2015

Year 12 Mathematics

Trial Examination

Teacher Setting Paper: Mrs T Finch
Head of Department: Mrs M Hill

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board approved calculator may be used
- Write you answers to Section 1 on the multiple choice answer sheet provided
- Write your answers to Section 2 in the answer booklets provided. Start a new booklet for each question
- Write your student number only on the front of each booklet
- A table of standard integrals is provided at the back of this paper
- In Questions 11-16, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section 1 Pages 2-4
10 marks

- Attempt all questions 1-10
- Allow 15 minutes for this section

Section 2 Pages 5 - 12
90 marks

- Attempt all questions 11-16
- Allow 2 hour and 45 minutes for this section

This examination paper does not necessarily reflect the content or format of the Higher School Certificate Examination in this subject

Section 1 (10 marks)

Attempt Questions 1 -10
Allow 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 -10

1. What is $\sqrt[3]{\dfrac{2.7-8.1\times 0.04}{3.4\times 10^{-2}}}$ correct to 3 significant figures?

   (A) 4.11

   (B) 4.12

   (C) 4.118

   (D) 4.119

2. If $3\sqrt{5}+\sqrt{20}=\sqrt{a}$, then $a=$?

   (A) 5

   (B) 25

   (C) 125

   (D) 15

3. What is the exact value of $\cos\dfrac{7\pi}{6}$?

   (A) $\dfrac{\sqrt{3}}{2}$

   (B) $-\dfrac{\sqrt{3}}{2}$

   (C) $\dfrac{1}{\sqrt{2}}$

   (D) $-\dfrac{1}{\sqrt{2}}$

4. $\int_1^4 f(x)\,dx=2$ then $\int_1^4 \left(2f(x)+3\right)\,dx$ is equal to

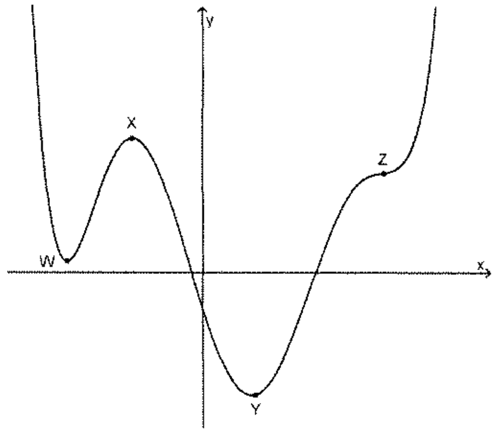
   (A) 2

   (B) 13

   (C) 7

   (D) 10

5. The diagram below shows the graph of $y = f(x)$. Points W, X, Y and Z are stationary points and Z is a point of inflexion.



Which of the points on $y = f(x)$ corresponds to the description

$y > 0 \qquad \frac{dy}{dx} = 0 \qquad \frac{d^2y}{dx^2} > 0$

(A) W

(B) X

(C) Y

(D) Z

6. $\lim_{\theta \to 0} \frac{\sin \theta}{4\theta} =$

(A) 1

(B) 4

(C) $\frac{1}{4}$

(D) 0

7. The function $f(x) = -3\cos\left(\frac{\pi x}{5}\right)$ has a period of

(A) $\frac{\pi}{5}$

(B) $\frac{\pi}{10}$

(C) 3

(D) 10

8. The limiting sum of the series $1-2p+4p^2......$ is $\frac{4}{7}$, find the value of p

(A) $\frac{3}{8}$

(B) $-\frac{3}{8}$

(C) $\frac{2}{7}$

(D) $-\frac{2}{7}$

9. What are the solutions to $3x^2-7x-3=0$

(A) $x=\frac{7\pm\sqrt{85}}{6}$

(B) $x=\frac{-7\pm\sqrt{85}}{6}$

(C) $x=\frac{7\pm\sqrt{13}}{6}$

(D) $x=\frac{-7\pm\sqrt{13}}{6}$

10. Which of the following represents the domain of the function: $f(x)=\sqrt{9-x^2}+\frac{1}{x-3}$

(A) $x\neq 3$

(B) $x<3$

(C) $-3<x<3$

(D) $-3\leq x<3$

End of multiple choice section
Section 2 is on the next page

Section 2 (90 marks)

Attempt Questions 11-16

Allow about 2 hour and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet

a) Solve for x: $\frac{6}{x} = x - 5$ **2**

b) Solve $|3x - 4| \leq 8$ **2**

c) If $\tan\theta = \frac{7}{9}$ and $\cos\theta < 0$, find the exact value of $\operatorname{cosec}\theta$ **2**

