Student Number:	
Class:	

JAMES RUSE AHS
GESTA NON VERBA

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION 2014

MATHEMATICS

General Instructions:

- Reading Time: 5 minutes.
- Working Time: 3 hours.
- Write in black pen.
- Board approved calculators & templates may be used
- A Standard Integral Sheet is provided.
- In Question 11 - 16, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged working.

Total Marks 100

Section I: 10 marks

- Attempt Question 1 – 10.
- Answer on the Multiple Choice answer sheet provided.
- Allow about 15 minutes for this section.

Section II: 90 Marks

- Attempt Question 11 - 16
- Answer on lined paper provided. Start a new page for each new question.
- Allow about 2 hours & 45 minutes for this section.

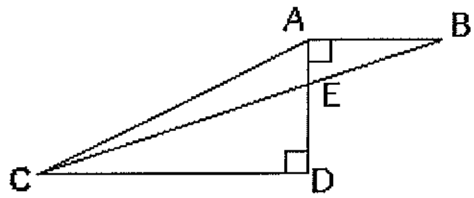
The answers to all questions are to be returned in separate *stapled* bundles clearly labelled Question 11, Question 12, etc. Each question must show your Candidate Number.

<u>Section I</u>   10 marks

Attempt Questions 1–10

Use the multiple-choice answer sheet for Questions 1–10.

Q1.



In the figure above AD = 4, AB = 3 and CD = 9. What is the area of triangle AEC?

A] 18   B] 13·5   C] 9   D] 4·5

Q2. n is an integer chosen at random from the set {5, 7, 9, 11}

p is chosen at random from the set {2, 6, 10, 14, 18}

What is the probability that $n + p = 23$?

A] 0·1   B] 0·2   C] 2·5   D] 0·3

Q3. A yacht is sailed from A to B on a bearing of 196^0T.
To sail from B directly back to A the bearing would be:

A] 074^0T B] 196^0T C] 164^0T D] 016^0T

Q4. After it is dropped a certain ball always bounces back to $\frac{2}{5}$ of the height of its previous bounce. After the first bounce it reaches a height of 125 cms. How high (in cms) will it reach after its fourth bounce?

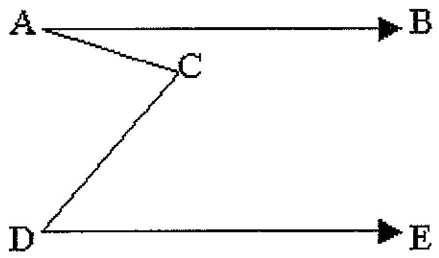
A] 20   B] 15   C] 8   D] 5

Q5. If $3x + y = 19$, and $x + 3y = 1$.

Find the value of $2x + 2y$

A] 20 B] 10 C] 11 D] 18

Q6.



(figure not to scale)

AB and DE are parallel. Angle BAC = 30°, angle CDE = 50°. What is the size of angle ACD?

A] 100° B] 90° C] 80° D] 70°

Q7. If $y = \ln(\ln x)$ then $\frac{dy}{dx}$ is :

A] $\frac{1}{x}$ B] $\frac{1}{lnx}$ C] $\frac{x}{lnx}$ D] $\frac{1}{xlnx}$

Q8. Evaluate $\int_0^5 x^{\frac{3}{2}}\,dx$:

A] $2\sqrt{5}$ B] $\sqrt{5}$ C] $5\sqrt{5}$ D] $10\sqrt{5}$

Q9. Find the value of $\lim\limits_{x \to 2} \frac{x^3-8}{x-2}$

A] 12 B] 4 C] 0 D] - 4

Q10. If $f(x) = \frac{1}{cot2x}$ then $f'(x)$ is:

A] $\cot 2x$ B] $\tan 2x$ C] $sec^2 2x$ D] $2sec^2 2x$

Section II

Q11. START A NEW PAGE **15 marks**

(a) Show that A(a,2b), B(2a,b) and C(-a, 4b) are collinear. **2**

(b) If $f(x) = \begin{cases} 2x+4 & for\ x>0 \\ -x-4 & for\ x\le 0 \end{cases}$ evaluate $f(2) + f(0) + f(-2)$. **2**

(c) The amount, A grams, of a radioactive element remaining after t years is given by the rule $A = 450(3^{-t})$.

(i) What is the initial amount of radioactive material? **1**

(ii) Determine when the amount will reach 50g. **1**

(d) The points A (0,0), B(4,3), C(0,6) and D(x,y) are the vertices of a rhombus ABCD respectively.

(i) Find the coordinates of D. **2**

(ii) Can the rhombus ABCD form a square? Give reasons. **2**

(e) (i) Find the locus of a point P which moves such that its distance from the point A(7, 2) is twice its distance from the point B(-2, -1). **3**

(ii) Describe the locus geometrically. **2**

Q12. START A NEW PAGE **15 marks**

(a) Solve: $\frac{5^{3n+3}}{25^{n-3}} = 125$ **3**

(b) Find in terms of r and θ the length of a chord in a circle of radius r, if the chord subtends an angle of θ degrees at the centre of the circle. **2**

(c) Find the equation of the curve which passes through the point (4, 1) and has a gradient function of $2x + 5$. **2**

(d) (i) Show that the equation of the tangent to the curve y = -2sin$x$ at the point $x = \frac{\pi}{6}$ is $y = -\sqrt{3}\,x + \frac{\pi\sqrt{3}}{6} - 1$ **3**

(ii) Neatly graph the curve y= -2sin$x$ and the tangent from part (i) for $0 \le x \le \pi$ **2**

(iii) Find the exact area between the tangent and the curve from $x = \frac{\pi}{6}$ to $x = \frac{\pi}{2}$ **3**

Q13. START A NEW PAGE **15 marks**

(a) Linda measures the angle of elevation to the top of a mountain to be 25^0. She walks 800m horizontally towards the mountain and finds that the angle of elevation has doubled.

