

Marking grid

Question	YO1	YO2	YO3	YO4	YO5	Total
MCQ	6 $\bar{1}$	2, 4, 7, 10 $\bar{4}$	8, 9 $\bar{2}$	1, 3 $\bar{2}$	5 $\bar{1}$	$\bar{10}$
11	a, c, d, e $\bar{9}$	b $\bar{1}$		f, g $\bar{5}$		$\bar{15}$
12	a $\bar{2}$	b, c(i), d $\bar{9}$	c(ii)(iii) $\bar{4}$			$\bar{15}$
13		a, b $\bar{8}$		c, d $\bar{7}$		$\bar{15}$
14			a $\bar{6}$		b $\bar{9}$	$\bar{15}$
15			a $\bar{3}$	b, c $\bar{12}$		$\bar{15}$
16	a(ii)(iii), b $\bar{10}$	a(i) $\bar{1}$	c $\bar{4}$			$\bar{15}$
Total	$\bar{22}$	$\bar{23}$	$\bar{19}$	$\bar{26}$	$\bar{10}$	$\bar{100}$

Section I

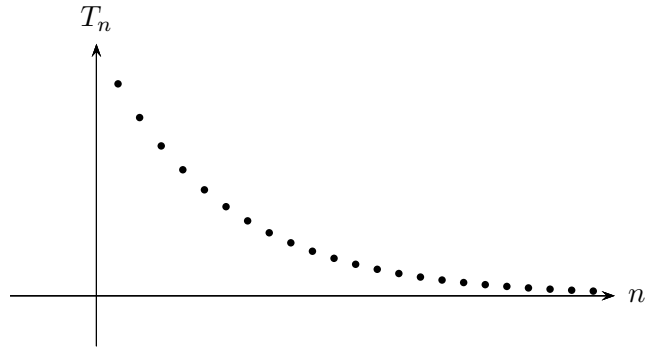
10 marks

Attempt Question 1 to 10

Mark your answers on the answer grid provided (labelled as page 13).

Questions	Marks
1. Find the value of $\lim_{x \rightarrow -3} \frac{x^3 + 27}{x^2 - x - 12}$.	1
(A) -27 (B) $-\frac{27}{7}$ (C) -9 (D) $-\frac{9}{7}$	
2. Which of the following represents the range of $y = \frac{1}{2} - \frac{1}{2} \cos 2x$?	1
(A) $-\frac{1}{2} \leq y \leq 0$ (B) $-\frac{1}{2} \leq y \leq \frac{1}{2}$ (C) $0 \leq y \leq \frac{1}{2}$ (D) $0 \leq y \leq 1$	
3. What is the primitive of $x^{-2} + 6x$?	1
(A) $-\frac{1}{x} + 3x^2$ (B) $\frac{1}{x} + 3x^2$ (C) $-\frac{2}{x^3} + 6$ (D) $\frac{2}{x^3} + 6$	
4. How many solutions are there to $\cos 2x = \frac{\sqrt{3}}{2}$ within the interval $0 \leq x \leq 2\pi$?	1
(A) 1 (B) 2 (C) 3 (D) 4	
5. For the curve $y = f(x)$, it is known that $f''(x) = (x + 4)^2(x - 2)$.	1
Which of the following statements is true?	
(A) $x = -4$ and $x = 2$ are both the x coordinates of a point of inflexion.	
(B) $x = -4$ is the only x coordinate of a point of inflexion.	
(C) $x = 2$ is the only x coordinate of a point of inflexion.	
(D) Neither $x = -4$ or $x = 2$ are the x coordinates of a point of inflexion.	

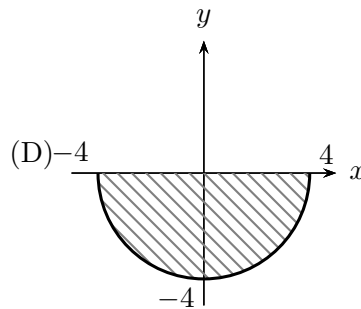
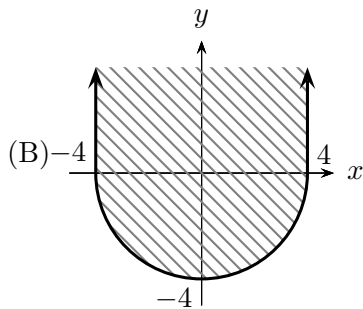
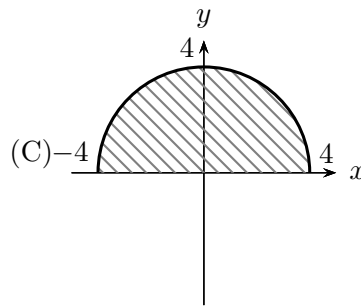
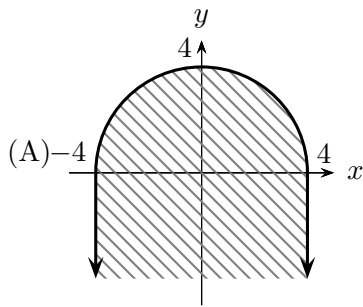
6. The graph shows consecutive terms of a sequence. Which of the following statements best describes this sequence? 1



- (A) geometric, $|r| < 1$ (C) arithmetic, $|d| < 1$
 (B) geometric, $|r| \geq 1$ (D) arithmetic, $|d| \geq 1$