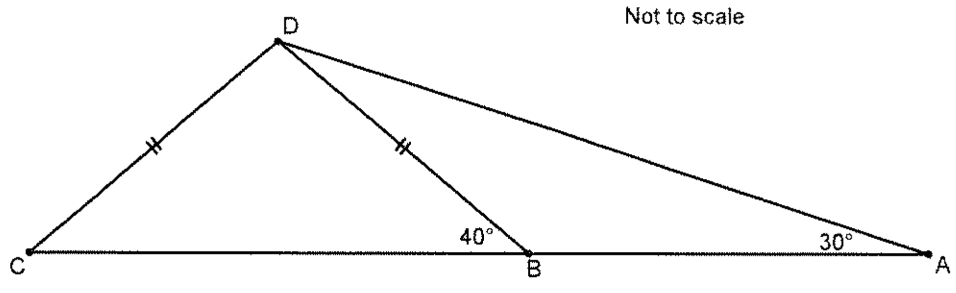
d) Solve for x: $9^{2x-3} = 27$ **2**

e) Differentiate with respect to x:

(i) $y = \cos\left(x^2 - 1\right)$ **2**

(ii) $y = \frac{1 - e^{2x}}{x^3}$ **2**

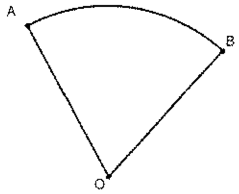
f) In the diagram below ABC is a straight line. If $\angle DBC = 40°$, $\angle DAB = 30°$, $BD = CD$ and $AB = 10\ cm$, what is the value of BC, to the nearest cm? **3**



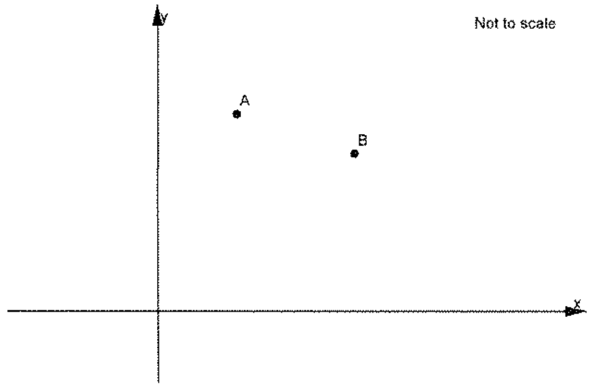
End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet

a) In the diagram AB is an arc of a circle with the centre O. The length of the arc AB is $\frac{5\pi}{2}\ cm$. The area of the sector AOB is $4\pi\ cm^2$. Find the radius of the sector. **2**



b) The diagram shows the points $A(2,5)$ and $B(5,4)$



(i) Show that the equation of the line AB is $x+3y-17=0$ **2**

(ii) Find the coordinates of M the midpoint of AB. **1**

(iii) Show that the equation of the perpendicular bisector of AB is $3x-y-6=0$ **1**

(iv) The perpendicular bisector of AB cuts the x-axis at C. Find the coordinates of C. **1**

(v) Find the area of ΔABC **2**

c) Give the exact value of $\log_3\left(\frac{1}{\sqrt{3}}\right)$ **1**

d) Find $\int \sin\frac{x}{2}\,dx$ **2**

e) Find the equation of the normal to $y=\left(x^3-1\right)^2$ at the point $x=-1$ **3**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet

a) Consider the function $f(x)=2x^3-3x^2-36x+26$

(i) Find the co-ordinates of the stationary points of the curve $y=f(x)$ and determine their nature. **3**

(ii) Find the co-ordinates of any point of inflexion. **1**

(iii) Hence sketch the graph of $f(x)=2x^3-3x^2-36x+26$ by showing the above information. **2**

(iv) For what values of x is the curve concave down and decreasing? **1**

b) There are three green lolly frogs and one red lolly frog in a jar. One frog is selected at random, eaten, and then a second frog is selected at random and also eaten. Find the probability that:

(i) The two frogs eaten are both green. **1**

(ii) The red frog is the second one eaten. **1**

c) Let A be the point $(-2,0)$ and $B(6,0)$. The point $P(x,y)$ is such that $AP \perp PB$.

(i) Find the gradient of PA. **1**

(ii) Hence, find the equation of the locus of P. **2**

d) If α and β are the roots of the quadratic equation $3x^2+8x+7=0$, find the value of

(i) $\alpha+\beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2+\beta^2$ **1**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet

a) Consider the parabola $2y = x^2 - 4x$

(i) Find the coordinates of the focus **2**

(ii) Find the equation of the directrix. **1**

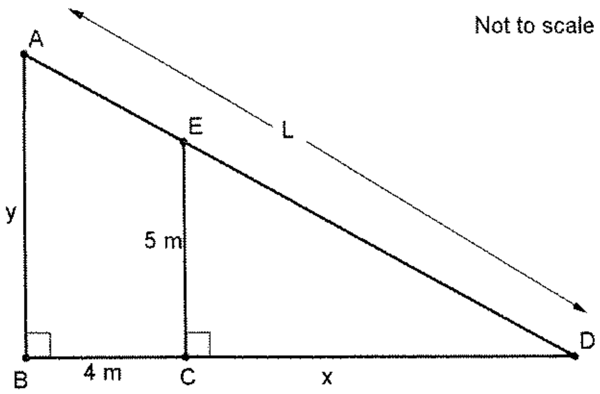
(iii) What is the length of the latus rectum? **1**

b) Solve: $2\sin\theta + \sqrt{3} = 0$ for $0 \le \theta \le 2\pi$ **2**

c) Evaluate: $\int_1^3 (6e^{3x} + 1)\ dx$ **2**

d) A sum of \$2 000 is deposited at the start of each year in an account that earns 10% interest per annum. Find the total value of the investment at the end of the 15th year, correct to the nearest dollar **2**

e) A 5 metre high fence stands 4 metres from the wall of a house. A farmer wishes to reach a point, A, on the wall by the use of a ladder, L, that can reach from the ground outside the fence to the wall as shown in the diagram below.