(i) Draw a neat diagram and show all of the given data. **1**

(ii) What is the height of the mountain to the nearest metre? Giving reasons. **3**

(b) Prove that $(1 - \cos\theta)(1 + \sec\theta) = \sin\theta\tan\theta$ **2**

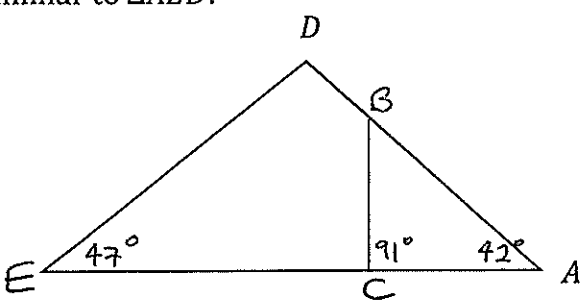
(c) Water is flowing into a container at the rate of $\frac{e^t}{e^t + 1}$ litres per second. Initially the container is empty.

(i) Find the volume of water in the container at time t. **2**

(ii) How much water is in the container after ln(31) seconds? (Write your answer in exact form). **2**

(d) (i) Prove that ΔABC is similiar to ΔAED. **3**

(Not drawn to scale)



(ii) If AC = 2 units and DA = 6 units, prove that AE = 3AB. **2**

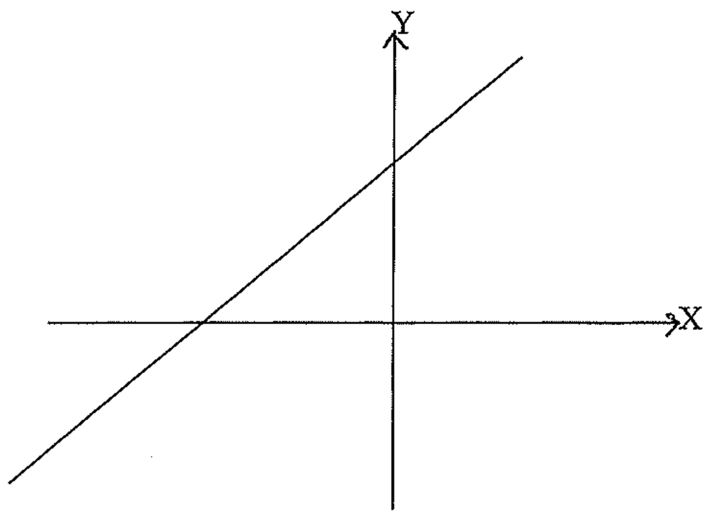
Q14. START A NEW PAGE **15 marks**

(a) Prove that $x^2 - 2x + 2 > 0$ for all values of x. **2**

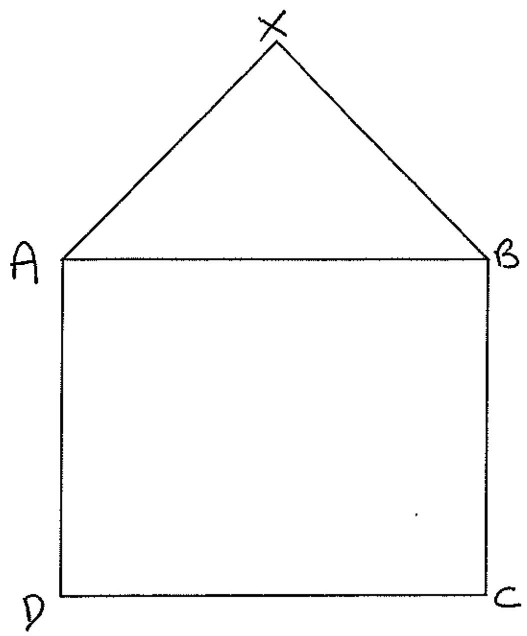
(b) Evaluate $\sum_{n=4}^{25}(2n - 6)$ **3**

(c) Find $f'(2)$ if $f(x) = \frac{2x-1}{x+1}$. **3**

(d) This diagram shows the graph of $y = f''(x)$. Use a ruler to make a neat copy of this graph and on the same diagram neatly sketch $y = f'(x)$ and $y = f(x)$, given that $f'(0) = a$ where $a > 0$, clearly labeling each graph. **4**



(e) ABCD is a square and ABX is an equilateral triangle standing on the side AB, as shown below. Copy this diagram neatly onto your answer booklet and prove that $\angle AXC = 3\angle BXC$. **3**



Q15. START A NEW PAGE **15 marks**

(a) Two dice are thrown.

