Section I
10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 10.

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1 Convert the angle measurement 76° into radians correct to 3 significant figures.

(A) 7.6

(B) 1.33

(C) 1.326

(D) 0.422

2 Which of the following is a factorisation of the expression $x^2 - 2 + \frac{1}{x^2}$?

(A) $\left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)$

(B) $\left(x + \frac{1}{x}\right)^2 - 2$

(C) $\left(x - \frac{1}{x}\right)^2$

(D) $\left(x + \frac{2}{x}\right)\left(x - \frac{1}{x}\right)$

3 What is the value of $\lim_{x\to 3}\left(\frac{x^2-9}{x-3}\right)$?

(A) 6

(B) 0

(C) 1

(D) undefined

4 A parabola has the equation $(y-1)^2 = 8(x+2)$. Which of the following is the correct statement?

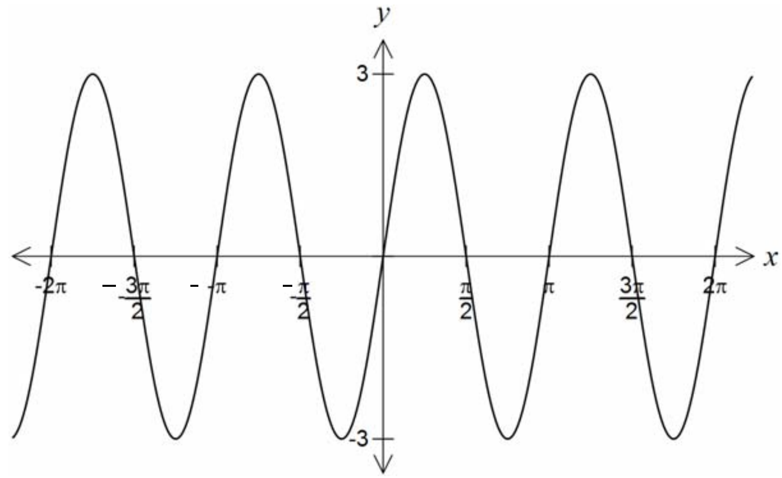
(A) The vertex is $(-2,1)$ and the focus is $(0,1)$

(B) The vertex is $(1,-2)$ and the focus is $(3,-2)$

(C) The vertex is $(1,-2)$ and the focus is $(-1,-2)$

(D) The vertex is $(-2,1)$ and the focus is $(-4,1)$

5 Consider the graph of $f(x)$ shown,



Which of the following functions describes $f(x)$?

(A) $f(x) = 3\cos x$

(B) $f(x) = 3\cos 2x$

(C) $f(x) = 3\sin 2x$

(D) $f(x) = 3\sin x$

6 $\frac{d}{dx}(\sin 2x) =$

(A) $2\sin 2x$

(B) $2\cos 2x$

(C) $2\tan 2x$

(D) $\frac{1}{2}\cos 2x$

7 What is the value of $\sum_{r=1}^{50}(2r-4)$?

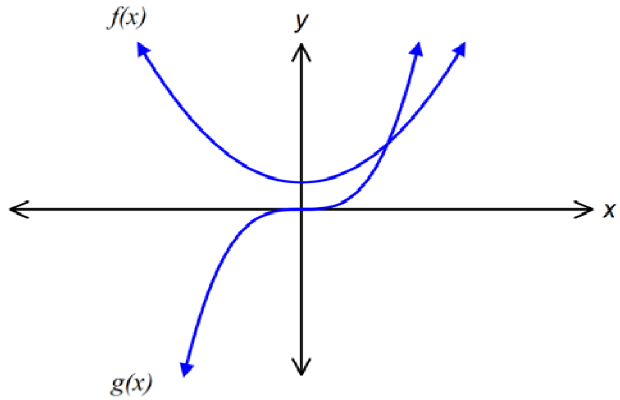
(A) 96

(B) 2350

(C) 2450

(D) 4700

8 Consider the functions $f(x)$ and $g(x)$ shown on the same pair of axes below:



Which statement is true?

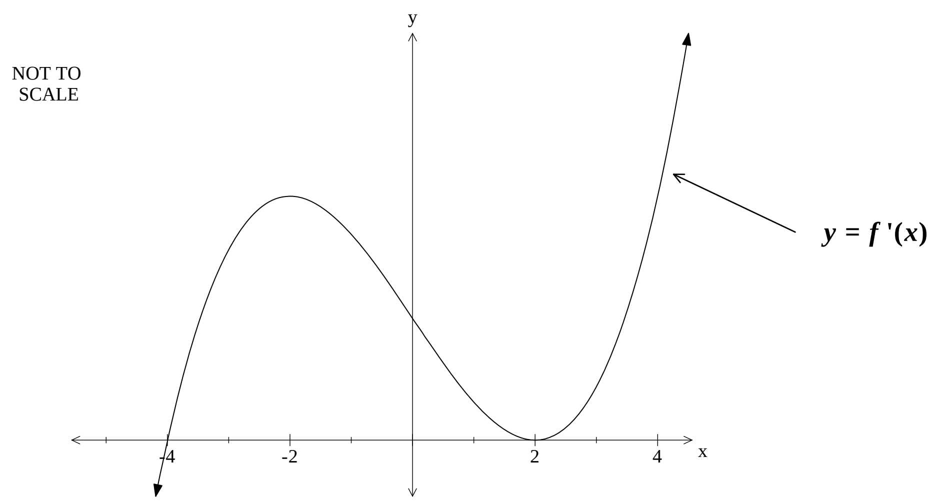
(A) $f(x)$ and $g(x)$ are both even functions.

(B) $f(x)$ and $g(x)$ are both odd functions.

(C) $f(x)$ is an even function and $g(x)$ is an odd function.

(D) $f(x)$ and $g(x)$ are both neither odd nor even functions.