(i) Prove that $y = \dfrac{5(x+4)}{x}$ **1**

(ii) Hence show that $L^2 = (x+4)^2\left(1 + \dfrac{25}{x^2}\right)$ **2**

(iii) Hence find the length of the shortest ladder that can reach from the ground outside the fence to the wall. (Answer to 1 decimal place) **2**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet

a) A particle moves along a smooth horizontal surface according to $x = 10 + 8t - 2t^2$, with x in metres and t in seconds.

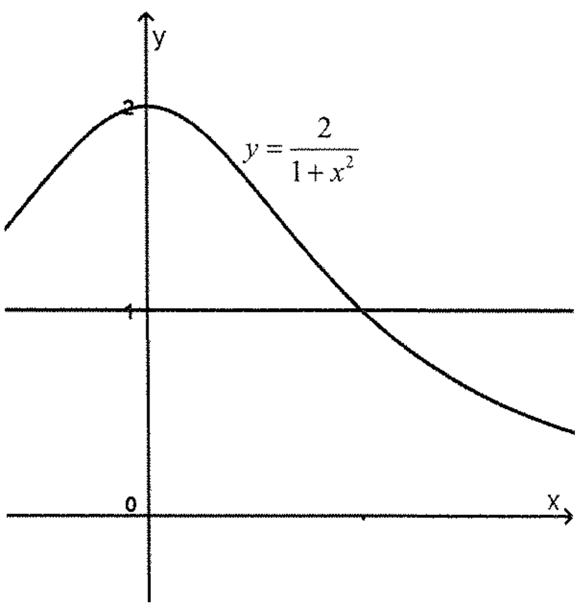
(i) Find when the particle is at the origin. **1**

(ii) Find where the particle is stationary. **2**

(iii) Show that the particle has constant acceleration. **1**

(iv) How far has the particle travelled in the first 6 seconds. **2**

b) The region between the curve $y = \dfrac{2}{1+x^2}$, the y-axis and the line $y = 1$ shown below, is rotated about the y-axis.



(i) Show that the volume of the solid formed is given by

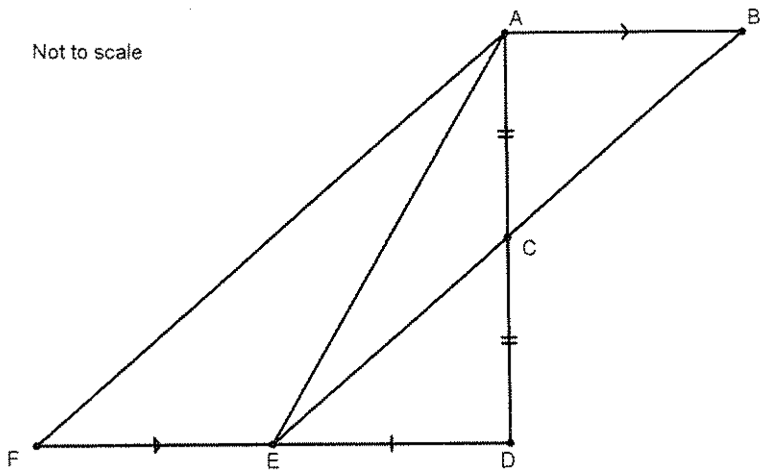
$$V = \pi \int_1^2 \left(\frac{2}{y} - 1 \right) dy$$ **1**

(ii) Hence find its volume. **2**

Question 15 is continued on page 10

Question 15 (continued)

c) In the diagram $AB \parallel FD$. ΔADF is right angled, C is the midpoint of AD and E is the midpoint of FD.



Copy or trace this diagram into your answer booklet.

(i) Explain why $\angle CED = \angle ABC$ **1**

(ii) Show that $\Delta CDE \equiv \Delta CAB$ **2**

(iii) Show that $AF = 2BC$ **1**

d) Consider the function $y = \log_e\left(x^2+1\right)$. Using Simpson's Rule with 5 function values to approximate $\int_0^4 \log_e\left(x^2+1\right)\,dx$ **2**

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet

a) The population of a species of bacteria P, at time t minutes, grows such that

$$P = 2000e^{kt} \text{ where } k \text{ is a positive constant.}$$

(i) Show that the rate of increase of the population is proportional to the size of the population at that time, that is $\frac{dP}{dt} = kP$. **1**

(ii) Given that the initial population doubles after 4 minutes, calculate the value of k correct to 3 significant figures. **2**

(iii) Find the population after 6 minutes, correct to the nearest whole number. **2**

b) Henry borrows \$200 000 which is to be repaid in equal monthly instalments of \$$M$.
The interest rate is 7.2% per annum reducible, calculated monthly.

