$

$= 8$ at $x = 1$

Equation has form $y - y_1 = m(x - x_1)$

Line passes through (1, 1)

$y - 1 = 8(x - 1)$

$y - 1 = 8x - 8$

$8x - y - 7 = 0$

Question 12

a)

∠AFE = ∠FBC (vert opp.)

∠ADC = ∠EAF (corr. angles, AB ‖ DC)

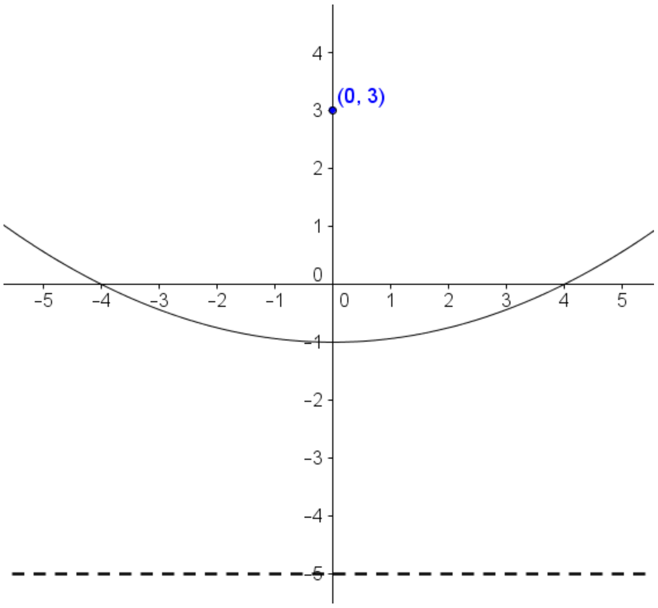
So ∠AEF = ∠BCF (remaining angles in triangles AEF and BCF)

Also, AE = AD (given)

Hence ΔAEF ≡ ΔBCF (AAS)

Hence AF = FB (corr. sides in congruent triangles)

b) i)



ii)

$x^2 = 16(y + 1)$

$x^2 = 16(0 + 1)$

$x^2 = 16$

$x = -4, 4$

P cuts the x axis at (-4, 0) and (4, 0)

iii)

$$y = \frac{x^2}{16} - 1$$

$$\text{Area} = \left| \int_{-4}^{4} \frac{x^2}{16} - 1 \, dx \right| = \left[\frac{x^3}{48} - x \right]_{-4}^{4}$$

$$= \left| \frac{64}{48} - 4 - \left(\left(\frac{-64}{48} \right) - (-4) \right) \right|$$

$$= \left| -\frac{16}{3} \right|$$

$$= \frac{16}{3} \text{ units}^2$$

c) i) L passes through (0, 4) and (3, 0)

$$m = \frac{0-4}{3-0} = -\frac{4}{3}$$

$$y - 4 = -\frac{4}{3}(x - 0)$$

$3y - 12 = -4x$

$4x + 3y - 12 = 0$ is the equation of L

ii) If B is midpoint then:

$$0 = \frac{x_D + 3}{2} \quad \text{and} \quad 4 = \frac{y_D + 0}{2}$$

$x_D = -3$ $\qquad$ $y_D = 8$

So D has co-ordinates (-3, 8)

iii)

$$m_{AC} = \frac{0-9}{3-0} \qquad m_{CD} = \frac{9-8}{0-(-3)}$$

$= -3$ $\qquad$ $= \frac{1}{3}$

Now $-3 \times \frac{1}{3} = -1$

So AC and CD are perpendicular

Therefore, ∠ACD is a right angle

iv)

$$\text{Distance}_{CD} = \sqrt{(-3-0)^2 + (8-9)^2}$$

$$= \sqrt{10}$$

$$\text{Distance}_{AC} = \sqrt{(3-0)^2 + (0-9)^2}$$

$$= \sqrt{90}$$

Area of ΔACD = $\frac{1}{2} \times \sqrt{90} \times \sqrt{10}$

$= 15$ units^2

Question 13

a) i)