9



The diagram above represents a sketch of the **gradient function** of the curve $y = f(x)$. Which of the following is a true statement ? For the curve $y = f(x)$

(A) a horizontal point of inflexion occurs at $x = -4$

(B) a horizontal point of inflexion occurs at $x = 2$

(C) a maximum turning point occurs at $x = -2$

(D) a maximum turning point occurs at $x = 2$

10 A quadratic equation has roots α and β. If $\alpha+\beta=6$ and $\alpha-\beta=4$, the quadratic equation could be written as

(A) $x^2-6x+13=0$

(B) $-x^2+6x-5=0$

(C) $3x^2-18x+4=0$

(D) $2x^2-12x+5=0$

END OF SECTION 1

Section II
90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section
Answer each question in a separate writing booklet.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) What is $\dfrac{1+\sqrt{5}}{7-2\sqrt{5}}$ as a fraction with a rational denominator? **1**

(b) Differentiate with respect to x and simplify fully:

(i) $\dfrac{\ln x}{x}$ **2**

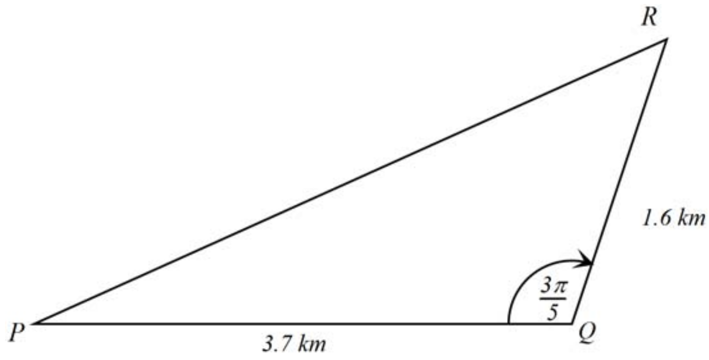
(ii) $5x(x^2-3)$ **2**

(c) Find $\displaystyle\int \frac{x^2+3x}{x}\,dx$ **2**

(d) If $f(x)=2\tan x$, find the exact value of $f'\left(\dfrac{\pi}{4}\right)$ **2**

(e) A pendulum oscillates through a distance of 3 cm on the first swing, 2 cm on the second swing, and on each successive swing through a distance of $\dfrac{2}{3}$ of the previous swing. How far will the pendulum travel before coming to a rest? **2**

(f) For triangle PQR,



(i) Find the length of PR correct to 1 decimal place. **2**

(ii) Using the sine rule, find the size of $\angle R$ in radians, correct to 3 significant figures. **2**

End question 11

Question 12 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) Solve $x^2 - 5x > 0$ **2**

(b) The length of an arc of a circle with radius 5cm is 14cm. Find the angle subtended at the centre by the arc. Give your answer to the nearest minute. **2**

(c) How many terms of the arithmetic sequence 66, 62, 58, ... are positive? **2**

(d) Graph on a number line the values of x for which $|x-2| \le 1$. **2**

(e) Find the equation of the tangent to the curve $y = x^2 - 5x$ at the point $(1, -4)$. **2**

(f) Top soil is tipped from a truck onto a pile. The rate, R kg/s, at which the top soil is flowing is given by the expression $R = t(100 - t^2)$ for $0 \le t \le A$, where t is the time in seconds after the top soil begins to flow.

(i) What is the rate of flow after 5 seconds? **1**

(ii) What is the largest value of A for which the expression for R is physically reasonable? **1**

(iii) Find the maximum rate of flow of the top soil. **3**

End question 12.

Question 13 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) The perpendicular distance from the point (1, 3) to the line $x - y + c = 0$ is $\frac{5}{\sqrt{2}}$. Find the possible value(s) of c. **2**

(b) A group of students decided to monitor the population of squirrels in a park. Initially there were 49 squirrels in the park. The population, P, of squirrels, after t weeks is given by $P = 49e^{kt}$

(i) After 7 weeks the population of squirrels has doubled. **2**

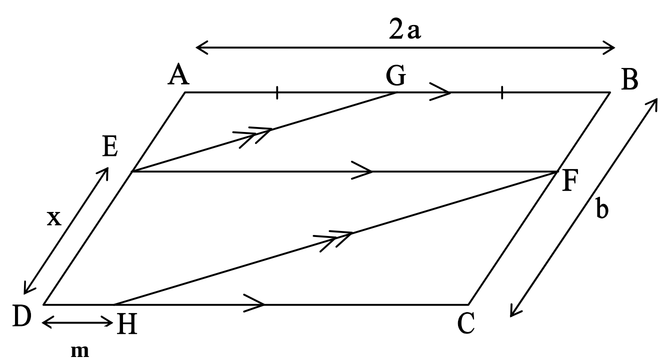
Show that $k = \frac{\ln 2}{7}$.

(ii) During which week did the population of squirrels exceed 180? **2**

(iii) What is the rate of change of the population of squirrels per week, after 14 weeks? **2**

(c) *ABCD* is a parallelogram with sides *AB* = *2a* and *BC* = *b* and *DE* = *x*. *EF* is parallel to *AB* and *DC* .

G is the midpoint of *AB*. The line parallel to EG from F intersects *CD* at *H* where *DH* = *m*.