(i) Show that the amount \$$A_n$, owing after the nth month is given by the formula: **2**

$$A_n = 200\,000 \times 1.006^n - M\left(1 + 1.006 + 1.006^2 + \ldots + 1.006^{n-1}\right)$$

(ii) The minimum monthly repayment is the amount required to repay the loan in 25 years. Find the minimum monthly repayment, \$$M$. **3**

Question 16 is continued on page 12

Question 16 (continued)

c) On a factory production line a tap opens and closes to fill empty containers with liquid. As the tap opens, the rate of flow of liquid, R, litres per second, increases for the first 10 seconds according to

$$R = \frac{6t}{50}$$

After 10 seconds, the rate of flow remains constant until the tap begins to close. As the tap closes, the rate of flow decreases at a constant rate for 10 seconds, after which the tap has fully closed and the rate of flow is zero.

(i) Show that, while the tap is fully open, the volume in the container, in litres, at any time t, is given by **2**

$$V = \frac{6}{5}(t-5)$$

(ii) For how many seconds must the tap remain fully open in order to exactly fill a 120 litre container with no spillage? **3**

End of Examination

2015 **Number: ______________**

Year 12 Mathematics

Trial Examination: Multiple Choice Answer Sheet

For multiple choice questions, choose the best answer A, B, C or D and fill in the correct circle.

1. Ⓐ Ⓑ Ⓒ Ⓓ
2. Ⓐ Ⓑ Ⓒ Ⓓ
3. Ⓐ Ⓑ Ⓒ Ⓓ
4. Ⓐ Ⓑ Ⓒ Ⓓ
5. Ⓐ Ⓑ Ⓒ Ⓓ
6. Ⓐ Ⓑ Ⓒ Ⓓ
7. Ⓐ Ⓑ Ⓒ Ⓓ
8. Ⓐ Ⓑ Ⓒ Ⓓ
9. Ⓐ Ⓑ Ⓒ Ⓓ
10. Ⓐ Ⓑ Ⓒ Ⓓ

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STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq 1; \quad x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right) \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \ x > 0$

2015 Maths Trial

Multiple Choice

1	B	6	C
2	C	7	D
3	B	8	A
4	B	9	A
5	A	10	D

(1) $4.1189 \approx 4.12$

(2) $3\sqrt{5} + \sqrt{20}$

$= 3\sqrt{5} + 2\sqrt{5}$

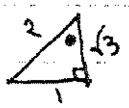
$= 5\sqrt{5}$

$\sqrt{a} = \sqrt{125}$

(3) $\cos\frac{7\pi}{6}$

S	A
✓T	C

$= -\cos\frac{\pi}{6}$



$= -\frac{\sqrt{3}}{2}$

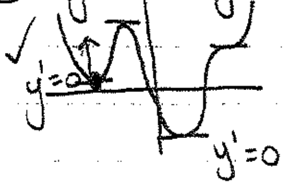
(4) $\int_1^4 2f(x) + 3\,dx$

$= 2\int_1^4 f(x)\,dx + \left[3x\right]_1^4$

$= 2 \times 2 + 12 - 3$

$= 13$

(5) $y > 0 \quad y' = 0 \quad y'' > 0$



(6) $\lim_{\theta \to 0} \frac{\sin\theta}{4\theta} = \frac{1}{4}\lim_{\theta \to 0}\frac{\sin\theta}{\theta}$

$= \frac{1}{4}$

(7) $f(x) = -3\cos\left(\frac{\pi x}{5}\right)$

Period $= \frac{2\pi}{n}$ $\quad n = \frac{\pi}{5}$

$= \frac{2\pi}{(\pi/5)}$

$= 2\pi \times \frac{5}{\pi}$

$= 10$

(8) $S_\infty = \frac{4}{7}$ $\quad a = 1 \quad r = \frac{-2p}{1}$

$\frac{a}{1-r} = \frac{4}{7}$

$\frac{1}{1 - {-2p}} = \frac{4}{7}$

$4(1 + 2p) = 7$

$4 + 8p = 7$

$8p = 3$

$p = 3/8$

(9) $3x^2 - 7x - 3 = 0$

$x = \frac{-{-7} \pm \sqrt{(-7)^2 - 4 \times 3 \times -3}}{2 \times 3}$

$= \frac{7 \pm \sqrt{85}}{6}$

(10) $f(x) = \sqrt{9 - x^2} + \frac{1}{x-3}$

Domain $-3 \leq x < 3$

Q11

a) $\frac{6}{x} = x - 5$

$6 = x^2 - 5x$

$x^2 - 5x - 6 = 0$ ✓

$(x-6)(x+1) = 0$

$x = 6, -1$ ✓

b) $|3x - 4| \leq 8$

$-8 \leq 3x - 4 \leq 8$ ✓

$-4 \leq 3x \leq 12$

$-\frac{4}{3} \leq x \leq 4$ ✓

c) $\tan\theta = \frac{7}{9}$ $\cos\theta < 0$

(triangle: hypotenuse x, opposite 7, adjacent 9, angle θ; quadrant diagram S | A ✓, T ✓ (circled) | C ×)