$f(x) = 7 + 4x^3 - 3x^4$

$f'(x) = 12x^2 - 12x^3$

$= 12x^2(1 - x)$

For stat. points, $f'(x) = 0$

$0 = 12x^2(1 - x)$

$x = 0, 1$

So stationary points at (0, 7) and (1, 8)

Consider point at (0, 7)

$f''(x) = 24x - 36x^2$

$f''(0) = 0$ Possible point of inflexion

$f''(x) < 0$ for $x < 0$ and $f''(x) > 0$ for $x > 0$

Therefore, horizontal point of inflexion at (0, 7)

Consider stationary point at (1,8)

$f''(1) = -12$ Concave down

Therefore maximum turning point at (1, 8)

ii)

$f''(x) = 24x - 36x^2$

Possible points of inflexion when $f''(x) = 0$

$0 = 12x(2 - 3x)$

$x = 0, \frac{2}{3}$

So possible points of inflexion at (0, 7) and $\left(\frac{2}{3}, \frac{205}{27}\right)$

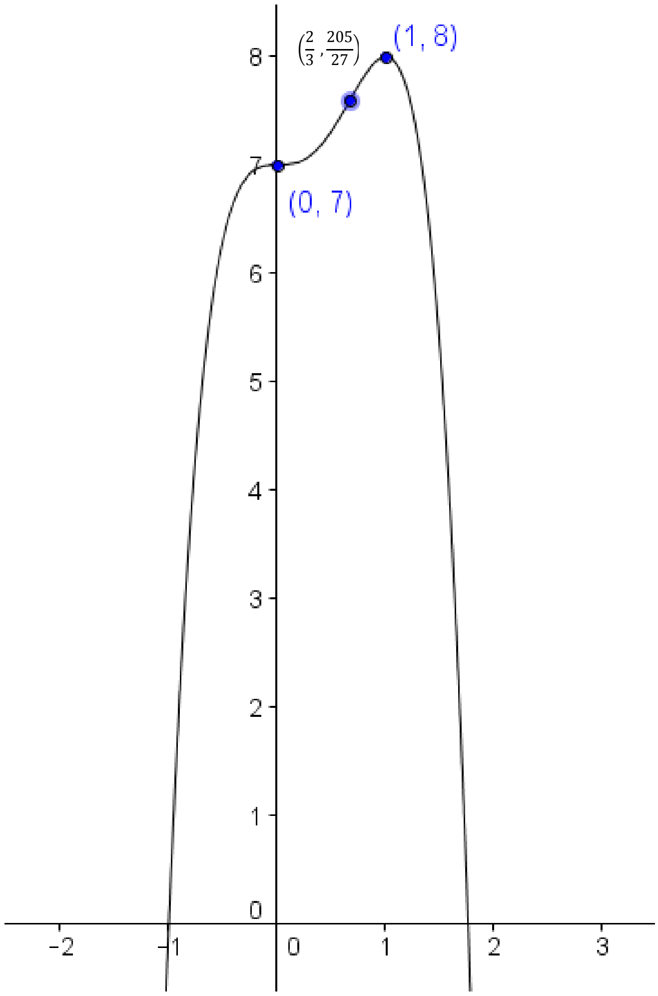
Consider $\left(\frac{2}{3}, \frac{205}{27}\right)$

When $x < \frac{2}{3}$, $f''(x) > 0$

When $x > \frac{2}{3}$, $f''(x) < 0$

So point of inflexion at $\left(\frac{2}{3}, \frac{205}{27}\right)$

iii)



b) $v = 7 - 6t - t^2$

i) $v' = a = -6 - 2t$

$= -6 - 2(2)$ at $t = 2$

$= -10 \text{ ms}^{-2}$

ii)