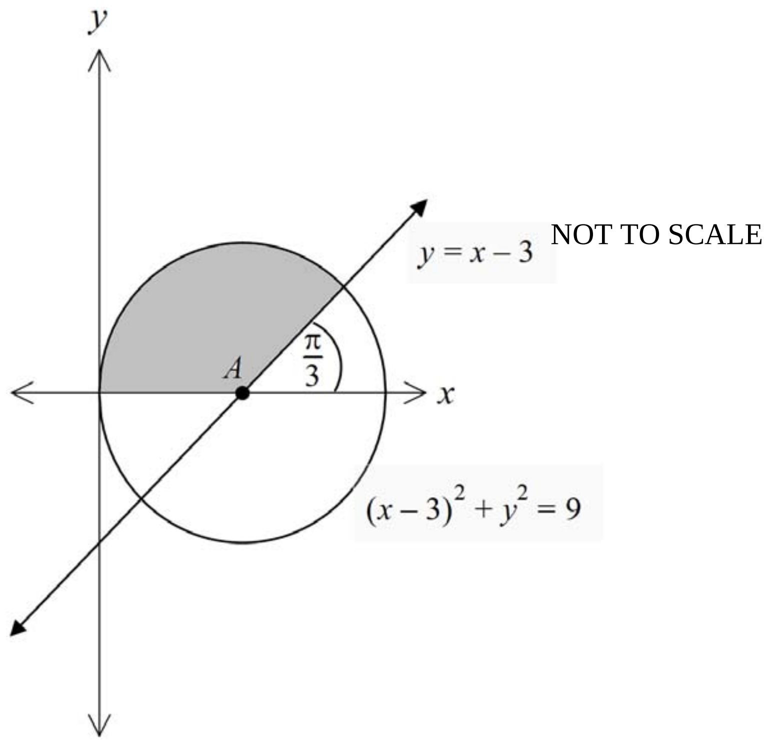


(i) Show that ΔAEG is similar to ΔCFH. **2**

(ii) Find an expression for x in terms of a, b and c. **2**

Question 13 continued next page

(d) The diagram shows the graphs $(x-3)^2+y^2=9$ and $y=x-3$.
The point A is the centre of the circle.



(i) Write down inequalities that describe the sector that is shaded in the diagram. **1**

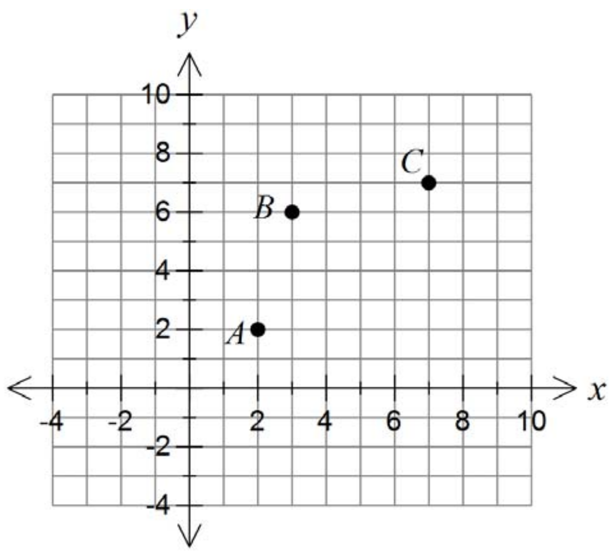
(ii) Find the exact area of the shaded sector. **2**

End question 13

Question 14 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) The points A, B and C have coordinates (2, 2), (3, 6) and (7, 7) respectively.

Point D is a point on the number plane so that $ABCD$ is a rhombus.



(i) Find the coordinates of point D. **1**

(ii) Find the exact length of the diagonal AC. **1**

(iii) Find the point of intersection of the diagonals AC and BD. **1**

(b) (i) Prove that $\dfrac{2\cot A}{2\left(\operatorname{cosec}^2 A\right)-2}=\tan A$ **2**

(ii) Hence or otherwise solve $\dfrac{2\cot A}{2\left(\operatorname{cosec}^2 A\right)-2}=\sqrt{3}$ for $0\le A\le 2\pi$. **2**

(c) The velocity of a particle moving along the x axis is given by, **2**

$$\dot{x}=b+\frac{c}{t+1} \quad \text{where } b \text{ and } c \text{ are constant.}$$

Given that the particle has initial velocity 2ms^{-1} and its initial acceleration was 4ms^{-2}, find the values of b and c.

Question 14 continued next page

Question 14 (continued) **Marks**

(d) A function is given by $f(x) = x^2 - \frac{x^3}{9}$.

(i) Find the coordinates of the stationary points of $y = f(x)$ and determine their nature. **3**

(ii) Hence, sketch the graph of $y = f(x)$ for $-3 \le x \le 12$ showing the stationary points. **2**

(iii) Find the maximum value of $y = -f(x)$ for $-3 \le x \le 12$. **1**

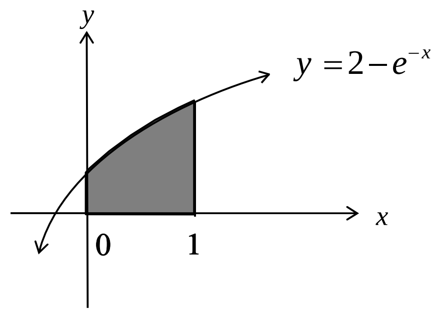
End question 14

Question 15 (15 marks) Use a SEPARATE writing booklet **Marks**

(a) State the range of $y=\sqrt{4-x^2}$. **1**

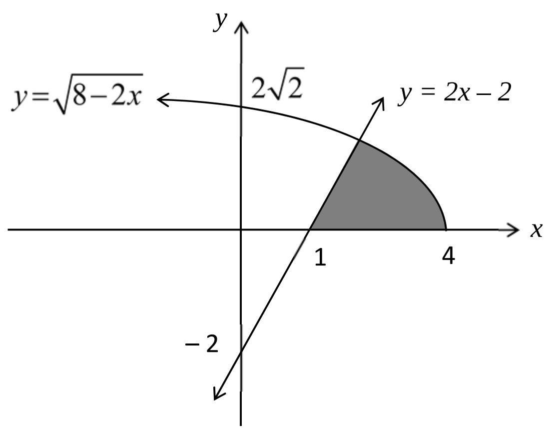
(b) Find the value of q, given that $\int_q^{\frac{2\pi}{3}} \cos\left(\frac{x}{2}\right) dx = \sqrt{3}-\sqrt{2}$. **3**

(c) The region shaded in the diagram is bounded by $y=2-e^{-x}$, the x-axis, the y-axis and the line $x = 1$. **3**



Find the exact volume of the solid formed when the shaded region is rotated about the x-axis.

(d) In the diagram, the line $y = 2x - 2$ intersects the curve $y=\sqrt{8-2x}$ at the point P.
The region bounded by this line, the curve $y=\sqrt{8-2x}$ and the x-axis is shaded as shown on the diagram.