(i) Draw a neat diagram to clearly illustrate all possible outcomes. **1**

What is the probability of throwing:

(ii) a double? **1**

(iii) a total less than eight? **1**

(b) Using the Trapezoidal Rule with 3 Functional values find an approximation to $\int_0^2 \sin x\,dx$ correct to three significant figures. **3**

(c) Find the area of the minor segment in a circle with radius 6.2cm and subtending an angle of 135 degrees at the centre. (Give your answer correct to 1 dec. pl) **2**

(d) A rectangular crate is required to have a volume of 24 cubic metres and its length is to be double its width. The crate is to be strengthened by a strip of angle iron running along all of its edges. Find the dimensions of the crate if the total length of angle iron is to be a minimum. **5**

(e) Neatly graph $y = -\sqrt{16 - x^2}$ **2**

Q16. START A NEW PAGE **15 marks**

(a) The velocity in *m/s* of a particle travelling in a straight line is given by $v = 4t^3 - 6t$ where t is the time taken in seconds. If the particle is initially 4m to the left of the origin find in terms of t:

(i) an expression for the displacement x m. **2**

(ii) an expression for the acceleration a m/s^2. **1**

(iii) When is the particle at the origin? **2**

PTO for Q15(b)

(b) Mitchell invests \$50 on the 1st of every month starting from January 2003 into a Superannuation Fund. Interest is paid at the rate of 5% p.a. compounded monthly.

(i) Show that his investment is worth \$ 85 923.96 by the end of 2044? **3**

(ii) Mitchell's employer pays 9% of his monthly income into the same fund for the same amount of time. If Mitchell earns \$43 000 p.a. how much is his fund worth in total at the end of 2044? **2**

(iii) Assuming that Mitchell and his boss pay as stated in parts (i) and (ii) calculate the value of Mitchell's fund at the end of 2044 if the interest rate increases to 6% at the beginning of 2020. **3**

(iv) Using the information from part (ii) and using the original interest rate only, find the amount that Mitchell must invest each month if he wishes to have a superannuation fund value of \$1 000 000 in total. **2**

END OF PAPER

(1)

MATHEMATICS: Question 11

Suggested Solutions	Marks Awarded	Marker's Comments
(a) $A(a,2b)$ $B(2a,b)$ and $C(-a,4b)$ $M_{AB} = \frac{2b-b}{a-2a} = \frac{b}{-a}$ $M_{BC} = \frac{4b-b}{-a-2b} = \frac{b}{-a}$ Gradient of AB = Gradient of BC $\therefore$ A, B and C are collinear Alternatively. Find gradient of AB $\left(\frac{-b}{a}\right)$ and form an equation with a point. $(y-y_1 = m(x-x_1))$ Then substitute a point and see if it is true.	2 marks ✓ gradients ✓ concluding	Must have / show a common point with equal gradients
(b) $f(x) = \begin{cases} 2x+4 & \text{for } x>0 \\ -x-4 & \text{for } x \leq 0 \end{cases}$ $f(2) + f(0) + f(-2)$ $= 8 + (-4) + (-2)$ $= 2$	2 marks. ✓ finding $f(2), f(0)$ and (-2) ✓ answer of 2	
(c)(i) $A = 450(3^{-t})$ when $t=0$ $A = 450 \times 3^0$ $A = 450$ $\therefore$ initial amount is 450g (ii) when $A = 50$ $50 = 450(3^{-t})$ $\frac{1}{9} = 3^{-t}$ $3^{-2} = 3^{-t}$ $\therefore t = 2$ It will take 2 years	c(i) 1 mark c(ii) 1 mark.	few forgot to write the units - g or years

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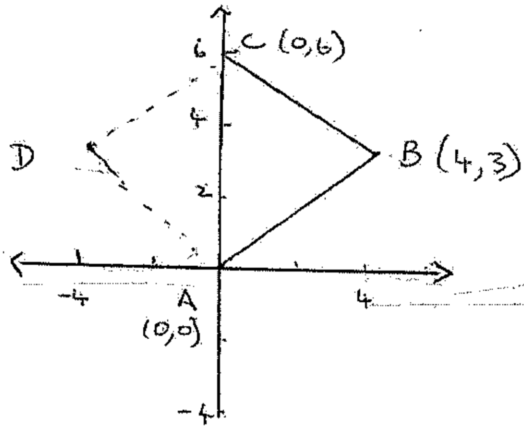
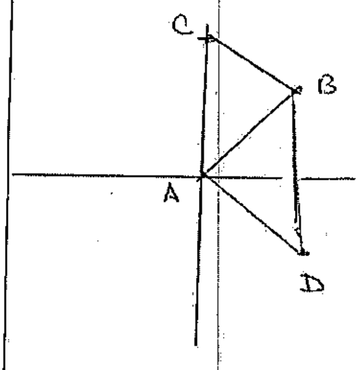
MATHEMATICS: Question 11

Suggested Solutions	Marks Awarded	Marker's Comments
d (i)  midpoint of AC is (0,3) and midpoint of BD is (0,3) $\frac{4+x}{2}=0$ and $\frac{y+3}{2}=3$ $x=-4$ $y=3$ $\therefore$ D is $(-4,3)$	2 marks to get coordinate D correct	 D cannot be $(4,-3)$ as the figure is not concyclic Similarly $(4,9)$ is not concyclic. - Some used the distance formula $(y-6)^2+(x-4)^2=(y-0)^2+(x-0)^2$ & got value of y and substituted it value to x.
c (ii). $M_{AB}=\frac{3-0}{4-0}=\frac{3}{4}$ $M_{BC}=\frac{6-3}{0-4}=\frac{-3}{4}$ product of gradients $M_{AB}\times M_{BC}$ $=\frac{3}{4}\times\frac{-3}{4}$ $=\frac{-9}{16}$ For line to be perpendicular (like in square) $M_{AB}\times M_{BC}=-1$ since $\frac{-9}{16}\neq -1$, there are no perpendicular lines $\therefore$ rhombus cannot be a square	2 marks	Must show all working to justify answer Some proved the figure cannot be a square by finding the distance of the diagonals They came up with 6 and 8.