Particle is at rest when $v = 0$

$7 - 6t - t^2 = 0$

$-(t^2 + 6t - 7) = 0$

$-(t - 1)(t + 7) = 0$

$t = -7, 1$ but $t \geq 0$

So particle is at rest after 1 second

iii)

Distance travelled =

$$\int_0^1 7 - 6t - t^2\,dt + \left|\int_1^4 7 - 6t - t^2\,dt\right|$$

$$= \left[7t - 3t^2 - \frac{t^3}{3}\right]_0^1 + \left[7t - 3t^2 - \frac{t^3}{3}\right]_1^4$$

$$= \left[\frac{11}{3} - 0\right] + \left|-\frac{124}{3} - \frac{11}{3}\right|$$

= $48\frac{2}{3}$ units

Alternatively,

$x = 7t - 3t^2 - \frac{t^3}{3} + C$

Since particle is initially at the origin

$0 = 7(0) - 3(0)^2 - \frac{(0)^3}{3} + C$

$C = 0$

$x = 7t - 3t^2 - \frac{t^3}{3}$

Now, particle comes to rest at t = 1

$x = 7(1) - 3(1)^2 - \frac{(1)^3}{3} = \frac{11}{3}$

At t = 4, particle is at

$x = 7(4) - 3(4)^2 - \frac{(4)^3}{3} = -\frac{124}{3}$

Total distance = $\frac{11}{3} + \left(\frac{11}{3} + \frac{124}{3}\right) = 48\frac{2}{3}$ units

c)

$$\text{Volume} = \pi \int_1^e \frac{1}{x}\,dx$$

$$= \pi\,[\ln x]_1^e$$

$= \pi\,[\ln e - \ln 1]$

$= \pi$ units^3

d)

$$\sqrt{3}\tan x = 1$$

$$\tan x = \frac{1}{\sqrt{3}}$$

tan > 0 in the 1st and 3rd quadrants

$x = 30°, 210°$

Question 14

a)

i) $A = \frac{1}{2} \times r^2 \times \theta$

$16 = \frac{1}{2} \times r^2 \times \theta$

$32 = r^2 \times \theta$

$\theta = \frac{32}{r^2}$

ii) Arc length = $r \times \theta$

$= r \times \frac{32}{r^2}$

$= \frac{32}{r}$

Perimeter = $r + r + \frac{32}{r}$

$= 2r + \frac{32}{r}$

iii) $P = 2r + 32r^{-1}$

$P' = 2 - 32r^{-2}$

$= 2 - \frac{32}{r^2}$

For turning point, P' = 0

$0 = 2 - \frac{32}{r^2}$

$0 = 2r^2 - 32$

$32 = 2r^2$

$16 = r^2$

$r = 4$ (since $r > 0$)

Now, $P'' = \frac{32}{r^3}$

> 0 at $r = 4$

So minimum value occurs at $r = 4$

Now,

$\theta = \frac{32}{r^2}$

$= \frac{32}{(4)^2}$ at $r = 4$

= 2 radians

So an angle of 2 radians gives minimum perimeter

b) i)

$N(t) = 4000e^{-.04t}$

$N(0) = 4000 \times e^0$

= 4000

So initially there are 4000 bacteria

ii) $N(5) = 4000 \times e^{-0.4(5)}$

= 541.3 (1 decimal place)