$x^2 = (7)^2 + 9^2$

$x = \sqrt{130}$ ✓

$\operatorname{cosec}\theta = -\frac{\sqrt{130}}{7}$ ✓

d) $9^{2x-3} = 27$

$(3^2)^{2x-3} = 3^3$ ✓

$4x - 6 = 3$

$4x = 9$

$x = \frac{9}{4}$ ✓

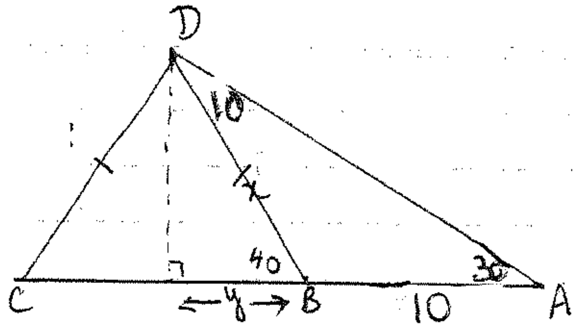
e) i) $y = \cos(x^2 - 1)$

$y' = -2x \sin(x^2 - 1)$ ✓ ✓

ii) $y = \frac{1 - e^{2x}}{x^3}$

$y' = \frac{x^3 \times -2e^{2x} - 3x^2(1 - e^{2x})}{x^6}$ ✓

$= \frac{-2xe^{2x} - 3 + 3e^{2x}}{x^4}$



$\frac{x}{\sin 30} = \frac{10}{\sin 10}$ ✓

$x = \frac{10 \sin 30}{\sin 10} \approx 28.79385\ldots$

$\cos 40 = \frac{y}{x}$ ✓

$y = \frac{10 \sin 30}{\sin 10} \times \cos 40$

$= 22.05737$

$\approx 22\text{cm}$ ✓

$\therefore BC \doteqdot 44\text{cm}$

OR Use cosine Rule $\angle CDB = \frac{180}{-80} \over 100$

$BC^2 = x^2 + x^2 - 2x \times x \cos\angle CDB$

$BC^2 = 2 \times \left(\frac{10 \sin 30}{\sin 10}\right)^2 - 2 \times \left(\frac{10 \sin 30}{\sin 10}\right)^2 \cos 100^\circ$

$BC \approx 44\text{cm}$

OR $\angle DCB = \angle DBC = 40$

$\angle CDB = 180 - 2 \times 40$

100

$\frac{BC}{\sin 100} = \frac{x}{\sin 40}$

$BC = \frac{28.79 \times \sin 100}{\sin 40}$

$= 44.$

Q12

a) $l = r\theta^c$ $\quad A = \frac{1}{2}r^2\theta^c$

$\frac{5\pi}{2} = r\theta$ ✓ $\quad 4\pi = \frac{1}{2}r^2\theta$

$\therefore r^2\theta = 8\pi$

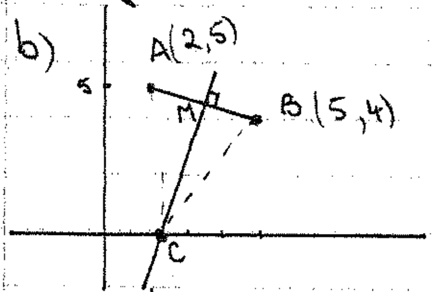
$\frac{r^2\theta}{r\theta} = \frac{8\pi}{\left(\frac{5\pi}{2}\right)}$

$r = \frac{16}{5}$ ✓ $\left(3\frac{1}{2}\right)$

$\left(\frac{16}{5} \times \theta = \frac{5\pi}{2}\right.$

$\left.\theta^c = \frac{25\pi}{32}\right)$

b)



i) $m = \frac{4-5}{5-2} = \frac{-1}{3}$ ✓

$y - 5 = -\frac{1}{3}(x-2)$ ✓

$3y - 15 = -x + 2$

$x + 3y - 17 = 0$

ii) $M\left(\frac{2+5}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$ ✓

iii) $m_\perp = 3$ ✓

$y - \frac{9}{2} = 3\left(x - \frac{7}{2}\right)$

$2y - 9 = 6x - 21$

$6x - 2y - 12 = 0$

$3x - y - 6 = 0$

iv) on x-axis $y = 0$

$\therefore 3x - 6 = 0$

$x = 2$ ✓

$C(2, 0)$

(b)(v) $d_{AB} = \sqrt{(2-5)^2 + (5-4)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$ ✓