7. Which of the following represents the inequality 1

$$y \leq \sqrt{4 - x^2}$$



8. Which of the following is numerically equivalent to $\log_3 15$? 1

(A) $\frac{\log_{15} 3}{\log_{15} 5}$ (B) $\frac{\log_5 3}{\log_5 15}$ (C) $\frac{\log_e 3}{\log_e 15}$ (D) $\frac{\log_5 15}{\log_5 3}$

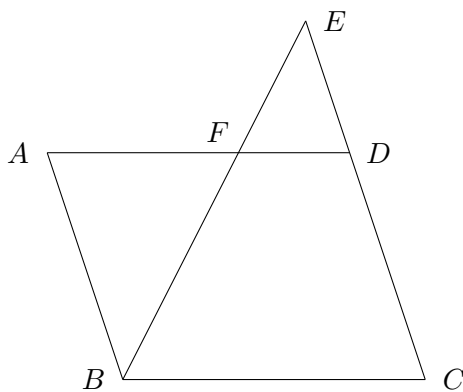
9. What is the value of q if 1

$$x^2 + p(x + 5) + q \equiv (x - 2)(x + 5)$$

(A) -25 (B) -10 (C) 3 (D) 5

10. In the following diagram, $ABCD$ is a parallelogram. F is a point lying on AD . BF produced and CD produced meet at E . 1

If $CD : DE = 2 : 1$, then what is the ratio of $AF : BC$?



(NOT TO SCALE)

- (A) $1 : 2$ (B) $2 : 3$ (C) $3 : 4$ (D) $8 : 9$

Examination continues overleaf...

Section II

90 marks

Attempt Questions 11 to 16

Allow approximately 2 hours and 45 minutes for this section.

Write your answers in the writing booklets supplied. Additional writing booklets are available. Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 Marks)	Commence a NEW booklet.	Marks
(a) Write $\frac{(\pi + e^4)^{\frac{3}{2}}}{\frac{4}{7}}$ correct to 2 significant figures.		2
(b) Write the <i>exact</i> value of $\tan 330^\circ$.		1
(c) Find the value of a and b if $\frac{1}{5 - \sqrt{12}} \equiv a + b\sqrt{3}$.		2
(d) Fully factorise: $x^4 - 81$.		2
(e) Solve $ x - 5 < 3$, and sketch the solution on a number line.		3
(f) Evaluate $\int (4x + 3)^6 dx$, expressing your answer in simplest form.		2
(g) Find the equation of the normal to the curve $y = 3e^{2x}$ at the point $x = 1$.		3

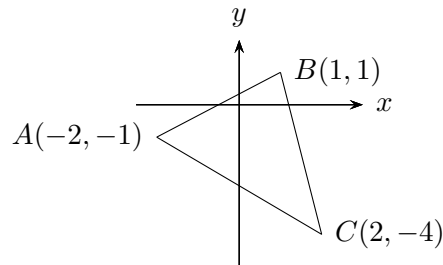
Question 12 (15 Marks)

Commence a NEW booklet.

Marks

(a) Evaluate $\sum_{k=2}^5 (-1)^k \left(\frac{1}{k}\right)$ **2**

(b) The diagram shows the points $A(-2, -1)$, $B(1, 1)$ and $C(2, -4)$.



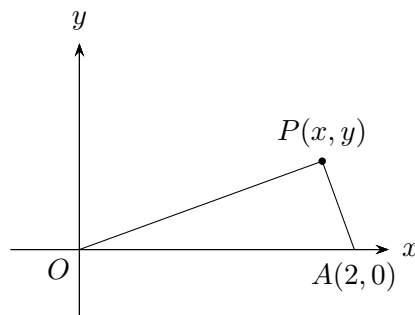
i. Calculate the length of the interval AB . **1**

ii. Find the equation of the line AB . **2**

iii. Show that the perpendicular distance from C to the line AB is $\frac{17}{\sqrt{13}}$. **1**

iv. Hence, calculate the area of $\triangle ABC$. **1**

(c) Refer to the following diagram:



i. Write down the gradient of AP in terms of x and y . **1**

ii. Show that the equation of the locus of all points P , such that $OP \perp AP$ is **2**

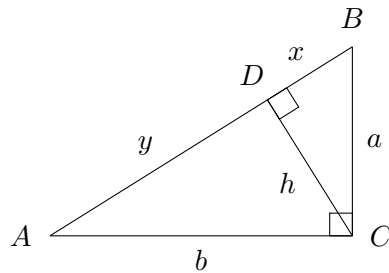
$$x^2 - 2x + y^2 = 0$$

iii. Deduce that the locus of all points P such that $OP \perp AP$ is a circle. Write down the centre and the radius of the circle. **2**

Question 12 continued on the next page...

Question 12 continued from the previous page...

- (d) In the diagram $\triangle ABC$ is a right angled at C . The perpendicular from C to AB meets AB at D .



Also, $BC = a$, $AC = b$, $DC = h$, $DB = x$ and $AD = y$.

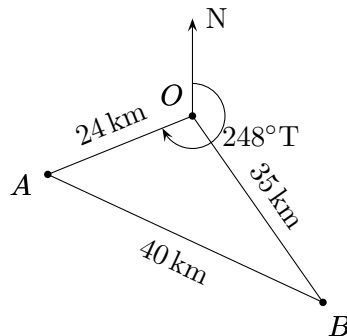
- i. Show that $h(x + y) = ab$. 2
- ii. Show that $h = \frac{ab}{\sqrt{a^2 + b^2}}$. 1

Question 13 (15 Marks)

Commence a NEW booklet.