MATHEMATICS: Question 11

Suggested Solutions	Marks Awarded	Marker's Comments
e (i) $PA = 2PB$ $PA^2 = 4PB^2$ $(x-7)^2 + (y-2)^2 = 4\left[(x+2)^2 + (y+1)^2\right]$ $x^2 - 14x + 49 + y^2 - 4y + 4 = 4x^2 + 16x + 16 + 4y^2 + 8y + 4$ $3x^2 + 30x + 3y^2 + 12 - 33 = 0$ $x^2 + 10x + y^2 + 4y - 11 = 0$	3 marks	
(ii) $x^2 + 10x + y^2 + 4y = 11$ $(x^2 + 10x + 25) + (y^2 + 4y + 4) = 11 + 25 + 4$ $(x+5)^2 + (y+2)^2 = 40$ $\therefore$ The locus is the circle with centre $(-5,-2)$ and radius $2\sqrt{10}$ or 40 units.	2 marks	

MATHEMATICS: Question 12 page 1

Suggested Solutions	Marks	Marker's Comments

(a) $\frac{5^{3n+3}}{25^{n-3}} = 125$

$\frac{5^{3n+3}}{(5)^{2n-6}} = 5^3$ ✓

$5^{3n+3-(2n-6)} = 5^3$

$3n+3-2n+6 = 3$ ✓

$n+9 = 3$ ✓

$n = -6$ ✓

(b)

method 1:

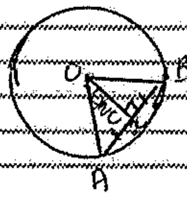
$x^2 = r^2 + r^2 - 2r^2\cos\theta$ ✓

$= 2r^2 - 2r^2\cos\theta$

Since $x > 0$

$x = r\sqrt{2 - 2\cos\theta}$ ✓

Note other correct answer. Method 1 and 2 were most common solutions. Could have also used Sine Rule.



Method 2: Using circle property OC is perp. bisector of chord

$\sin\frac{\theta}{2} = \frac{AC}{r}$

$AC = r\sin\frac{\theta}{2}$

$\therefore AB = 2r\sin\frac{\theta}{2}$

(c) Gradient function $\frac{dy}{dx} = 2x+5$

$y = \int 2x+5\,dx$

$y = \frac{2x^2}{2} + 5x + c$ ✓

$y = x^2 + 5x + c$

Subst pt $(4,1)$ $1 = 4^2 + 20 + c$

$\therefore c = -35$

$\therefore$ Eqn of curve is $y = x^2 + 5x - 35$ ✓

MANY students used $y - y_1 = m(x - x_1)$

$(y-1) = (2x+5)(x-4)$ etc

This is not correct because it can only be used for straight lines.

(d) (i) $y = -2\sin x$

$\frac{dy}{dx} = -2\cos x$

When $x = \frac{\pi}{6}$

$y = -2\sin\frac{\pi}{6}$

$y = -1$

When $x = \frac{\pi}{2}$ $\frac{dy}{dx} = -2 \times \frac{\sqrt{3}}{2}$ $\therefore m = -\sqrt{3}$ ✓ ✓

Eqn of tangent at $(\frac{\pi}{6}, -1)$ is

$y + 1 = -\sqrt{3}(x - \frac{\pi}{6})$

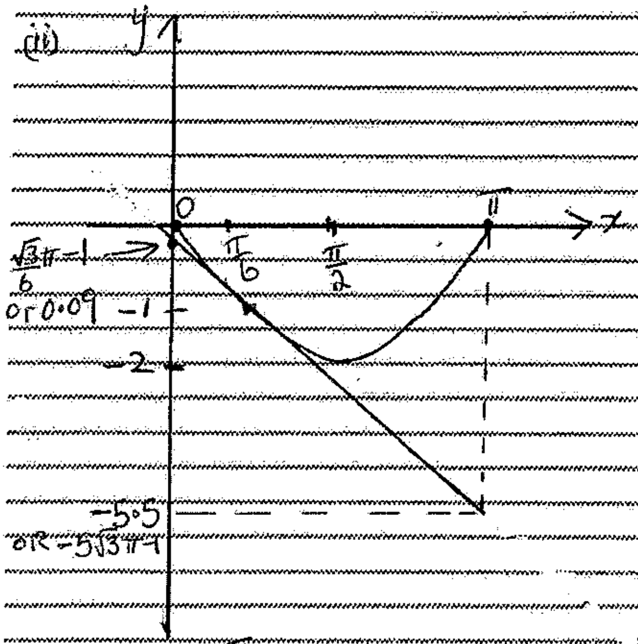
$y + 1 = -\sqrt{3}x + \frac{\sqrt{3}\pi}{6}$ ✓

$\therefore y = -\sqrt{3}x + \frac{\sqrt{3}\pi}{6} - 1$

MATHEMATICS: Question ...12 page 2.

Suggested Solutions	Marks	Marker's Comments

(ii)



1 for sin curve in domain $0 \leq x \leq \pi$. Needed to label endpoints & min value.

1 for tangent in domain $0 \leq x \leq \pi$. Many students did not extend the tangent for the whole domain.