So after 5 minutes there are 541.3 bacteria

iii) 50% of initial bacteria is 2000

$2000 = 4000 \times e^{-0.4t}$

$½ = e^{-0.4t}$

ln ½ = - 0.4t

$t = \frac{\ln\frac{1}{2}}{-0.4}$

= 1.7 mins

iv)

$$\frac{dN}{dt} = kN$$

$$= -0.4 \times 4000 \times e^{-0.4t}$$

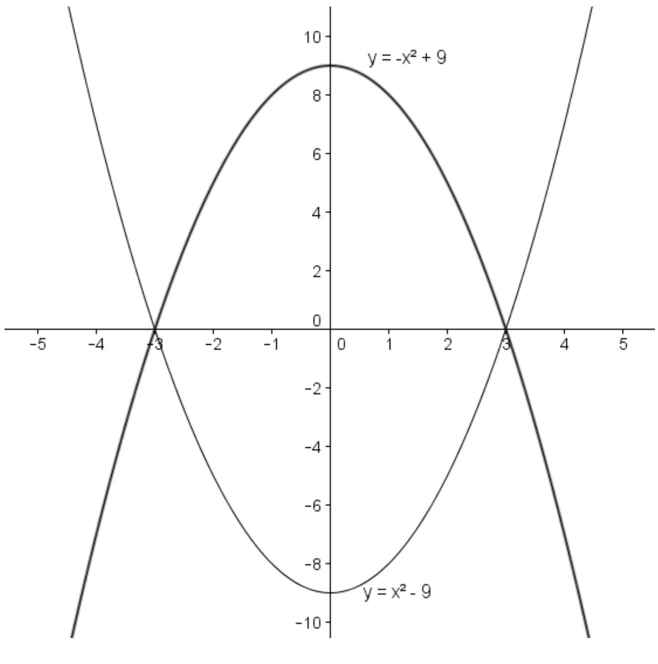
$$= -0.4 \times 4000 \times e^{-0.4(1.7)}$$

= -810.6 (1 decimal place)

So after 1.7 mins, when 50% of the bacteria have been killed, the population is decreasing at a rate of 810.6 bacteria per minute.

c)

i)



ii) Points of intersection at (-3, 0) and (3, 0)

iii)

$$\text{Area} = \int_{-3}^{3} [(9 - x^2) - (x^2 - 9\,)]dx$$

$$= \int_{-3}^{3} 18 - 2x^2 dx$$

$$= \left[18x - \frac{2x^3}{3}\right]_{-3}^{3}$$

$$= \left[54 - \frac{54}{3}\right] - \left[-54 + \frac{54}{3}\right]$$

= 72 units ^{2}

Question 15

a)

ln10 = *x* ln7 = *y*

$$\ln 0.07 = \ln\left(\frac{7}{100}\right)$$

$$= \ln 7 - \ln 100$$

= ln7 – ln$(10)^2$

= ln7 – 2ln10

= *y* – 2*x*

b) i)

$$\int_0^1 e^{2x}\,dx = \left[\frac{1}{2}e^{2x}\right]_0^1$$

$$= \frac{1}{2}e^2 - \frac{1}{2}e^0$$

$$= \frac{1}{2}[e^2 - 1]$$

ii)

$$\int_1^{2e} \frac{x}{x^2+1}\,dx = \frac{1}{2} \times \int_1^{2e} \frac{2x}{x^2+1}\,dx$$

$$= \frac{1}{2} \times [\ln(x^2+1)]_1^{2e}$$

$$= \frac{1}{2}[\ln(4e^2+1) - \ln 2]$$

c)

Arithmetic series with a = ln2 and d = ln2

$$\text{Sum} = \frac{n}{2}[a + \text{L}]$$

$$= \frac{21}{2}(\ln 2 + 21\ln 2)$$

$$= \frac{21}{2}(22\ln 2)$$

= 231ln2

= 160.12 (2 decimal places)

d)