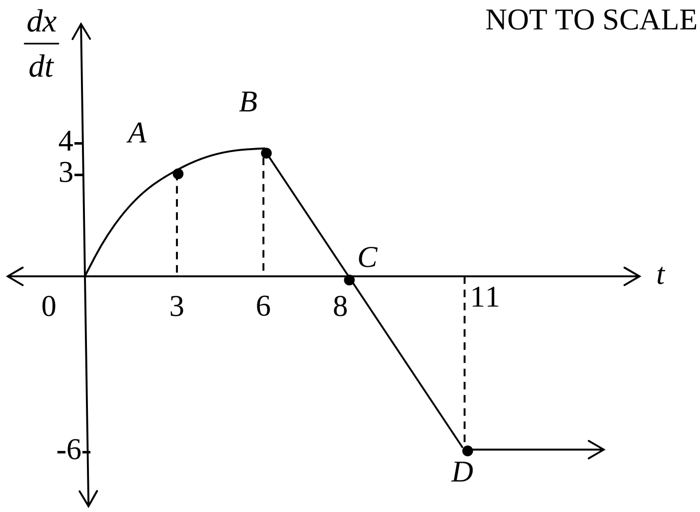


(i) Find the x coordinate of the point P. **1**

(ii) Find the area of the shaded region. **3**

Question 15 continued next page

(e) An object is moving along the x – axis. The graph shows the velocity, $\frac{dx}{dt}$ of the object, as a function of time, t. The coordinates of the points shown on the graph are A (3, 3), B (6, 4), C (8, 0) and D (11, -6). The object is initially at the origin and the velocity is constant at -6m/s for $t \geq 11$.



(i) Use the Trapezoidal rule with 3 function values to calculate the distance travelled by the object between t = 0 and t = 6. **2**

(ii) Find the displacement of the object when t = 11. **1**

(iii) During which time(s) is the object slowing down? **1**

End Question 15

Question 16 (15 Marks) Use a SEPARATE writing booklet **Marks**

(a) Given that $f''(x) = 2$, find the equation of a curve $y = f(x)$ which has a stationary point at the point (1,3). **3**

(b) Jack opened an investment account with an initial deposit of $5000 on 1st January 2007. He did this so that he could provide his daughter with $800 at the start of January each year for university text books. The account earned interest at a rate of 3% per annum, compounding annually and paid at the end of each year.

The first $800 withdrawal was made one year after the investment was set up.

(i) Calculate the account balance immediately after the first withdrawal has been made. **1**

(ii) Let A_n be the amount of money in the account after n years (when n withdrawals will have been made). **2**

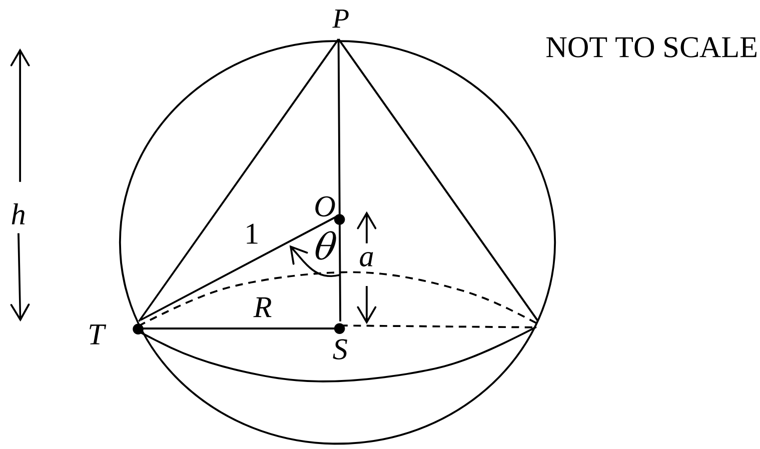
Show that $A_n = \frac{1}{3}\left[80000 - 65000(1.03)^n\right]$.

(iii) After withdrawing $800 on 2nd January 2010, Jack's daughter advised him that she would need $900 per year from 2011 onwards. **3**

How many more years of textbook fees can come out of the fund?

Question 16 continued next page

(c) The diagram below shows a cone of radius, R, and height, h, inscribed in a sphere with a radius of 1cm and centre at O. The distance, a, is from the centre of the sphere to the base of the cone. OT is inclined at angle θ from SO, where $0 < \theta < \frac{\pi}{2}$ and $SP = h$.



(i) Show that the volume, V, of the cone is given by **2**

$$V = \frac{1}{3}\pi \sin^2\theta(1 + \cos\theta).$$

(ii) Find the angle θ, in radians that gives the maximum volume of the cone. **4**

End of paper

2013 CTHS Mathematics Section I - Answer Sheet

Student Name ______________________________

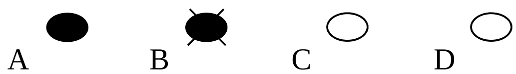
Class________________

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

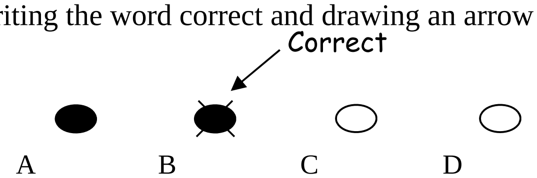
Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

- If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.



- If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.