$\perp d_{MC} = \frac{|1\times 2 + 3\times 0 - 17|}{\sqrt{1^2+3^2}}$ $\quad$ Or use $d_{MC} =$

$= \frac{|-15|}{\sqrt{10}}$

Area $= \frac{1}{2}\sqrt{10} \times \frac{15}{\sqrt{10}}$ ✓

$= 7.5\ u^2$

c) $\log_3\left(\frac{1}{\sqrt{3}}\right) = \log_3 3^{-1/2}$

$= -\frac{1}{2}$ ✓

d) $\int \sin\frac{x}{2}\,dx = -2\cos\frac{x}{2} + c$ ✓ ✓

e) $y = (x^3 - 1)^2$ at $x = -1$

$y' = 2(x^3-1) \times 3x^2$

$= 2\left((-1)^3 - 1\right) \times 3 \times (-1)^2$

$= 2 \times -2 \times 3$ ✓

$= -12$

$m_\perp = \frac{1}{12}$ $\quad y = \left((-1)^3 - 1\right)^2 = 4$

$y - 4 = \frac{1}{12}(x - -1)$ ✓

$12y - 48 = x + 1$

$x - 12y + 49 = 0$ ✓

Q13

a) $f(x) = 2x^3 - 3x^2 - 36x + 26$

i) $f'(x) = 6x^2 - 6x - 36$

$f'(x) = 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$ ✓

$x = 3 \quad x = -2$

$f(3) = -55 \quad f(-2) = 70$ ✓

$f''(x) = 12x - 6$

$f''(3) = 36 - 6 = 30 > 0 \quad f''(-2) = -24 - 6 = -30 < 0$

$\therefore (3, -55)$ is a Min. ✓ $\quad \therefore (-2, 70)$ is a Max

(ii) $f''(x) = 0$

$12x - 6 = 0$

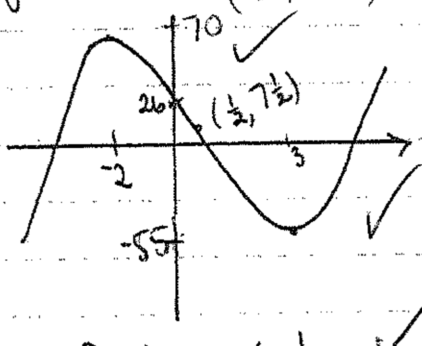
$x = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = 2\times\left(\frac{1}{2}\right)^3 - 3\times\left(\frac{1}{2}\right)^2 - 36\left(\frac{1}{2}\right) + 26$

$= 7\frac{1}{2}$

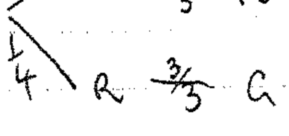
Inflexion at $\left(\frac{1}{2}, 7\frac{1}{2}\right)$ ✓

iii)



iv) $-2 < x < \frac{1}{2}$ ✓

(b)



(i) $P(GG) = \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$ ✓

(ii) $P(GR) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$ ✓

c) $A(-2,0)$ $B(6,0)$ $P(x,y)$

i) $m_{PA} = \frac{y-0}{x--2} = \frac{y}{x+2}$ ✓

ii) $m_{PA} \times m_{PB} = -1$

$\frac{y}{(x+2)} \times \frac{y}{(x-6)} = -1$ ✓

$y^2 = -(x+2)(x-6)$

$y^2 = -x^2 - 2x + 6x + 12$

$x^2 - 4x + y^2 = 12$ ✓

or $(x-2)^2 + y^2 = 16$

d) $3x^2 + 8x + 7 = 0$

i) $\alpha + \beta = \frac{-b}{a}$

$= \frac{-8}{3}$ ✓

ii) $\alpha\beta = \frac{c}{a}$

$= \frac{7}{3}$ ✓

iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(-\frac{8}{3}\right)^2 - 2 \times \frac{7}{3}$