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos 2x\,dx = \left[\frac{1}{2}\sin 2x\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$$

$$= \frac{1}{2}\sin\left(\frac{\pi}{2}\right) - \frac{1}{2}\sin\left(\frac{\pi}{3}\right)$$

$$= \frac{1}{2} \times 1 - \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{2} - \frac{\sqrt{3}}{4}$$

$$= \frac{2 - \sqrt{3}}{4}$$

e) i)

$$y = \sqrt[3]{x}$$

$$y^2 = x^3$$

$$y^{\frac{2}{3}} = x$$

$$(y^{\frac{2}{3}})^2 = x^2$$

$$x^2 = y^{\frac{4}{3}}$$

ii)

$$\text{Volume} = \pi \times \int_0^1 y^{\frac{4}{3}} \, dy$$

$$= \pi \left[\frac{3}{7} y^{\frac{7}{3}}\right]_0^1$$

$$= \pi \left[\frac{3}{7} - 0\right]$$

$$= \frac{3}{7}\pi \quad \text{units}^3$$

f)

$$\sqrt{80} + \sqrt{5} = \sqrt{y}$$

$$\text{LHS} = 4\sqrt{5} + \sqrt{5}$$

$$= 5\sqrt{5}$$

$$= \sqrt{125}$$

So $y = 125$

Question 16

a)

$AB^2 = a^2 + h^2$ (Pythagoras in ΔABD)

$AC^2 = h^2 + b^2$ (Pythagoras in ΔADC)

Now,

$AB^2 + AC^2 = (a + b)^2$ (Pythagoras in ΔABC)

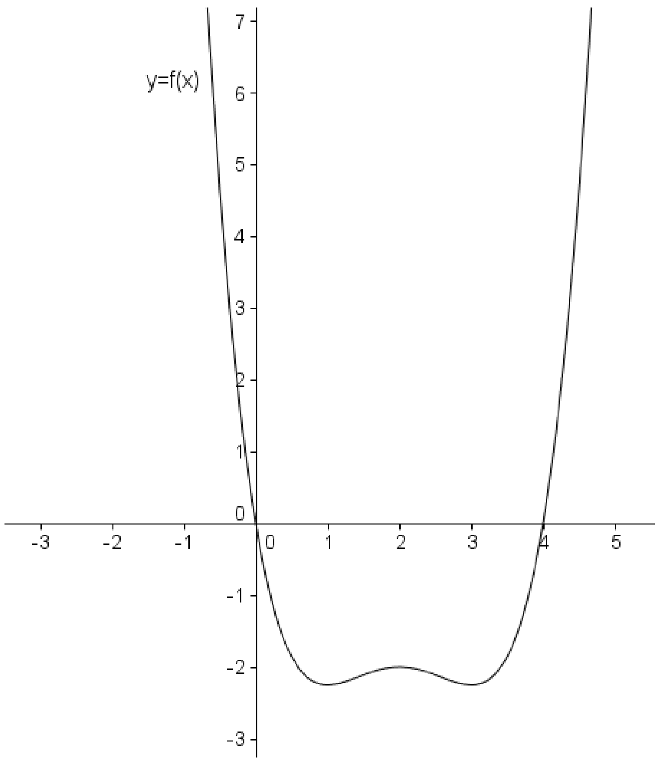
$(a^2 + h^2) + (h^2 + b^2) = (a + b)^2$

$2h^2 + a^2 + b^2 = a^2 + 2ab + b^2$

$2h^2 = 2ab$

$h = \sqrt{ab}$ $(h > 0)$

b)



Graph could also be shifted up or down

c)

h = 1

$y_1 = f(1) = \frac{1}{2}$ $y_2 = f(2) = \frac{1}{5}$ $y_3 = f(3) = \frac{1}{10}$