(iii) $A = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \left(-2\sin x + \sqrt{3}x - \frac{\sqrt{3}\pi}{6} + 1\right) dx$

$= \left[2\cos x + \frac{\sqrt{3}}{2}x^2 - \frac{\sqrt{3}\pi}{6}x + x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ ✓

$= 0 + \frac{\sqrt{3}\pi^2}{8} - \frac{\sqrt{3}\pi^2}{12} + \frac{\pi}{2} - \left(\sqrt{3} + \frac{\sqrt{3}\pi^2}{72} - \frac{\sqrt{3}\pi^2}{36} + \frac{\pi}{6}\right)$ ✓

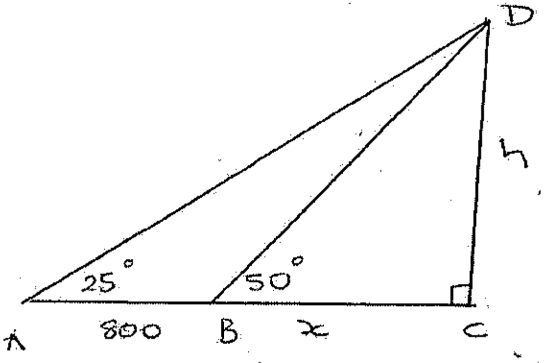
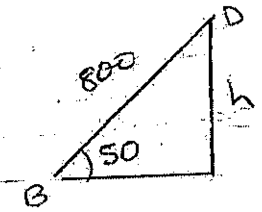
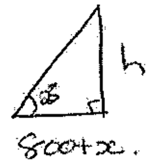
$= \left(\frac{\sqrt{3}\pi^2}{24} + \frac{\pi}{2}\right) - \left(\sqrt{3} - \frac{\sqrt{3}\pi^2}{72} + \frac{\pi}{6}\right)$

$= \left(\frac{\sqrt{3}\pi^2}{18} + \frac{\pi}{3} - \sqrt{3}\right)$ units^2 ✓ *

Note: *

If you got the answer $\sqrt{3} - \frac{\pi}{3} - \frac{4\sqrt{3}}{72}\pi^2$ then you needed to have $|\ |$. The answer as it stands is −0·26!

MATHEMATICS: Question 13 (1)

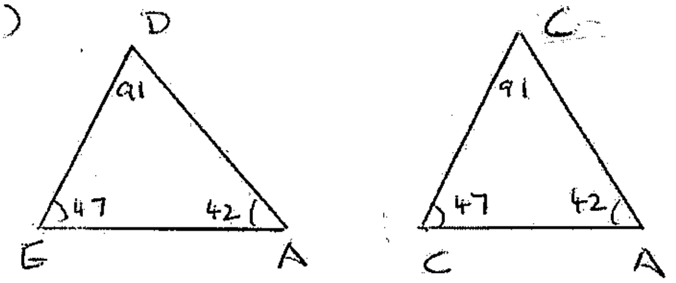
Suggested Solutions	Marks Awarded	Marker's Comments
(i) 	1 mark	show 800 m 25° and 50°
(ii) $\angle ABD = 130°$ (adjacent angles on a straight line are supplementary) $\therefore \angle ADB = 25°$ (angle sum of $\triangle ABD$ is 180°) $\therefore DB = 800$ m (equal sides are opposite equal angles in $\triangle ABD$) $\therefore$  $\sin 50 = \frac{h}{800}$ $h = 800 \times \sin 50$ $h = 612.8355545$ $\therefore$ The height is 613 m.	3 marks. ✓ finding DB ✓ reasons ✓ $h = 613$	Most students attempted  $\tan 25 = \frac{h}{800+x}$ $h = \tan 25(800+x)$ $h = 800\tan 25 + x\tan 25$ $x\tan 25 = h - 800\tan 25$ $\therefore x = \frac{h - 800\tan 25}{\tan 25}$ then $\tan 50 = \frac{h}{x}$ $\therefore x = \frac{h}{\tan 25}$ and solved simultaneously Some calculated BD using sine rule to get 800 - used sine rule again to get h

MATHEMATICS: Question..13..

Suggested Solutions	Marks Awarded	Marker's Comments
b) LHS $= (1-\cos\theta)(1+\sec\theta)$ $= 1+\sec\theta-\cos\theta-1$ $= \dfrac{1}{\cos\theta} - \cos\theta$ ✓ $= \dfrac{1-\cos^2\theta}{\cos\theta}$ $= \dfrac{\sin^2\theta}{\cos\theta}$ $= \dfrac{\sin\theta}{\cos\theta}\cdot\sin\theta$ $= \sin\theta\tan\theta$ $=$ RHS. ✓	2 marks 1 mark for expanding 1 mark prove LHS = RHS with correct working	
c(i) $\dfrac{dV}{dt} = \dfrac{e^t}{e^t+1}$ $V = \ln(e^t+1)+C$ ✓ initially $V=0$ $0 = \ln(2)+C$ $\therefore C = -\ln 2$ $\therefore V = \ln(e^t+1) - \ln 2$ ✓	2 marks	
(ii) when $t = \ln 3$ $V = \ln(e^{\ln 3}+1) - \ln 2$ ✓ $= \ln 32 - \ln 2$ $= \ln 16$ $V = 4\ln 2$. ✓	2 marks	