Marks

- (a) A section of rainforest is to be designated for a species count. The shape is shown below. The bearing of landmark A from landmark O is 248°T and is 24 km in distance. The distance from landmark A to B is 40 km and from landmark B to O is 35 km.



- i. Show that $\angle AOB = 83^\circ$, to the nearest degree. 2
 - ii. Calculate the area of the rainforest, correct to the nearest square kilometre. 2
 - iii. What is the bearing of landmark O from landmark B ? 1
- (b) Solve for x for $0^\circ \leq x \leq 360^\circ$: $2 \cos^2 x + 3 \sin x \cos x + \sin^2 x = 0$ 3
- (c) Express in simplest form:
- i. Find: $\frac{d}{dx}(\operatorname{cosec} x)$. 2
 - ii. Evaluate: $\int_0^{\frac{\pi}{4}} \sin 3x \, dx$. 2
- (d) By differentiating $\sin^3 x + 1$ or otherwise, evaluate: 3

$$\int \frac{\cos x - \cos^3 x}{\sin^3 x + 1} \, dx$$

Question 14 (15 Marks) Commence a NEW booklet. **Marks**

- (a) A parabola has equation $x = -y^2 + 4y - 6$.
- Find the coordinates of its vertex. **2**
 - Find the coordinates of its focus and the equation of its directrix. **2**
 - Sketch the parabola, showing all relevant features. **2**

- (b) Carbon emissions since 2010 have been growing exponentially according to the differential equation

$$\frac{dC}{dt} = kC$$

where C is the number of gigatonnes of carbon (GtC) emitted by burning fossil fuels, cement and land use change. (Source: co2now.org)

During 2010, 9.19 GtC was emitted throughout the entire world. In 2013, this had risen to 9.9 GtC.

- Show that $C = C_0 e^{kt}$ is a solution to this differential equation. **1**
- Find the value of C_0 and k , correct to 2 decimal places. **3**
- Find the number of gigatonnes of carbon emitted in 2030 if no action is taken and assuming that emissions continues to grow at this rate, correct to 2 decimal places. (You may use your 2 decimal place values from previous parts) **1**
- Find the rate of increase of carbon emitted during the year 2030. **1**
- Assuming no reduction in emissions, predict the nearest year when catastrophic climate change occurs if in 2010, there remains a total of 1 000 GtC of carbon that can be emitted into the atmosphere. **3**

Question 15 (15 Marks)

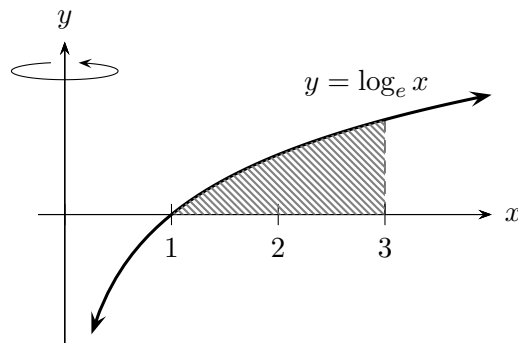
Commence a NEW booklet.

Marks

- (a) If α and β are the roots to the equation $2x^2 + 3x + 7 = 0$, evaluate:
- $\alpha + \beta$. **1**
 - $(\alpha - 2)(\beta - 2)$. **2**
- (b) A function is defined by $f(x) = 2x^3 - 6x + 3$.
- Find the coordinates of the turning points of the graph $y = f(x)$ and determine their nature. **3**
 - Hence sketch the graph of $y = f(x)$, showing the turning points and the y intercept. **2**
- (c) Using Simpson's rule with five function values, find an estimate for the definite integral, correct to 2 decimal places: **3**

$$\int_2^6 2^{x-1} dx$$

- (d) The area bounded by the curve $y = \log_e x$, the x axis and the line $x = 3$ is rotated about the y axis. **4**



Find the exact volume that is generated.

Question 16 (15 Marks)

Commence a NEW booklet.

Marks

- (a) A particular bus route saw patronage of 50 passengers on its first day of operation. As the route's popularity grew with passengers, patronage grew steadily by 14 passengers every subsequent day.
- How many passengers did the bus route carry on its 28th day of operation? **1**
 - After how many days of operation did the bus route carry over 800 passengers on one single day of operation? **1**
 - On which day did the 10 000th passenger use the bus route? **2**

- (b) Consider the geometric series

$$\sin^2 \theta + \sin^2 \theta \cos^2 \theta + \sin^2 \theta \cos^4 \theta + \dots$$

where $0 < \theta < \frac{\pi}{2}$.