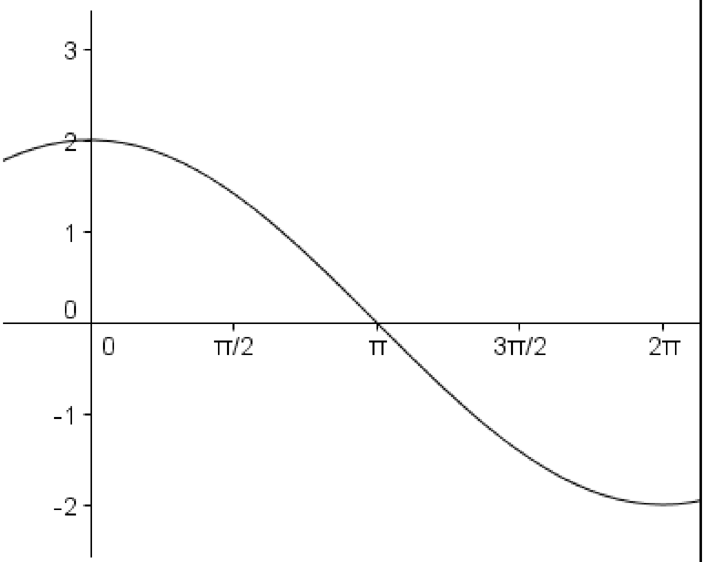
$y_4 = f(4) = \frac{1}{17}$ $y_5 = f(5) = \frac{1}{26}$

$$\text{Area} = \frac{1}{2}\left[\frac{1}{2} + \frac{1}{26} + 2 \times \left(\frac{1}{5} + \frac{1}{10} + \frac{1}{17}\right)\right]$$

$$= \frac{1}{2}\left[\frac{7}{13} + \frac{61}{85}\right]$$

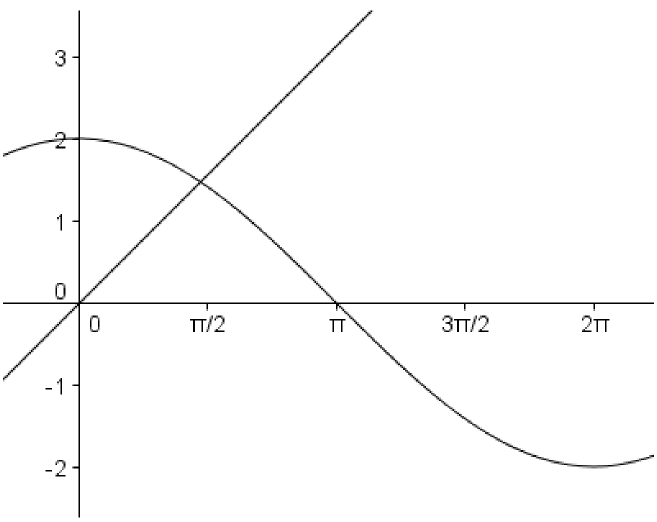
$=\frac{69}{1105}$ = 0.628 (3 decimal places)

d) i)



Amplitude = 2 Period = 4π

ii) One solution, seen by drawing y = x on the graph



e)

$$f'(x) = \frac{1}{\pi^2}8x + cos2x$$

$$f(x) = \frac{1}{\pi^2} \times 4x^2 + \frac{1}{2}sin2x + \text{C}$$

$$= \frac{4x^2}{\pi^2} + \frac{sin2x}{2} + \text{C}$$

Now $f(\frac{\pi}{4}) = \frac{3}{4}$

$$\frac{3}{4} = \frac{4(\frac{\pi}{4})^2}{\pi^2} + \frac{\sin(2 \times \frac{\pi}{4})}{2} + \text{C}$$

$$\frac{3}{4} = \frac{4\pi^2}{16\pi^2} + \frac{1}{2} + \text{C}$$

$$\frac{3}{4} = \frac{1}{4} + \frac{1}{2} + \text{C}$$

C = 0

So,

$$f(x) = \frac{4x^2}{\pi^2} + \frac{sin2x}{2}$$

f)

$$\frac{d}{dx}\left[\ln\frac{x^2-4}{x+1}\right] = \frac{\frac{x^2+2x+4}{(x+1)^2}}{\frac{x^2-4}{x+1}}$$

$$= \frac{x^2+2x+4}{(x+1)^2} \times \frac{x+1}{x^2-4}$$

$$= \frac{x^2+2x+4}{(x+1)(x-2)(x+2)}$$