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1. A ○ B ○ C ○ D ○
2. A ○ B ○ C ○ D ○
3. A ○ B ○ C ○ D ○
4. A ○ B ○ C ○ D ○
5. A ○ B ○ C ○ D ○
6. A ○ B ○ C ○ D ○
7. A ○ B ○ C ○ D ○
8. A ○ B ○ C ○ D ○
9. A ○ B ○ C ○ D ○
10. A ○ B ○ C ○ D ○

TRIAL 2013 SOLUTIONS

M-Choice Summary

1. B 2. C 3. A 4. A 5. C 6. B

7. B 8. C 9. B 10. B

WORKED SOLUTIONS

#1 $7.6 \times \frac{\pi}{180} = 1.32645\ldots$

$= 1.33$ 2dp

2. $x^2 - 2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2$

3. $\lim_{x \to 3} \left(\frac{x^2 - 9}{x - 3}\right) = \lim_{x \to 3} \frac{(x+3)(x-3)}{(x-3)}$

$= 3 + 3$

$= 6$

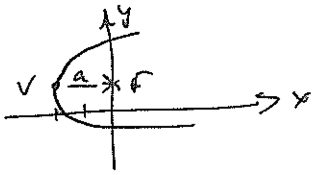
4. $(y-1)^2 = 8(x+2)$

Vertex $(-2, 1)$

$y^2 = 4a(x)$

$\therefore 4a = 8$

$a = 2$



$\therefore V(-2, 1)$ focus $(0, 1)$

5. amplitude $= 3$

period $= \pi = \frac{2\pi}{n}$ $\therefore n = 2$

$f(x) = 3\sin 2x$

6. $\frac{d}{dx} \sin 2x = 2\cos 2x$

7. $\sum_{r=1}^{50} (2r - 4) = -2 + 0 + 2 + \cdots + 96$

AP: $a = -2$ $d = 2$

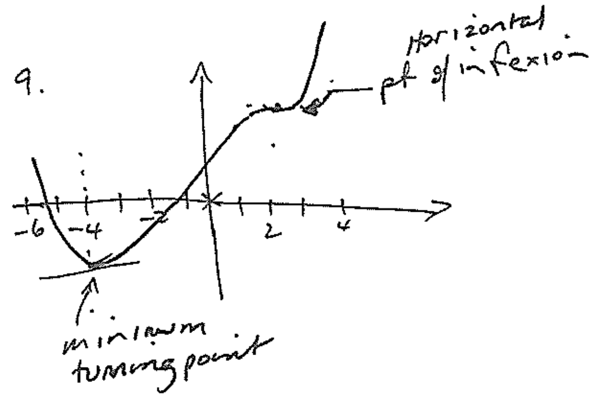
$l = 96$

$S_{50} = \frac{50}{2}\{-2 + 96\}$

$= 2350$

8. note $\left\{ \begin{array}{ll} f(x) = -f(-x) & \text{odd} \\ f(x) = f(-x) & \text{even} \end{array} \right.$

9.



10. $\alpha + \beta = 6$ $\alpha - \beta = 4$

$(\alpha + \beta)^2 = 36$ ① $(\alpha - \beta)^2 = 16$ ②

② − ①

$4\alpha\beta = 20$

$\alpha\beta = 5.$

So for Ⓐ $\alpha + \beta = \frac{-6}{1}$ $\alpha\beta = 13$ ✗

Ⓑ $\alpha + \beta = \frac{-6}{-1}$ $\alpha\beta = \frac{-5}{-1}$ ✓

Ⓒ $\alpha + \beta = \frac{18}{3}$ $\alpha\beta = \frac{4}{3}$ ✗

Ⓓ $\alpha + \beta = \frac{12}{2}$ $\alpha\beta = \frac{5}{2}$ ✗

Question 11

Ap4 2013

a) $\frac{1+\sqrt{5}}{7-2\sqrt{5}} \times \frac{7+2\sqrt{5}}{7+2\sqrt{5}}$

$= \frac{7+2\sqrt{5}+7\sqrt{5}+10}{49-20}$

$= \frac{17+9\sqrt{5}}{29}$

b) i) $u = \ln x \quad u' = \frac{1}{x}$

$v = x \quad v' = 1$

$\frac{d}{dx}\left(\frac{\ln x}{x}\right) = \frac{x \cdot \frac{1}{x} - \ln x \cdot 1}{x^2}$

$= \frac{1-\ln x}{x^2}$

b) ii) $y = 5x(x^2-3)$

$= 5x^3 - 15x$

$\therefore y' = 15x^2 - 15$

OR

$y' = 5(x^2-3) + 5x \times 2x$

$= 5x^2 - 15 + 10x^2$

$= 15x^2 - 15.$

c) $\int \frac{x^2+3x}{x}\,dx$

$= \int x+3 \; dx$

$= \frac{x^2}{2} + 3x + c.$

d). $f(x) = 2\tan x$

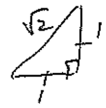
$f'(x) = 2\sec^2 x$

$f'\left(\frac{\pi}{4}\right) = 2\sec^2\frac{\pi}{4}$

$= 2(\sqrt{2})^2$

$= 4.$

[note $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$



]

e)

$3, 2, \frac{4}{3}, \ldots$

$a = 3 \quad r = \frac{2}{3}$

note $|r| < 1$

$\therefore$ distance $= S_\infty$

$S_\infty = \frac{a}{1-r}$

$= \frac{3}{1-\frac{2}{3}}$

$= 9$ cm.

11 f i) $PR^2 = 1.6^2 + 3.7^2 - 2 \times 1.6 \times 3.7 \times \cos\frac{3\pi}{5}$

$= 19.9087621$

$\therefore PR = 4.461923\ldots$

$= 4.5$ km (1dp)

ii) $\frac{\sin\frac{3\pi}{5}}{4.46\ldots} = \frac{\sin R}{3.7}$

$\sin R = 0.7889930741$

$R = 0.909168\ldots$

$\therefore R = 0.909$ Radians.