- i. Show that the partial sum S_n of the first n terms is given by **2**

$$S_n = 1 - \cos^{2n} \theta$$

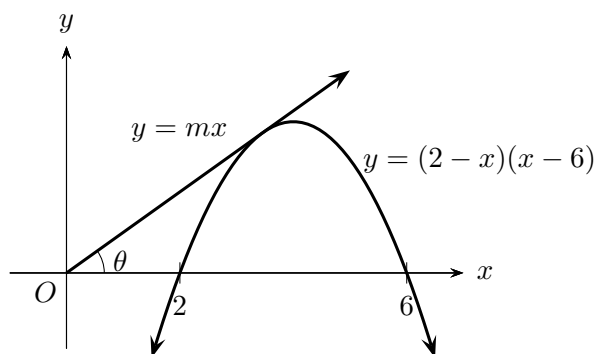
- ii. Explain why this geometric series always has a limiting sum. **1**
- iii. Let S be the limiting sum of this geometric series. Show that **2**

$$S - S_n = \cos^{2n} \theta$$

- iv. If $\theta = \frac{\pi}{3}$, find the least value of n for which **2**

$$S - S_n < 10^{-6}$$

- (c) The line $y = mx$ represents the flight path of a plane which has just taken off from the airport at O . The parabola $y = (2 - x)(x - 6)$ represents a hill that a plane must fly over.



- i. Explain carefully, why the solution(s) to **1**

$$x^2 + (m - 8)x + 12 = 0$$

represents the x coordinate where the plane will just clear the hill.

- ii. Find the angle of inclination θ which the plane must ascend in order to just clear the hill. **3**

End of paper.

Suggested Solutions

Section I

1. (B) 2. (D) 3. (A) 4. (D) 5. (C)
6. (A) 7. (A) 8. (D) 9. (A) 10. (B)

Section II

Question 11 (Lam)

- (a) (2 marks)

$$\frac{(\pi + e^4)^{\frac{3}{2}}}{\frac{4}{7}} = 767.80\dots = 770 \text{ (2 s.f.)}$$

- (b) (1 mark)

$$\tan 330^\circ = -\frac{1}{\sqrt{3}}$$

- (c) (2 marks)

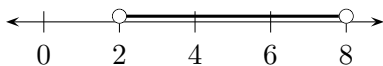
$$\begin{aligned} \frac{1}{5 - \sqrt{12}} \times \frac{5 + \sqrt{12}}{5 + \sqrt{12}} &= \frac{5 + \sqrt{12}}{25 - 12} \\ &= \frac{5}{13} + \frac{1}{13}\sqrt{12} \\ &= \frac{5}{13} + \frac{1}{13} \times 2\sqrt{3} \\ &= \frac{5}{13} + \frac{2}{13}\sqrt{3} \\ \therefore a &= \frac{5}{13} \quad b = \frac{2}{13} \end{aligned}$$

- (d) (2 marks)

$$\begin{aligned} x^4 - 81 &= (x^2 - 9)(x^2 + 9) \\ &= (x - 9)(x + 9)(x^2 + 9) \end{aligned}$$

- (e) (3 marks)

$$\begin{aligned} |x - 5| &< 3 \\ -3 &< x - 5 < 3 \\ +5 & \quad +5 \quad +5 \\ 2 &< x < 8 \end{aligned}$$



- (f) (2 marks)

$$\begin{aligned} \int (4x + 3)^6 dx &= \frac{(4x + 3)^7}{7 \times 4} + C \\ &= \frac{1}{28}(4x + 3)^7 + C \end{aligned}$$

- (g) (3 marks)

$$y = 3e^{2x}$$

Finding the gradient of the tangent at $x = 1$:

$$\begin{aligned} \frac{dy}{dx} &= 6e^{2x} \Big|_{x=1} = 6e^2 \\ \therefore m_{\perp} &= -\frac{1}{6e^2} \end{aligned}$$

At $x = 1$, $y = 3e^2$. Apply the point-gradient formula,

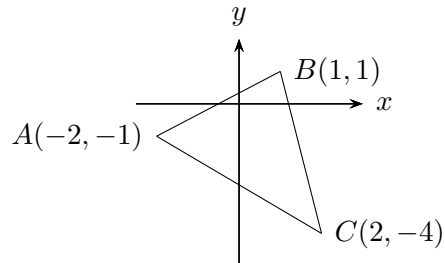
$$\begin{aligned} \frac{y - 3e^2}{x - 1} &= -\frac{1}{6e^2} \\ 6e^2 y - 18e^4 &= -x + 1 \\ \therefore x + 6e^2 y - 18e^4 - 1 &= 0 \end{aligned}$$

Question 12 (Tan)

- (a) (2 marks)

$$\begin{aligned} \sum_{k=2}^5 (-1)^k \binom{1}{k} &= \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} \\ &= \frac{13}{60} \end{aligned}$$

- (b) i. (1 mark)



$$\begin{aligned} AB &= \sqrt{(-2 - 1)^2 + (-1 - 1)^2} \\ &= \sqrt{(-3)^2 + (-2)^2} = \sqrt{13} \end{aligned}$$

- ii. (2 marks)

$$m_{AB} = \frac{-1 - 1}{-2 - 1} = \frac{2}{3}$$

Applying the point gradient formula,

$$\begin{aligned} \frac{y - 1}{x - 1} &= \frac{2}{3} \\ 3y - 3 &= 2x - 2 \\ 2x - 3y + 1 &= 0 \end{aligned}$$

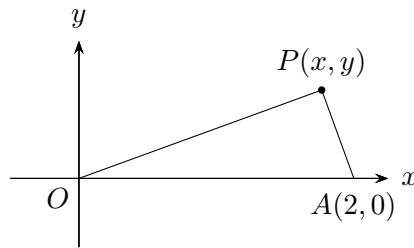
- iii. (1 mark)
 ✓ [1] for *showing*, via perpendicular distance formula, the required distance.

