Introduction. An important concept in mathematical statistics is the concept of "probability". How likely are we to get the results that we want, and how many tries should we take to definitely get those results? Unfortunately in the HSC, we won't be covering those later applications, however an introduction to the field is covered. Later, if you go on to a degree in commerce, science or engineering, you may find yourself happy we took these first few steps.

Defining a Probability Space. Before we can go into extensive applications of probability, or how to compute a probability, it is important we understand how we define a probability.

Consider the simple case of the coin - when you flip, either two things will happen. Either you get heads, or tails. So, in total, two things can happen, and you can get either one equally. Common sense tells us that we have a 50% chance of getting a heads or tails. This leads to the definition of a probability.

Since each event is equally likely, the probability of a single event is defined to be $\frac{1}{n}$, where n is the number of outcomes. In this case, there are two equally likely outcomes, so the probability of any one occuring is $\frac{1}{2}$. Similarly, with a die, there are 6 equally likely events, so the probability of any one occuring is $\frac{1}{6}$.

Mutually Exclusive Outcomes. Of course, after considering the die, we naturally ask ourselves - what's the probability of not just getting one number, but multiple? For example, what if we wanted an even number?

Well, there are 3 even numbers, and 6 total numbers, so the probability of getting an even number is $\frac{3}{6} = \frac{1}{2}$. However, we could also see this as getting a 2, 4 or 6. This way, the probability of an even number is the sum of these events. We could write $\mathbb{P}(\text{even number}) = \mathbb{P}(2) + \mathbb{P}(4) + \mathbb{P}(6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$.

"But the first method was so much easier and quicker!" you cry. "Why should I worry about something harder and with more work?!" you ask? The answer is simple - this allows us to define addition in terms of probabilities.

Before we considered simple cases, however what about something a little more difficult? Let's say we have a die that's weighted such that the number on each side is that much more likely to be attained. For example, a 6 is 6x more likely than a 1, and 6/2=3x more likely than a 2. This defines a relative probability, where $p(2) = 2\mathbb{P}(1)$ and so on, but what about an absolute probability?

From an earlier definition, we know that all probabilities are out of 1. Using addition as we defined it earlier, if we add up all **mutually exclusive outcomes**, they will come to 1. However, what do we define as **mutually exclusive**? Well, two events are mutually exclusive if they cannot occur together. A simple scenario is getting heads AND tails from a single coin toss. Or, getting an even AND an odd number on a die. An example of two events that aren't **mutaully exclusive** are rolling that die and getting a 1 and an odd number.

So, back to those absolute probabilities. If we add up a set of mutually exclusive events, they must come to 1. So, let's consider the obvious set of mutually exclusive events - getting a 1, 2, 3, 4, 5 or 6. Obviously if I get a 2, I can't also get a 3. So, this gives us the following mathematical solving:

$$\begin{aligned}
\mathbb{P}(\Omega) &= \mathbb{P}(1) + \mathbb{P}(2) + \mathbb{P}(3) + \mathbb{P}(4) + \mathbb{P}(5) + \mathbb{P}(6) \\
&= \mathbb{P}(1) + 2\mathbb{P}(1) + 3\mathbb{P}(1) + 4\mathbb{P}(1) + 5\mathbb{P}(1) + 6\mathbb{P}(1) \\
&= 21\mathbb{P}(1) = 1 \\
\therefore \mathbb{P}(1) &= \frac{1}{21}
\end{aligned}$$

Summary Notes. Finally, it's important to summarise some important concepts defined through these notes. You will probably (all puns intended) find these formula useful to contain on any summary sheet:

- For any event A, $0 \leq \mathbb{P}(A) \leq 1$
- For any event E in some sample space Ω, $\mathbb{P}(E) = \frac{\text{number of times} E \text{occurs in} \Omega}{\text{number of events in } \Omega}$
- For any set of mutually exclusive events $E_1, E_2, ..., E_n$, $\sum_{i=1}^{n} \mathbb{P}(E_i) = 1$ and $\mathbb{P}(E_1 \cup ... \cup E_m) = \mathbb{P}(E_1) + ... + \mathbb{P}(E_m)$