AP4 2013

Question 12

a) $x^2 - 5x > 0$

$x(x-5) > 0$

$\therefore\ x < 0$ or $x > 5$

b) 

$l = r\theta$

$14 = 5\theta$

$\theta = \frac{14}{5} \times \frac{180}{\pi}$

$= 160.42818$

$= 160^\circ 26'$

c) $66, 62, 58 \ldots$

$a = 66 \quad d = -4 \quad T_n > 0$

$T_n = a + (n-1)d$

$= 66 - 4n + 4$

$= 70 - 4n$

for $T_n > 0$ $\quad 70 - 4n > 0$

$70 > 4n$

$17.5 > n$

$\therefore$ there are 17 positive terms

d) $|x-2| \leq 1$

ie $-1 \leq x - 2 \leq 1$

$1 \leq x \leq 3$

e) $y = x^2 - 5x$

$y' = 2x - 5$

at $x = 1$ $\quad y' = 2 \times 1 - 5$

$= -3 \quad (= m)$

equation of tangent

$y + 4 = -3(x - 1)$

$y + 4 = -3x + 3$

$\therefore\ 3x + y + 1 = 0$ or $y = -3x - 1$

f) i) $R = t(100 - t^2)$

when $t = 5$

$R = 5(100 - 25)$

$= 5 \times 75$

$= 375$ kg/sec

ii) initially no flow; when empty no flow.

so largest value of A when $R = 0$

$\therefore\ 0 = t(100 - t^2)$

ie $t = 0$ or $t = 10$

hence largest value of t is 10 secs $\quad \therefore\ A = 10$

Qu 12

f iii) max Rate of flow when $\frac{dR}{dt} = 0$ and

$\frac{d^2R}{dt^2} < 0.$

$\therefore \frac{dR}{dt} = 100 - 3t^2$

for $\frac{dR}{dt} = 0 \quad 0 = 100 - 3t^2$

$t = \pm \frac{10}{\sqrt{3}}$

as $t \geq 0 \quad t = \frac{10}{\sqrt{3}}$ sec.

$\frac{d^2R}{dt^2} = -6t$

at $t = \frac{10}{\sqrt{3}} \quad \frac{d^2R}{dt^2} = \frac{-6\times10}{\sqrt{3}}$

< 0

$\therefore$ max.

the max Rate is when $t = \frac{10}{\sqrt{3}}$

$\therefore R = 100\times\frac{10}{\sqrt{3}} - \left(\frac{10}{\sqrt{3}}\right)^3$

$= \frac{1000}{\sqrt{3}} - \frac{1000}{3\sqrt{3}}$

$= \frac{2000}{3\sqrt{3}}$

max Rate of flow.

$= \frac{2000\sqrt{3}}{9}$ kg/sec.

Question 13

a) $x - y + c = 0 \qquad \overset{x\ y}{(1,3)}$

$a=1 \quad b=-1 \qquad$ dist $= \frac{5}{\sqrt{2}}$

$\therefore \left|\frac{1\times1 - 1\times3 + c}{\sqrt{1^2+1^2}}\right| = \frac{5}{\sqrt{2}}$

ie $\frac{5}{\sqrt{2}} = \left|\frac{-2+c}{\sqrt{2}}\right|$

hence $-2+c = 5$

$c = 7$

OR $-2+c = -5$

$c = -3$

ie $c = 7$ OR $c = -3$

b) i) $P = 49e^{kt} \qquad t=0 \ P_0=49$

$t=7 \ P=98$

$\therefore 98 = 49e^{7k}$

$2 = e^{7k}$

$\ln 2 = 7k$

$\therefore k = \frac{\ln 2}{7}$

ii) $P > 180 \quad \therefore 180 < 49\,e^{\frac{\ln 2}{7}\cdot t}$

$\ln\left(\frac{180}{49}\right) < t\cdot\frac{\ln 2}{7}$

$t = \frac{7}{\ln 2}\times\ln\left(\frac{180}{49}\right)$

$= 13\cdot14$

$\therefore$ 180 squirrels in 14th week

APr 2013

Qu 13

b iii) Rate of change $= \frac{dP}{dt}$

$$\frac{dP}{dt} = 49ke^{kt}$$

$$= 49 \frac{\ln 2}{7} e^{14 \times \frac{\ln 2}{7}}$$

$$= 19.40812106$$

$= 19.4$ squirrels per week.

c) i) In ΔAEG and ΔCFH

$\angle AEG = \angle FCH$ (opposite $\angle$'s in a parallelogram)

$\angle AGE = \angle GEF$ (alternate $\angle$'s $AG \parallel EF$)

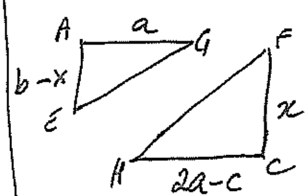
$\angle GEF = \angle EFH$ (alternate $\angle$'s $EG \parallel HF$)

$\angle EFH = \angle FHC$ (alternate $\angle$'s $EF \parallel DC$)

$\therefore \angle AGE = \angle FHC$

Hence $\Delta AEG \sim \Delta CFH$ (equiangular)

c ii) as $\Delta AEG \sim \Delta CFH$



$$\frac{a}{2a-c} = \frac{b-x}{x}.$$

(corresponding sides of similar Δs are in proportion)

$$ax = (b-x)(2a-c)$$

$$ax = 2ab - 2ax - bc + cx$$

$$\therefore ax + 2ax - cx = 2ab - bc$$

$$x(3a-c) = b(2a-c)$$

$$x = \frac{b(2a-c)}{(3a-c)}$$

d) i)

$$\left.\begin{array}{l} y \geq 0 \\ (x-3)^2 + y^2 \leq 9 \\ y \geq x - 3 \end{array}\right\}$$

ii) $\angle A = \pi - \frac{\pi}{3}$

$$= \frac{2\pi}{3}$$

Area $= \frac{1}{2} r^2 \theta$

$$= \frac{1}{2} \times 3^2 \times \frac{2\pi}{3}$$

$= 3\pi$ units^2.

Question 14

a) i) by inspection from A move 4 across and 1 up

D $(6,3)$

ii) $AC = \sqrt{(7-2)^2 + (7-2)^2}$

$= \sqrt{50}$

$= 5\sqrt{2}$ units

iii) Diagonals of Rhombus bisect at right angles.