$= \frac{64}{9} - \frac{14}{3}$ ✓

$= 2\frac{4}{9}$

14 a) $2y = x^2 - 4x$

i) $x^2 - 4x + 4 = 2y + 4$

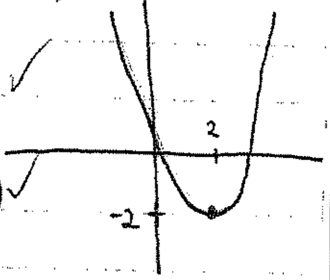
$(x-2) = 2(y+2)$

$V(2, -2)$

$4a = 2$

$a = \frac{1}{2}$ ✓

Focus $(2, -1\frac{1}{2})$ ✓

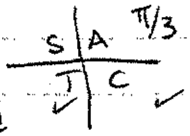


ii) $y = -2\frac{1}{2}$ ✓

iii) $4a = 2$ ✓

b) $2\sin\theta + \sqrt{3} = 0 \qquad 0 \le \theta \le 2\pi$

$\sin\theta = -\frac{\sqrt{3}}{2}$ ✓



$\theta = \pi + \frac{\pi}{3}, \; 2\pi - \frac{\pi}{3}$

$= \frac{4\pi}{3}, \frac{5\pi}{3}$ ✓

c) $\int_1^3 (6e^{3x} + 1)\, dx$

$= \left[\frac{6e^{3x}}{3} + x\right]_1^3$ ✓

$= [2e^9 + 3] - \{2e^3 + 1\}$

$= 2e^9 - 2e^3 + 2$ ✓

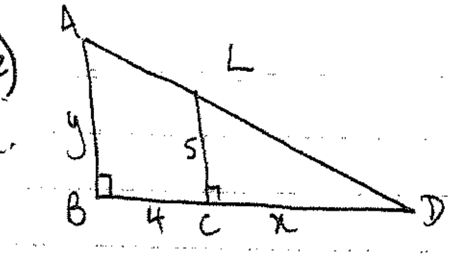
(d) $2000 \times 1.1^{15} + 2000 \times 1.1^{14} + \dots + 2000 \times 1.1$

$= 2000\,[1.1 + 1.1^2 + \dots + 1.1^{15}]$

$= 2000 \times \frac{1.1(1.1^{15} - 1)}{1.1 - 1}$

$\approx \$69899$ ✓

e)



i) $\triangle ABD \,|||\, \triangle LCD$ equiangular

$\therefore$ corresponding sides n same ratio

$\frac{y}{5} = \frac{x+4}{x}$

$y = \frac{5(x+4)}{x}$

ii) $AD^2 = AB^2 + BD^2$

$L^2 = y^2 + (x+4)^2$

$= \frac{25(x+4)^2}{x^2} + (x+4)^2$

$= (x+4)^2\left[1 + \frac{25}{x^2}\right]$

iii) $L = (x+4)(1 + 25x^{-2})^{1/2}$

$\frac{dL}{dx} = (x+4) \times \frac{1}{2}(1 + 25x^{-2})^{-1/2} \times -50x^{-3} + (1 + 25x^{-2})^{1/2} \times 1$

$0 = \frac{-25(x+4)}{x^3\sqrt{1+25x^{-2}}} + (1+25x^{-2})^{1/2} \frac{x^3(1+25x^{-2})^{1/2}}{x^3(1+25x^{-2})^{1/2}}$

$0 = -25x - 100 + x^3\left(1 + \frac{25}{x^2}\right)$

$0 = -25x - 100 + x^3 + 25x$

$x^3 = 100$

$x = \sqrt[3]{100}$

$L = (\sqrt[3]{100} + 4)\left(1 + \frac{25}{(\sqrt[3]{100})^2}\right)^{1/2}$

$= 12.701657\dots$

≈ 12.7 m

15.

a) $x = 10 + 8t - 2t^2$

i) $x = 0$ $\quad 2t^2 - 8t - 10 = 0$

$t^2 - 4t - 5 = 0$

$(t - 5)(t + 1) = 0$

$t = 5, -1$

$t \geq 0 \therefore$ <u>t = 5 seconds</u>

ii) $\dot{x} = 8 - 4t = 0$

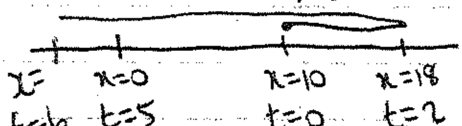
$4t = 8$

<u>t = 2 seconds</u>

$x = 10 + 8 \times 2 - 2 \times 2^2$
$= 18$

iii) $\ddot{x} = -4 = \text{constant}$

iv) $t = 2 \quad x = 10 + 8 \times 2 - 2 \times 2^2$
$= 18$



$x = 10 + 8 \times 6 - 2 \times 6^2$
$= -14$

$d = 8 + 18 + 14$

$= 40 \text{ m}$

b) $y = \frac{2}{1 + x^2}$ $\quad V = \pi \int_a^b x^2 \, dy$

$1 + x^2 = \frac{2}{y}$

$x^2 = \frac{2}{y} - 1$

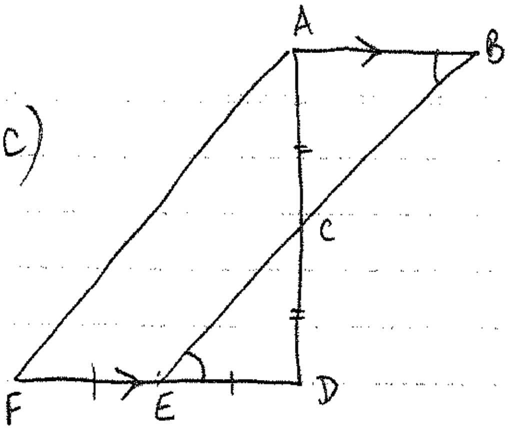
$V = \pi \int_1^2 \left(\frac{2}{y} - 1\right) dy$