$$\begin{aligned} d_{\perp} &= \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}} \\ &= \frac{|2(2) - 3(-4) + 1|}{\sqrt{2^2 + 3^2}} \\ &= \frac{17}{\sqrt{13}} \end{aligned}$$

- iv. (1 mark)

$$\begin{aligned} A &= \frac{1}{2} \times AB \times d_{\perp} \\ &= \frac{1}{2} \sqrt{13} \times \frac{17}{\sqrt{13}} = \frac{17}{2} \end{aligned}$$

- (c) i. (1 mark)



$$m_{AP} = \frac{y - 0}{x - 2} = \frac{y}{x - 2}$$

- ii. (2 marks)

$$m_{OP} = \frac{y}{x}$$

If $OP \perp AP$, then

$$\begin{aligned} m_{OP} \times m_{AP} &= -1 \\ \frac{y}{x - 2} \times \frac{y}{x} &= -1 \\ \frac{y^2}{x(x - 2)} &= -1 \\ y^2 &= -x^2 + 2x \\ \therefore x^2 - 2x + y^2 &= 0 \end{aligned}$$

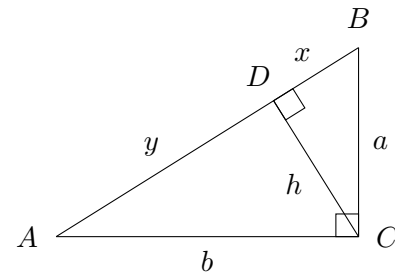
- iii. (2 marks)

Completing the square,

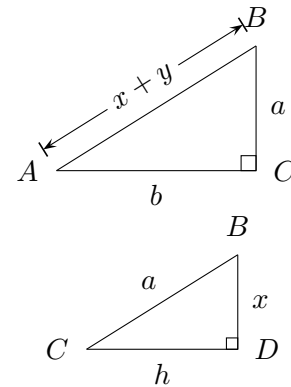
$$\begin{aligned} x^2 - 2x + 1 + y^2 &= 1 \\ (x - 1)^2 + y^2 &= 1 \end{aligned}$$

which represents a circle of centre (1, 0) and radius 1.

- (d) i. (2 marks)



Redrawing with $\triangle BCD$ reoriented:



In $\triangle ABC$ and $\triangle BCD$,

- $\angle BCA = \angle BDC = 90^\circ$ (given)
- $\angle ABC = \angle CBD$ (common)

$\therefore \triangle ABC \parallel \triangle BCD$ (equiangular).

Hence all corresponding sides are in the same ratio, i.e.

$$\begin{aligned} \frac{h}{b} &= \frac{a}{x + y} \\ h(x + y) &= ab \end{aligned}$$

Alternatively, find via area of of $\triangle ABC$:

$$\begin{aligned} A &= \frac{1}{2}h(x + y) \\ A &= \frac{1}{2}ab \\ \therefore \frac{1}{2}h(x + y) &= \frac{1}{2}ab \\ \therefore h(x + y) &= ab \end{aligned}$$

Alternatively, use trigonometric ratios. In $\triangle ABC$,

$$\sin \angle ABC = \frac{b}{(x+y)}$$

$$\sin \angle CBD = \frac{h}{a}$$

$$\sin \angle ABC = \sin \angle CBD$$

$$\therefore \frac{b}{x+y} + \frac{h}{a}$$

$$ab = h(x+y)$$

ii. (1 mark)

✓ [1] for showing Pythagoras' Theorem usage in $\triangle ABC$ and thus result required.

In $\triangle ABC$, $a^2 + b^2 = (x+y)^2$, i.e.

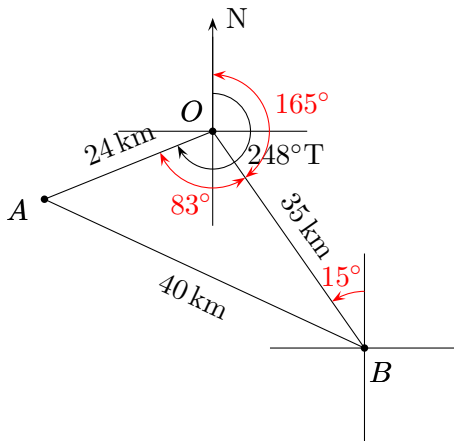
$$x+y = \sqrt{a^2 + b^2}$$

Hence,

$$h = \frac{ab}{x+y} = \frac{ab}{\sqrt{a^2 + b^2}}$$

Question 13 (Wall)

(a) i. (2 marks)



In $\triangle AOB$, apply the cosine rule:

$$\cos \angle AOB = \frac{24^2 + 35^2 - 40^2}{2(24)(35)} = \frac{67}{560}$$

$$\therefore \angle AOB = 83^\circ \text{ (1 d.p.)}$$

ii. (2 marks)

$$\begin{aligned} A &= \frac{1}{2}ab \sin C \\ &\approx \frac{1}{2} \times 24 \times 35 \times \sin 83^\circ \\ &\approx 417 \text{ km}^2 \end{aligned}$$

iii. (1 mark)

$$345^\circ$$

(See red text in diagram)