$\therefore M = \left(\frac{7+2}{2}, \frac{7+2}{2}\right)$

$= (4\cdot5, 4\cdot5)$

b) i) $LHS = \frac{2\cot A}{2(\operatorname{cosec}^2 A) - 2}$

$= \frac{2\cot A}{2(\operatorname{cosec}^2 A - 1)}$

$= \frac{2\cot A}{2\cot^2 A}$

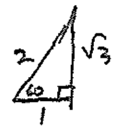
$(\because \cot^2 A + 1 = \operatorname{cosec}^2 A)$

$= \frac{1}{\cot A}$

$= \tan A$

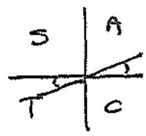
$= RHS.$

ii) $\frac{2\cot A}{2(\operatorname{cosec}^2 A) - 2} = \sqrt{3}$



$\therefore \tan A = \sqrt{3}$

$A = \frac{\pi}{3}$ OR $\frac{\pi}{3} + \pi$



$\therefore A = \frac{\pi}{3}, \frac{4\pi}{3}$

c. $\dot{x} = b + \frac{c}{t+1}$ ... ①

$\ddot{x} = -c(t+1)^{-2}$ ... ②

when $v=2$ $t=0$

from ①

$2 = b + c$ ... ③

from ② $a = 4$ $t = 0$

$4 = -c \times 1^{-2}$

$\therefore c = -4$ ... ④

sub into ③

$2 = b - 4$

$b = 6$

soln $b = 6$ $c = -4$

Q14 AT4 2013

a) $f(x) = x^2 - \frac{x^3}{9}$

i) $f'(x) = 0$ stationary pts

$\therefore 2x - \frac{x^2}{3} = 0$

$x\left(2 - \frac{x}{3}\right) = 0$

$x = 0$ or $2 - \frac{x}{3} = 0$

$2 = \frac{x}{3}$

$x = 6$

$f''(x) = 2 - \frac{2x}{3}$

$f''(0) = 2 > 0$ $\therefore$ min

$f''(6) = 2 - \frac{12}{3}$

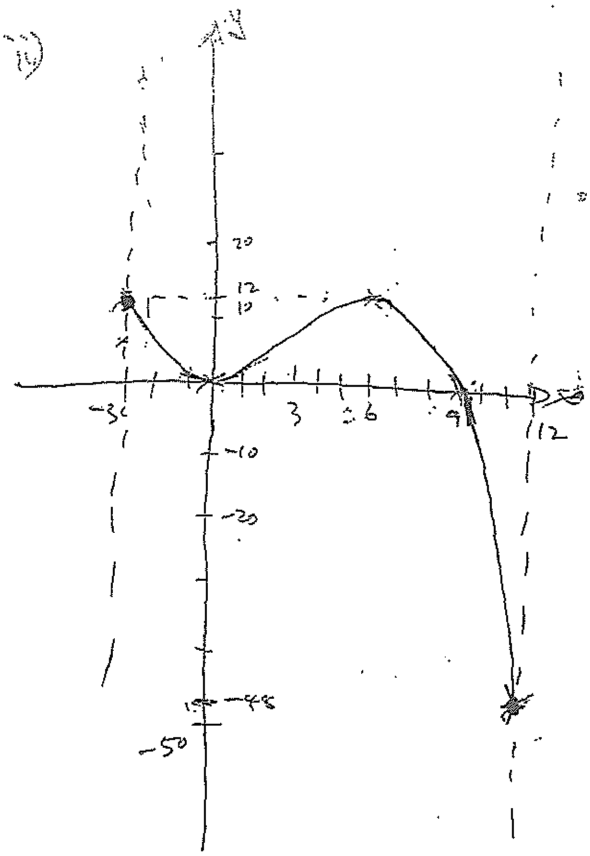
$= -2 < 0$ max.

$\therefore$ Stationary pts

(0,0) is a min (1)

$y = 6^2 - \frac{6^3}{9} = 12$ (6, 12) is a max

ii)

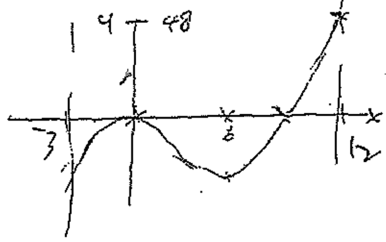


(12, −48) f(12)

(−3, 12) f(−3)

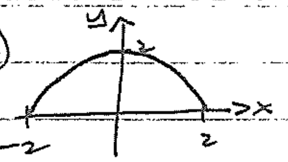
f(9) = 0

iii) $y = -f(x)$ has max 48 (1)



(function flips)

Question 15 AP4 2013

a) 

$0 \le y \le 2.$

b) $\int_q^{\frac{2\pi}{3}} \cos\left(\frac{x}{2}\right) dx$

$= 2\left[\sin \frac{x}{2}\right]_q^{2\pi/3}$

$= 2\left[\sin \frac{\pi}{3} - \sin \frac{q}{3}\right]$

hence

$\sqrt{3} - \sqrt{2} = 2\left[\frac{\sqrt{3}}{2} - \sin\left(\frac{q}{2}\right)\right)$

$\sqrt{3} - \sqrt{2} = \sqrt{3} - 2\sin\frac{q}{2}$

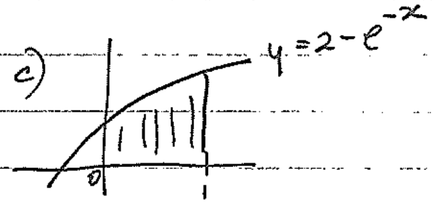
$2\sin\frac{q}{2} = \sqrt{2}$

$\sin\frac{q}{2} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\frac{q}{2} = \frac{\pi}{4}$

$q = \frac{\pi}{2}$

(or 1·57)

c) 

$V = \pi\int_0^1 y^2\, dx$

$= \pi\int_0^1 4 - 4e^{-x} + e^{-2x}\, dx$

$= \pi\left[4x + 4e^{-x} - \frac{e^{-2x}}{2}\right]_0^1$

$= \pi\left(\left\{4 + \frac{4}{e} - \frac{1}{2e^2}\right\} - \left\{0 + 4 - \frac{1}{2}\right\}\right)$

$= \pi\left\{\frac{4}{e} - \frac{1}{2e^2} + \frac{1}{2}\right\}$

or $\pi\left[\frac{e^2 + 8e - 1}{2e^2}\right].$

d i) $\sqrt{8-2x} = 2x - 2$

$8 - 2x = 4x^2 - 8x + 4$

$0 = 4x^2 - 6x - 4$

$0 = 2x^2 - 3x - 2$

$0 = (2x+1)(x-2)$

$x = -\frac{1}{2},\ x = 2$

ii) Area $= \text{area}\,\Delta + \int_2^4 \sqrt{8-2x}\, dx$

$= \frac{1}{2}\cdot(1)(2) + \int_2^4 (8-2x)^{1/2}\, dx$

$= 1 + \left[\frac{2}{3}\frac{(8-2x)^{3/2}}{-2}\right]_2^4$

$= 1 - \frac{1}{3}\left[0 - 8\right]$

$= 1 + \frac{8}{3}$

$= 3\frac{2}{3}$ units^2.