MATHEMATICS: Question 13

3

Suggested Solutions	Marks Awarded	Marker's Comments
d (i) 		
$E\hat{D}A = 91^\circ$ (angle sum of $\triangle ADE$ is 180°)	3 marks	
In $\triangle$'s ADE and ABC	✓ reason	
$\hat{A}$ is common ✓	✓ 2 angles	
$E\hat{D}A = B\hat{C}A$ (both 91°) ✓	✓ equiangular	
$\therefore \triangle ABC \parallel\!\mid \triangle AED$ (equiangular) ✓		
(ii) $\frac{AC}{AD} = \frac{AB}{AE}$ (corresponding sides of similar triangles $\triangle ABC$ and $\triangle AED$ are in same ratio)	✓ reason ✓ proving	Some used Sine Rule after mentioning similarity
$\frac{2}{6} = \frac{AB}{AE}$		
$\frac{1}{3} = \frac{AB}{AE}$		
$\therefore AE = 3.AB$ QED		

MATHEMATICS Extension 2: Question…….		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
14. a) $a > 0$, $\therefore$ the function concaves up	1	
$b^2 - 4ac = (-2)^2 - 4(1)(2)$		
$= -4$		
< 0	1	
$\therefore x^2 - 2x + 2 = 0$ has no solution		
$\therefore x^2 - 2x + 2 > 0$ for all values of x		
or		
$x^2 - 2x + 2$		
$= (x-1)^2 + 1$		
Since $(x-1)^2 \geq 0$	1	
$(x-1)^2 + 1 \geq 1$		
> 0	1	
b) $a = 2$, $d = 2$, $n = 22$	1	
$T_{22} = 2 + (22-1)2$		
$= 44$		
$\therefore l = 44$		
$S_{22} = \frac{22}{2}(2 + 44) = \boxed{506}$	1	

MATHEMATICS Extension 2: Question……..

Suggested Solutions	Marks Awarded	Marker's Comments

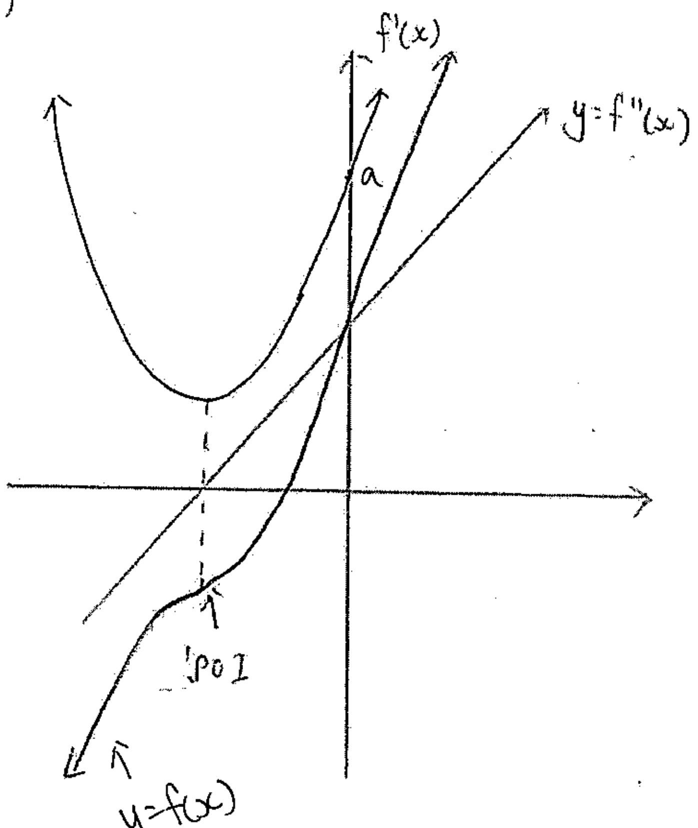
c) $f(x) = \dfrac{2x-1}{x+1}$

$f'(x) = \dfrac{(x+1).2 - (2x-1).1}{(x+1)^2}$ —— 1

$= \dfrac{3}{(x+1)^2}$ —— 1

$\therefore f'(2) = \dfrac{1}{3}$ —— 1

d)



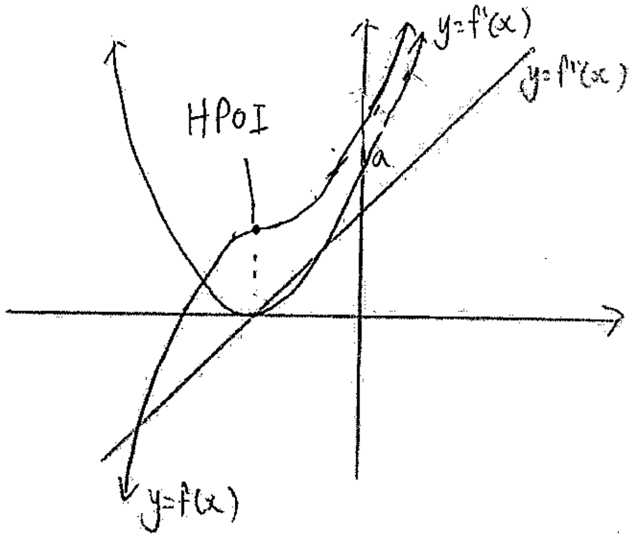
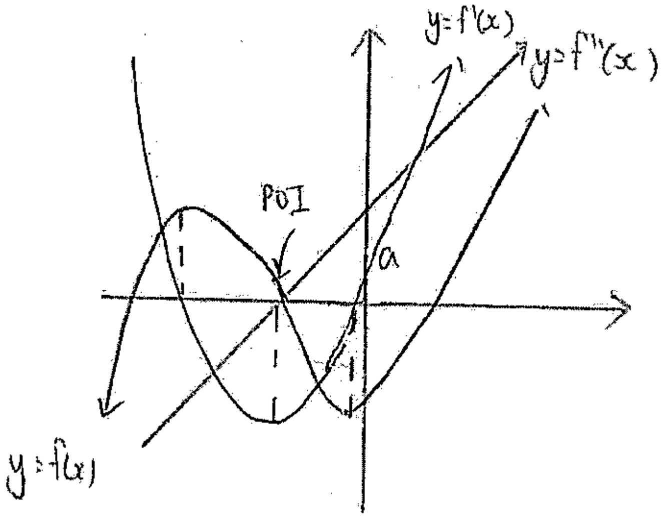
1 for turning point of $f'(x)$