(b) (3 marks)

$$2 \cos^2 x + 3 \sin x \cos x + \sin^2 x = 0$$

$$(2 \cos x + \sin x)(\cos x + \sin x) = 0$$

$$2 \cos x + \sin x = 0$$

$$\sin x = -2 \cos x$$

$$\tan x = -2$$

$$x = 116^\circ 34', 296^\circ 34' \text{ when } \tan x = -2$$

$$= 135^\circ, 315^\circ \text{ when } \tan x = -1$$

$$\cos x + \sin x = 0$$

$$\sin x = -\cos x$$

$$\tan x = -1$$

(c) i. (2 marks)

$$\frac{d}{dx} (\operatorname{cosec} x) = \frac{d}{dx} ((\sin x)^{-1})$$

$$= -(\sin x)^{-2} \cos x$$

$$\left(= -\frac{\cos x}{\sin^2 x} \right)$$

$$\left(= \cot x \operatorname{cosec} x \right)$$

ii. (2 marks)

$$\begin{aligned} \int_0^{\frac{\pi}{4}} \sin 3x \, dx &= -\frac{1}{3} [\cos 3x]_0^{\frac{\pi}{4}} \\ &= -\frac{1}{3} \left(\cos \frac{3\pi}{4} - \cos 0 \right) \\ &= -\frac{1}{3} \left(-\frac{1}{\sqrt{2}} - 1 \right) \\ &= \frac{1}{3} \left(\frac{1}{\sqrt{2}} + 1 \right) \end{aligned}$$

(d) (3 marks)

$$\frac{d}{dx} (\sin^3 x + 1) = \frac{d}{dx} ((\sin x)^3 + 1)$$

$$= 3 \sin^2 x \cos x$$

$$= 3(1 - \cos^2 x) \cos x$$

$$= 3(\cos x - \cos^3 x)$$

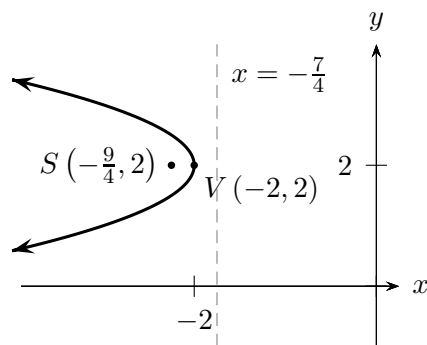
$$\therefore \int \frac{\cos x - \cos^3 x}{\sin^3 x + 1} \, dx = \frac{1}{3} \int \frac{3(\cos x - \cos^3 x)}{\sin^3 x + 1} \, dx$$

$$= \frac{1}{3} \ln(\sin^3 x + 1) + C$$

Question 14 (Bhamra)

(a) i. (2 marks)

$$\begin{aligned}x &= -y^2 + 4y - 6 \\x + 6 &= -(y^2 - 4y + 4 - 4) \\x + 6 &= -(y - 2)^2 + 4 \\ \therefore x + 2 &= -4 \left(\frac{1}{4}\right) (y - 2)^2\end{aligned}$$

Hence vertex is at $(-2, 2)$.

ii. (2 marks)

$$\begin{aligned}S &\left(-\frac{9}{4}, 2\right) \\ \text{Directrix } x &= -\frac{7}{4}\end{aligned}$$

iii. (2 marks)

- ✓ [1] for shape & direction.
 - ✓ [1] for directrix, S and V .
- See above.

(b) i. (1 mark)

$$\begin{aligned}\frac{dC}{dt} &= kC \\ \frac{dC}{C} &= k dt\end{aligned}$$

Integrating,

$$\begin{aligned}\int \frac{dC}{C} &= \int k dt \\ \log_e C &= kt + C_1 \\ C &= e^{kt+C_1} = e^{kt} e^{C_1} \\ &= C_0 e^{kt}\end{aligned}$$

where $e^{C_1} = C_0$ (constant).

Alternatively differentiate $C = C_0 e^{kt}$ to show required result.

ii. (3 marks)

- ✓ [1] for value of C_0 .
- ✓ [2] for working and value of k .

Year 2010 corresponds to $t = 0$.

$$\begin{aligned}C &= 9.19 = C_0 e^0 \\ \therefore C_0 &= 9.19\end{aligned}$$

When $t = 3$, $C = 9.9$:

$$\begin{aligned}9.9 &= 9.19 e^{3k} \\ e^{3k} &= \frac{9.9}{9.19} \\ 3k &= \log_e \frac{9.9}{9.19} \\ \therefore k &= \frac{1}{3} \log_e \frac{9.9}{9.19} \\ &\approx 0.0248 \dots = 0.025 \text{ (3 d.p.)}\end{aligned}$$

iii. (1 mark) When $t = 20$,

$$\begin{aligned}C &= 9.19 e^{0.025 \times 20} \\ &= 15.15 \dots\end{aligned}$$

iv. (1 mark)

$$\begin{aligned}\frac{dC}{dt} &= kC \\ &= 0.025 \times 15.15 \dots \\ &= 0.3787 \text{ GtC/year}\end{aligned}$$

v. (3 marks)

- ✓ [1] only for attempts to make a substitution and find the amount emitted during that year.
- ✓ [2] only for attempting to use some form of arithmetic/geometric series (sum)
- ✓ [3] (full marks) for integrating and attempts to find t_1 .