ii) $V = \pi \left[2 \ln y - y\right]_1^2$

$= \pi \left[(2 \ln 2 - 2) - (2 \ln 1 - 1)\right]$

$= 2\pi \ln 2 - \pi \text{ u}^3$

c)



i) Alternate angles are equal
AB || FD

ii) In ΔCDE + ΔCAB

$AC = CD$ given

$\angle ECD = \angle ACB$ vertically opposite angles equal

$\angle DEC = \angle ABC$ Alternate angles in parallel lines

$\therefore \Delta CDE \equiv \Delta CAB$ AAS

iii) $CE = BC$ corresponding sides in congruent triangles

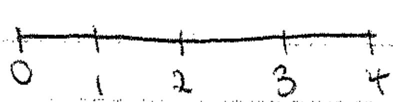
$\Delta CED \;|||\; \Delta AFD$

$\frac{AF}{CE} = \frac{AD}{CD} = \frac{2}{1}$

$\therefore AF = 2 \times CE$

So $AF = 2BC$

d)



$f(x) = \ln(x^2 + 1)$

$\int_0^4 \log_e(x^2 + 1) \, dx \approx \frac{1}{3}\left[f(0) + f(4) + 4(f(1) + f(3)) + 2f(2)\right]$

$= \frac{1}{3}\left[\ln 1 + \ln 17 + 4(\ln 2 + \ln 10) + 2\ln 5\right]$

$= \frac{1}{3}\left[\ln 17 + \ln 20^4 + \ln 5^2\right]$

$= \frac{1}{3} \ln(17 \times 20^4 \times 5^2) \approx 6.01167$

$= \frac{1}{3} \ln 68000000$

16a) $P = 2000e^{kt}$

i) $\frac{dP}{dt} = 2000 \times k e^{kt}$

$= k \times 2000e^{kt}$

$\frac{dP}{dt} = kP$ ✓

ii) $P = 2P_0$ when $t = 4$

$4000 = 2000e^{k \times 4}$

$2 = e^{4k}$ ✓

$\ln 2 = 4k$

$k = \frac{1}{4}\ln 2$ ✓

≈ 0.173

iii) $t = 6$

$P = 2000 \times e^{0.173 \times 6}$ ✓

$= 5656.85$

≈ 5657 (if exact k) ✓

or $P \approx 5647$ for rounded k.

b) $r = 7.2\% = \frac{0.072}{12} = 0.006$

$A_1 = 200000 + 0.006 \times 200000 - M$

$= 200000 \times 1.006 - M$

$A_2 = A_1 \times 1.006 - M$ ✓

$= 200000 \times 1.006^2 - M \times 1.006 - M$

$A_n = 200000 \times 1.006^n - M \times 1.006^{n-1} \ldots - M \times 1.006 - M$ ✓

$A_n = 200000 \times 1.006^n - M(1 + 1.006 + 1.006^2 + \ldots + 1.006^{n-1})$

$n = 25 \times 12 = 300$

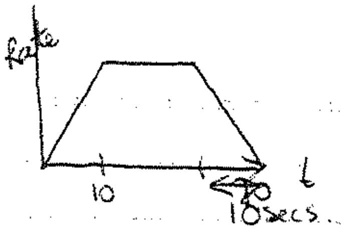
ii) $0 = 200000 \times 1.006^{300} - M\left[\frac{1(1.006^{300} - 1)}{1.006 - 1}\right]$ ✓

$M = 200000 \times 1.006^{300} \times \frac{0.006}{(1.006^{300} - 1)}$

$= \$1439.18$ ✓

✓

c) $R = \frac{6t}{50}$



$\frac{dV}{dt} = \frac{6t}{50}$

$V = \frac{6}{50} \times \frac{t^2}{2} + c$

$V = \frac{3t^2}{50} + c$ initially $V = 0$, $t = 0$

$0 = \frac{3 \times 0^2}{50} + c$ $c = 0$

$V = \frac{3t^2}{50}$ for $0 \le t \le 10$ ✓

At $t = 10$ $V = \frac{3 \times 10^2}{50} = 6L$

At $t = 10$ $R = \frac{6 \times 10}{50} = \frac{6}{5} L/s$

$\therefore \frac{dV}{dt} = \frac{6}{5}$ ✓

In this section $t = 10$ $V = 6$

$V = \frac{6}{5}t + c$

$6 = \frac{6}{5} \times 10 + c$ $c = -6$

$V = \frac{6}{5}t - 6$

$= \frac{6}{5}(t - 5)$ ✓ for $t \ge 10$

ii) First 10 seconds Volume $= 6L$

last 10 seconds $R = -\frac{6}{5} L/s$

$\therefore V = \frac{6}{5} \times 10 = 12L$ ✓

$\therefore 120 - (6 + 12) = 102$

$102 = \frac{6}{5}(t - 5)$ ✓

$\frac{102 \times 5}{6} + 5 = t$ $t = 90$ seconds