Qu 15

d ii) OR

$$\int_1^2 2x-2\,dx + \int_2^4 \sqrt{8-4x}\,dx$$

$$= \left[x^2-2x\right]_1^2 + \left[-\frac{2}{3}(8-2x)^{3/2}\right]_2^4$$

$$= 3\tfrac{2}{3}\,u^2.$$

15e) (i)

x_n	0	3	6
y_n	0	3	4

$h = 3$

Distance $= \frac{h}{2}\left[y_0 + y_n + 2(y_1 + y_2 \cdots)\right]$

$= \frac{3}{2}\left[0 + 4 + 2(3)\right] = \frac{3}{2} \times 10$

$= 15$ metres.

Area Δ

$\frac{1}{2}\times 3 \times 6 = 9$

Q15e ii) Change in Displacement $= \int_0^{11} V\,dt$

$= 15 + 4 - 9$

$= 19 - 9$

$= 10$ (considering areas above & below x-axis)

Since object is initially at the origin the particle is at $x = 10$m when $t = 11$

iii) Object is slowing when

$V > 0$ and $a < 0$

$\therefore\ 6 < t < 8$ (between B and C)

Question 16

a) $f''(x) = 2$

$f'(x) = 2x + c$

at $(1,3)$ $0 = 2\times 1 + c$

$c = -2$

$\therefore f'(x) = 2x - 2$

$f(x) = x^2 - 2x + k$

at $(1,3)$ $3 = 1 - 2 + k$

$4 = k$

$f(x) = x^2 - 2x + 4$

b) i) $A_1 = 5000(1\cdot 03) - 800$

$= \$4350$

ii) $A_2 = A_1(1\cdot 03) - 800$

$= 5000(1\cdot 03)^2 - 800(1\cdot 03) - 800$

$A_3 = 5000(1\cdot 03)^3 - 800(1\cdot 03)^2 - 800\times 1\cdot 03 - 800$

$= 5000(1\cdot 03)^3 - 800(1 + 1\cdot 03 + 1\cdot 03^2)$

$\therefore A_n = 5000(1.03)^n - 800(\underbrace{1 + 1.03 + 1.03^2 + \dots + 1.03^{n-1}}_{G.P.})$

$a = 1$
$r = 1.03$
there are n terms

using $S_n = \dfrac{a(r^n - 1)}{r - 1}$

$= 5000(1.03)^n - 800\left[\dfrac{1\times(1.03)^n - 1}{1\cdot 03 - 1}\right]$

$= 5000(1.03)^n - \dfrac{800}{0.03}\left[(1.03)^n - 1\right]$

$= 5000(1.03)^n - \dfrac{80000}{3}(1.03)^n + \dfrac{80000}{3}$

$= \frac{1}{3}\left[80000 + 3\times 5000 - 80000(1\cdot 03)^n\right]$

$= \frac{1}{3}\left[80000 - 65000(1.03)^n\right]$

Question 16

6 iii) $A_3 = \frac{1}{3}\left[80000 - 65000(1.03)^3\right]$

$= \$2990.915$

$A_n = 2990.915(1.03)^n - 900\left(\frac{1.03^n - 1}{0.03}\right)$

$30000(1.03^n - 1) = 2990.915(1.03)^n$

$30000(1.03^n) - 2990.915(1.03)^n = 30000$

$27009.085(1.03)^n = 30000$

$1.03^n = 1.110737$

$n = \frac{\ln 1.110737}{\ln 1.03}$

$= 3.55$

∴ only 3 years left.

c i) $R = \sin\theta$

$(h-1)^2 + \sin^2\theta = 1$

$(h-1)^2 = 1 - \sin^2\theta$

$(h-1)^2 = \cos^2\theta$

$h = \cos\theta + 1$

OR $\cos\theta = \frac{a}{1}$

$h = a + 1$

$h = \cos\theta + 1$.

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \sin^2\theta(\cos\theta + 1)$

c ii) $\frac{dv}{d\theta} = \frac{1}{3}\pi\left[(\cos\theta+1)(2\sin\theta\cos\theta)+\sin^2\theta(-\sin\theta)\right]$

$= \frac{\pi}{3}\left[2\sin\theta\cos^2\theta + 2\sin\theta\cos\theta - \sin^3\theta\right]$

at max/min $\theta = 2\sin\theta\cos^2\theta + 2\sin\theta\cos\theta - \sin^3\theta$

$= \sin\theta\left[2\cos^2\theta + 2\cos\theta - (1-\cos^2\theta)\right]$

$= \sin\theta\left[3\cos^2\theta + 2\cos\theta - 1\right]$

$\therefore \sin\theta = 0 \qquad (3\cos\theta - 1)(\cos\theta + 1) = 0$

$\cos\theta = \frac{1}{3} \qquad \cos\theta = -1$

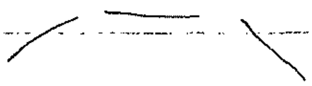
$\theta = 0 \qquad \theta = \pi$

$\theta = 1{\cdot}23$

Test $\theta = 1{\cdot}2$

$\theta = 1{\cdot}3$

θ	1·2	1·23	1·3
V'	0·4088	0	-0·2526
	>0	$=0$	<0



$\therefore V$ is max when $\theta = 1{\cdot}23$