Total emitted (1 000 GtC) is the integral from $t = 0$ to $t = t_1$, where

$k \approx 0.025$:

$$\begin{aligned} 1\,000 &= \int_0^{t_1} 9.19e^{kt} dt \\ &= \frac{9.19}{k} [e^{kt}]_0^{t_1} \\ &= \frac{9.19}{k} (e^{kt_1} - e^0) \\ \frac{1\,000k}{9.19} &= e^{kt_1} - 1 \\ e^{kt_1} &= \frac{1\,000k}{9.19} + 1 \\ kt_1 &= \log_e \left(\frac{1\,000k}{9.19} + 1 \right) \\ t_1 &= \frac{1}{k} \log_e \left(\frac{1\,000k}{9.19} + 1 \right) \\ &= 52.734 \dots \end{aligned}$$

Catastrophic climate change would occur approx 53 years after 2010, i.e. in 2063.

Question 15 (Tan)

(a) i. (1 mark)

$$\begin{aligned} 2x^2 + 3x + 7 &= 0 \\ \alpha + \beta &= -\frac{b}{a} = -\frac{3}{2} \end{aligned}$$

ii. (2 marks)

$$\begin{aligned} \alpha\beta &= \frac{c}{a} = \frac{7}{2} \\ \therefore (\alpha - 2)(\beta - 2) &= \alpha\beta - 2(\alpha + \beta) + 4 \\ &= \frac{7}{2} - 2\left(-\frac{3}{2}\right) + 4 \\ &= \frac{21}{2} \end{aligned}$$

(b) i. (3 marks)

$$\begin{aligned} f(x) &= 2x^3 - 6x + 3 \\ f'(x) &= 6x^2 - 6 \end{aligned}$$

Stationary pts occur when $f'(x) = 0$:

$$\begin{aligned} 6(x^2 - 1) &= 0 \\ (x - 1)(x + 1) &= 0 \\ \therefore x &= \pm 1 \\ f''(x) &= 12x \end{aligned}$$

• When $x = -1$:

$$\begin{aligned} f(-1) &= -2 + 6 + 3 = 7 \\ f''(-1) &= -12 < 0 \end{aligned}$$

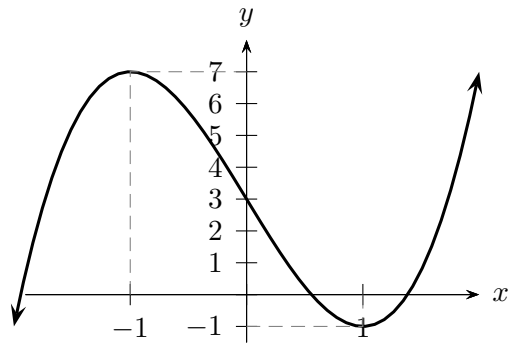
Hence $(-1, 7)$ is a local maximum.

• When $x = 1$:

$$\begin{aligned} f(1) &= 2 - 6 + 3 = -1 \\ f''(1) &= 12 > 0 \end{aligned}$$

Hence $(1, -1)$ is a local minimum.

ii. (2 marks)



(c) (3 marks)

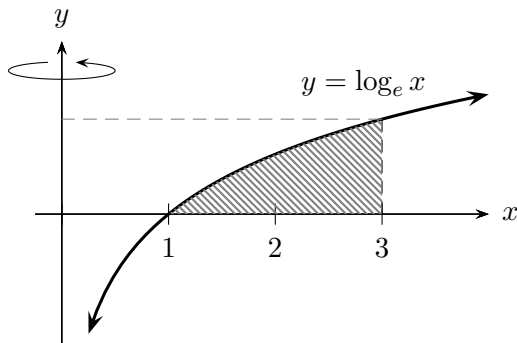
- ✓ [1] for table
- ✓ [1] for substitution into Simpson's Rule.
- ✓ [1] for final answer.

x	2	3	4	5	6
2^{x-1}	2	4	8	16	32

$$\begin{aligned} \int_2^6 2^{x-1} dx &\approx \frac{1}{3} (2 + 32 + 4(4 + 16) + 2(8)) \\ &= \frac{130}{3} \end{aligned}$$

(d) (4 marks)

- ✓ [1] for volume of cylinder
- ✓ [1] for changing subject from y to x
- ✓ [1] for finding volume generated by rotation about y axis.
- ✓ [1] for subtracting volumes.



Volume generated =

Volume of cylinder – Volume around y axis

- Volume of cylinder with radius 3, height $h = \ln 3$:

$$V = \pi r^2 h = 9\pi \ln 3$$

- Volume generated about y axis:

$$\begin{aligned} y &= \log_e x \\ x &= e^y \\ V &= \pi \int_0^{\ln 3} x^2 dy \\ &= \pi \int_0^{\ln 3} e^{2y} dy \\ &= \frac{\pi}{2} [e^{2y}]_0^{\ln 3} \\ &= \frac{\pi}{2} (e^{2 \ln 3} - e^0) \\ &= \frac{\pi}{2} (9 - 1) = 4\pi \end{aligned}$$

- Volume generated when shaded area is rotated:

$$V = 9\pi \ln 3 - 4\pi$$

Question 16 (Lam)

- (a) i. (1 mark)

$$50, 50 + 14, 50 + 14(2) \dots$$

$$a = 50 \quad d = 14$$

$$\begin{aligned} T_n &= a + d(n - 1) \\ &= 50 + 14(n - 1) \\ &= 50 + 14n - 14 \\ &= 36 + 14n \end{aligned}$$

$$\begin{aligned} \therefore T_{28} &= 36 + 14(28) \\ &= 428 \end{aligned}$$

- ii. (1 mark)

$$\begin{aligned} T_n &> 800 \\ \therefore 36 + 14n &> 800 \\ 14n &> 764 \\ n &> 54.57 \dots \end{aligned}$$

Carries over 800 passengers on one day on the 55th day.