1 for $f'(0) = a$

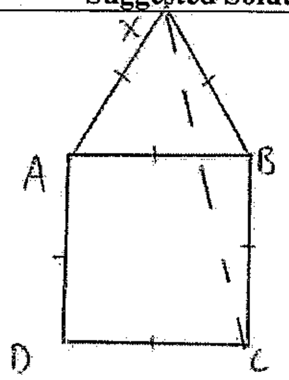
1 for POI for $f(x)$.

1 for correct shape of $f(x)$

or

MATHEMATICS Extension 2: Question……..		
Suggested Solutions	**Marks Awarded**	**Marker's Comments**
 or 		

MATHEMATICS Extension 1 : Question……..

Suggested Solutions	Marks Awarded	Marker's Comments
e)  Construct CX. $AB = BC$ (sides of square are equal) $AB = BX$ (sides of equilateral triangles are equal) $\therefore BX = BC$ $\therefore \triangle CBX$ is Isosceles (2 equal sides) $\angle XBC = 90^\circ$ (angles of squares are 90°) $\angle XBA = 60^\circ$ (angles of equilateral triangles are 60°) $\therefore \angle XBC = 90^\circ + 60^\circ = 150^\circ$ $\angle BXC = \angle BCX$ (equal sides are opposite to equal angles on Isosceles triangles) $\therefore \angle BXC = \frac{180 - 150}{2} = 15^\circ$ (angle sum of $\triangle BXC$ equals 180°) $\angle AXB = 60^\circ$ (angles on equilateral triangles are 60°) $\therefore \angle AXC = \angle AXB - BXC$ $= 60^\circ - 15^\circ$ $= 45^\circ$ $\therefore \angle AXC = 3 \times \angle BXC$.		Just about all students got the full marks. Few reasoning marks deducted E.g needed explanation why angles are 90° or 60° etc… before using them.

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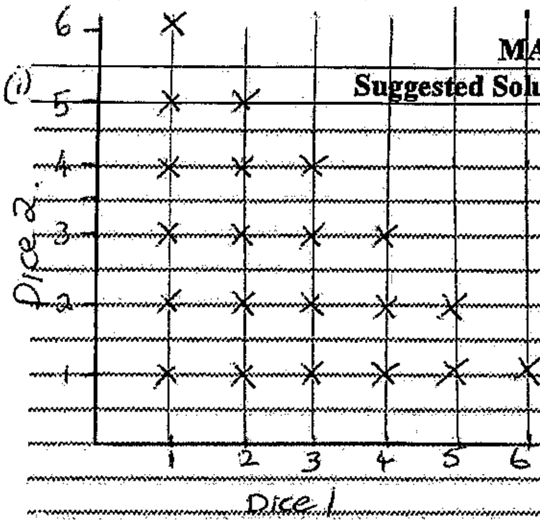
Suggested Solutions	Marks	Marker's Comments

(i)

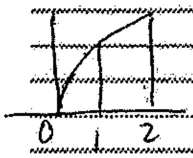


36 ✓

(ii) $P(\text{double}) = \frac{6}{36}$ ✓

(iii) $P(\text{total} < 8) = \frac{21}{36} = \frac{7}{12}$ ✓

(b) $\int_0^2 \sin x \, dx \doteqdot \frac{1-0}{2}\left(f(0)+f(1)\right)+\frac{2-1}{2}\left(f(1)+f(2)\right)$ ✓



OR $\frac{1}{2}\left(f(0)+2f(1)+f(2)\right)$

$= \frac{1}{2}\left(0+2\sin(1)+\sin(2)\right)$ ✓

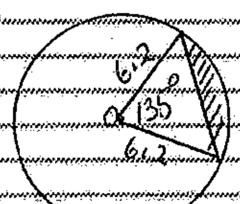
$= 1.296119698$

Need to use radians <u>not</u> degrees.

$\int_0^2 \sin x \, dx \doteqdot 1.30$ (3 sig. figs.) ✓

Many students need to revise significant figures.