- iii. (2 marks)

$$\begin{aligned} S_n &= 10\,000 \\ 10\,000 &= \frac{n}{2} (2a + d(n - 1)) \\ &= \frac{n}{2} (100 + 14n - 14) \\ &= \frac{n}{2} (86 + 14n) \\ \therefore 7n^2 + 43n &= 10\,000 \\ 7n^2 + 43n - 10\,000 &= 0 \\ n &= \frac{-43 \pm \sqrt{43^2 - 4(7)(-10\,000)}}{2 \times 7} \\ &= \frac{-43 \pm \sqrt{281\,849}}{14} \end{aligned}$$

As $n > 0$,

$$n = \frac{-43 + \sqrt{281\,849}}{14} \approx 34.85$$

The route carried its 10 000th passenger on the 35th day.

- (b) i. (2 marks)

$$\sin^2 \theta + \sin^2 \theta \cos^2 \theta + \sin^2 \theta \cos^4 \theta + \dots$$

$$a = \sin^2 \theta \quad r = \cos^2 \theta$$

$$\begin{aligned} S_n &= \frac{a(1 - r^n)}{1 - r} \\ &= \frac{\sin^2 \theta (1 - (\cos^2 \theta)^n)}{1 - \cos^2 \theta} \\ &= 1 - \cos^{2n} \theta \end{aligned}$$

- ii. (1 mark)

- As the common ratio is $\cos^2 \theta$, and $-1 \leq \cos \theta \leq 1$, then

$$0 \leq \cos^2 \theta \leq 1$$

Since the magnitude of the common ratio is less than 1, a limiting sum exists.

iii. (2 marks)

$$\begin{aligned} S &= \frac{a}{1-r} \\ &= \frac{\sin^2 \theta}{1 - \cos^2 \theta} \\ &= 1 \end{aligned}$$

$$\therefore S - S_n = 1 - (1 - \cos^{2n} \theta) = \cos^{2n} \theta$$

iv. (2 marks) When $\theta = \frac{\pi}{3}$

$$(\cos \theta)^{2n} = \left(\cos \frac{\pi}{3}\right)^{2n} = \left(\frac{1}{2}\right)^{2n}$$

$$\therefore \left(\frac{1}{2}\right)^{2n} < 10^{-6}$$

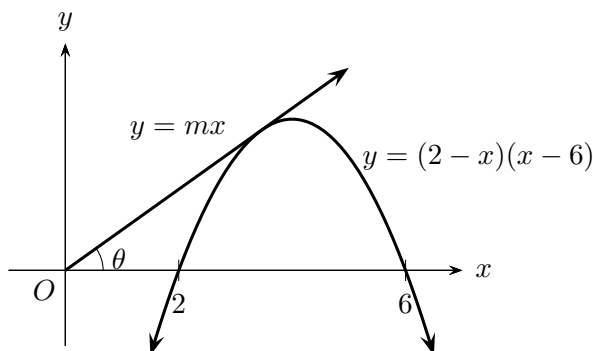
$$2n \log \frac{1}{2} < \log 10^{-6}$$

$$2n > \frac{\log 10^{-6}}{\log \frac{1}{2}}$$

$$n > \frac{\log 10^{-6}}{2 \log \frac{1}{2}} \approx 9.96$$

$\therefore n = 10$ for this to first occur.

(c) i. (1 mark)



$$\begin{cases} y = mx \\ y = -x^2 + 8x - 12 \end{cases}$$

Solving simultaneously to find the point of intersection,

$$\begin{aligned} mx &= -x^2 + 8x - 12 \\ x^2 + x(m-8) + 12 &= 0 \end{aligned}$$

ii. (3 marks)

✓ [1] for identifying $\Delta = 0$ for the plane to 'just clear' the mountain.

✓ [1] for both values of $\tan \theta$

✓ [1] for final answer.

$\Delta = 0$ in the quadratic in x , involving m :

$$\Delta = (m-8)^2 - 4(1)(12) = 0$$

$$(m-8)^2 = 48$$

$$m-8 = \pm 4\sqrt{3}$$

$$\therefore m = 8 \pm 4\sqrt{3}$$

As $m = \tan \theta$,

$$\therefore \tan \theta = 8 \pm 4\sqrt{3}$$

$$\tan \theta = 1.071 \text{ or } 14.928$$

$$\theta = 46.98^\circ \text{ or } 86.16^\circ$$

Checking the first derivative at $x = 8 + 4\sqrt{3}$:

$$y = -x^2 + 8x - 12$$

$$\frac{dy}{dx} = -2x + 8$$

If $m = 8 + 4\sqrt{3}$,

$$8 + 4\sqrt{3} = -2x + 8$$

$$\therefore -2x = 4\sqrt{3}$$

$x = -2\sqrt{3}$ (which is not possible as $x > 0$)

If $m = 8 - 4\sqrt{3}$,

$$8 - 4\sqrt{3} = -2x + 8$$

$$\therefore 2x = 4\sqrt{3}$$

$$x = 2\sqrt{3}$$

Hence angle of inclination θ is 46.98° .