(c)



$135° = \frac{3\pi}{4}$ ✓

$A = \frac{1}{2}r^2(\theta - \sin\theta)$

$= \frac{1}{2} \times 6.2^2\left(\frac{3\pi}{4} - \sin\frac{3\pi}{4}\right)$

$= 0.5 \times 6.2^2 \times (1.649)$

$= 31.69546577$ ✓

$= 31.7 \text{ cm}^2$ (1dp)

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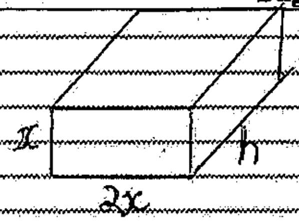
Suggested Solutions	Marks	Marker's Comments

(d) 

$V = 2x^2h = 24m^3$

$h = \frac{24}{2x^2}$

$h = \frac{12}{x^2}$ ✓

$L = 4\times 2x + 4\times x + 4\times h$

$= 8x + 4x + 4\left(\frac{12}{x^2}\right)$ ✓

$= 12x + \frac{48}{x^2}$

$= 12x + 48x^{-2}$

$\frac{dL}{dx} = 12 + -96x^{-3}$

$= 12 - \frac{96}{x^3}$

Possible max/min when $\frac{dL}{dx} = 0$

ie $12 - \frac{96}{x^3} = 0$

$\frac{96}{x^3} = 12 \quad x \neq 0$

$x^3 = 8$

$x = 2$ ✓

$\frac{dL^2}{dx^2} = 288x^{-4}$

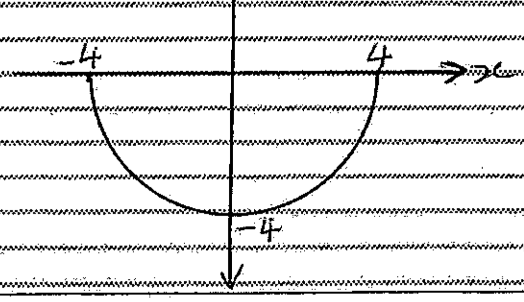
$= \frac{288}{x^4}$ When $x = 2$

$\frac{dL^2}{dx^2} = \frac{288}{2^4} > 0$ ✓

∴ concave up ∴ min value.

∴ Minimum length occurs when $x = 2$ ✓

ie Dimensions are 2m by 4m by 3m.

(e) 

✓ for neat semi-circle

✓ for axis labeled.

MATHEMATICS Extension 1 : Question……..

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16. a) $v = 4t^3 - 6t$		
i) $\frac{dx}{dt} = 4t^3 - 6t$		
$x = \int 4t^3 - 6t \, dt$		
$= t^4 - 3t^2 + c$	1	
When $t = 0$, $x = -4$		
$\therefore -4 = (0)^4 - 3(0)^2 + c$		
$c = -4$		
$\therefore x = t^4 - 3t^2 - 4$	1	
ii) $v = 4t^3 - 6t$		
$a = \frac{dv}{dt}$		
$= 12t^2 - 6$	1	
iii) $x = 0 \Rightarrow t^4 - 3t^2 - 4 = 0$		
$(t^2 - 4)(t^2 + 1) = 0$	1	
$\therefore t^2 = 4$ or -1		
Since $t^2 \geq 0$, $t^2 = -1$ is not a solution		
$\therefore t = \pm 2$. Since $t \geq 0$		
$t = 2$.	1	

MATHEMATICS: Question……..

Suggested Solutions	Marks	Marker's Comments
16. b) i Let I_n be the amount in the investment after n months.		
42 years = 504 months	1	
5% p.a $= \frac{1}{240}$ per month	1	
$I_1 = 50 \times \frac{241}{240}$		
$I_2 = 50 \times \left(\frac{241}{240}\right)^2 + 50 \times \left(\frac{241}{240}\right)$		Student lost marks for inconsistent working out. E.g decided to take \$50 away at the end to turn the sum into the required amount when earlier working does not indicate a need to do so.
$\vdots$		
$I_{504} = 50 \times \left(\frac{241}{240}\right)^{504} + 50 \times \left(\frac{241}{240}\right)^{503} + \ldots 50 \times \left(\frac{241}{240}\right)$		
$= 50\left[\left(\frac{241}{240}\right)^{504} + \left(\frac{241}{240}\right)^{503} + \ldots \left(\frac{241}{240}\right)\right]$		
$= 50 \times \left[\frac{\left(\frac{241}{240}\right)\left[\left(\frac{241}{240}\right)^{504} - 1\right]}{\frac{241}{240} - 1}\right]$	1	
$= \$85923.96$		
ii) \$43000 p.a		
$0.09 \times \frac{43000}{12} = \$322.50$	1	
$\text{Total} = (\$322.50 + 50)\left[\frac{\frac{241}{240}\left[\left(\frac{241}{240}\right)^{504} - 1\right]}{\left(\frac{241}{240}\right) - 1}\right]$		
$= \$640133.54$	1	

MATHEMATICS: Question……..

Suggested Solutions	Marks	Marker's Comments

16 b) iii) $I_{204} = 372.5\left[\dfrac{\frac{241}{240}\left[\left(\frac{241}{240}\right)^{204}-1\right]}{\frac{241}{240}-1}\right]$

$= \$119892.8656$ (1)

$I_{504} = I_{204} \times \left(\dfrac{201}{200}\right)^{300} + 372.5\left[\dfrac{\frac{201}{200}\left[\left(\frac{201}{200}\right)^{300}-1\right]}{\frac{201}{200}-1}\right]$

$\doteq \$535318.0256 + \boxed{\$259430.9523}$ (1)

$= \$794748.98$ (1)

iv) $1000000 = (322.50 + x)\left[\dfrac{\frac{241}{240}\left[\left(\frac{241}{240}\right)^{504}-1\right]}{\frac{241}{240}-1}\right]$ (1)

$322.50 + x = \dfrac{1000000}{\left[\dfrac{\frac{241}{240}\left[\left(\frac{241}{240}\right)^{504}-1\right]}{\frac{241}{240}-1}\right]}$

$x = \$259.41c.$ (1)